

NOTES AND REFERENCES

Acronymic references to texts are keyed to bibliographical entries.

Frontispiece: [The God of Zero] after a photograph by Justin Kerr of the head of the God of the Number Zero (Copan, Honduras). Maya: Late Classic period (A. D. 770-780), stone, in the Cleveland Museum of Art. Reproduced in Sch Plate 110.

Epigraph: Deputy Prime Minister John Prescott on the radio news program "The World at One", BBC Radio 4, July 23, 1998, speaking about the desire (expressed by Michael Meecham) to have zero nuclear contamination in the sea. Prescott continued: "That's something we'll be interpreting in the future." It is pleasing to think that the acuity of our ironic tropes has kept pace with the ever-growing finesse of our number sense, so that a mere 'zero' hardly suffices any longer (among those who give 110%) to stand for the nadir, but we must revert to expressions such as one I heard recently in Berlin: "Null comma nix" - which Ellen Kaplan aptly translates as "Zero point zilch". Remember too that Molesworth knew "0 but less than 0" about the third declension - or anything.

ZERO: THE LENS

2: [An O without a figure] King Lear I.iv. 212.

2: [Some claim that $0 \times 0 = 0$ proves numbers are real] A. W.

Moore, in his review in the Times Literary Supplement (September 11, 1998, pg. 34) of Jerrold Katz' Realistic Rationalism (MIT Press, 1998), writes: "Given that $0 \times 0 = 0$ and that $1 \times 1 = 1$, it follows that there are numbers that are their own squares. But then it follows in turn that there are numbers. In a single step of artless simplicity, we seem to have advanced from a piece of elementary arithmetic to a startling and highly controversial philosophical conclusion: that numbers exist... many philosophers of mathematics, Jerrold J. Katz among them, base their belief in the existence of numbers and other abstract objects on just such arguments."

ONE: MIND PUTS ITS STAMP ON MATTER

5: [X perhaps from a hand-signal] Ellen Kaplan's conjecture. cf. M 240-2.

5: [V as upper half of X] M 242.

5: [3/4 frame around symbol to increase value by 100,000] M 28, 44.

5: [Tiberius, Livia, Galba] The story of the will is in Suetonius, Lives of the Twelve Caesars (Sue VII 5.2):

Sestertium namque quingenties praecipuum inter legatarios

habuit, sed quia notata non perscripta erat summa, herede
Tiberio legatum ad quingenta revocante, ne haec quidem
accepit.

Mooney translates (Sue 59):

[Livia Augusta] left him a legacy of fifty million sesterces in
priority to the other legacies; but since this sum was only
marked in figures and not written fully in words, Tiberius,
who was her heir, reduced it to five hundred thousand
sesterces, and even this amount Galba did not receive.

There is considerable confusion at M 28, where $5 \times 10^2 \times 10^5$
 $= 5 \times 10^7$ is written as 5×10^8 by Menninger (or his English
translator); and where Menninger claims that since the three-
quarter frame increased the value of the figure it enclosed by a
factor of 10^5 , Tiberius paid Galba only 500 (= D) sesterces.
Instead it appears that Tiberius agreed to pay him the lordlier
 5×10^5 sesterces, because he had D read as D, where the bar
increases the value of the symbol it stands over by a factor of
 10^3 . So D = 500, D = 500,000 and D = 50,000,000. The three-
quarter frame thus indeed increases the value of the symbol it
encloses by a factor of $100,000 = 10^5$. The justification for these
readings lies in Mooney's notes (Sue 204-5):

"Quingenties: sc. 50,000,000 sesterces. Casaubon regarded this
sum as incredibly large, and would read quinquagies, i.e.,

5,000,000.

"Notata: i.e., merely marked in figures or symbols and not written fully in words; the difference between HS D = 500,000 sesterces and HS D = 50,000,000 sesterces is very slight (sic!).

"Quingenta: sc. sestertia; 500,000 sesterces."

This will not be the only occasion we shall have for seeing how numbers, which we think of as timeless, wobble and grow indistinct as they recede in time.

5: [Tiberius not liking Galba] EB article 'Tiberius': "...it is doubtful he ever liked, or was liked by, any man." See Suetonius (op. cit., pg. 285): "Tiberius too, when he heard that Galba was destined to be emperor, but in his old age, said: 'Well, let him live then, since that does not concern me.' "

6: [No attempt will automatically produce the answer] By some imaginative manipulations, such as those David Nelson (Ne) suggests, you could in fact manage the following:

XL III

XX IV

XLXX IVIII

Now cancel an X before with the X after the L in the left-hand

group, leaving you with LX; and the I before with a I after the V in the right-hand group, leaving VII, or LXVII, the correct answer 67. But this is far from automatic, requiring first a thoughtful separation of symbols into distinct groups, and second a notion of cancelling balanced pairs of symbols around a symbol of higher order. Nor did the Romans consistently write their numbers in this way that Menninger calls "back-counting", with XL, for example, indicating ten *before* 50: this usage, he says (M 76) was late.

6: [Story began 5000 years ago with Sumerians] K 4-5 (4000 years ago is safer); D 10-11.

6: [Story of father and son] Kr 1 12.

7: [Sumerian sexagesimals for trade with another culture] K 6, M 164-5 (Kline says "Now suppose there is another system with a different unit but the same ratios, and political or social forces compel the fusing of the two systems." The inference to a neighboring people is Kewitsch's, cited in I 51; see there for Thureau-Dangin's objection; but see vdW 41 for his reply to this objection.

7: [Government records kept of equivalent values, even in barter] F 72, W 117.

7: [Sumerians pressed circular and semi-circular shapes into clay

with tip of reeds] M 163.

7: [Circles yield to wedges and hooks; and what the Sumerian symbols were] K 5-6; M 164-7; Mc 30; D 11; vdW 38; N1 15.
According to Nissen (Nis 119), the round stylus was used for numerical signs even after wedges were used for words (c. 3100 B. C.)

7: [Punchcards] The cards may largely be gone but their name has only been partially forgotten. A letter from Jerome S. Shipman of Potomac, Md. tells me that the correct term is "punched cards", and refers me to pg. 12 of the 75th Anniversary edition of IBM's house journal Think, where the products of its predecessor company C-T-P are said to have "ranged from commercial scales... to tabulators and punched cards." A caption on pg. 28 of the same issue describes, he says, "an advanced machine for high-speed production of punched cards." These cards may have flicked through their processors with what at the time was lightning speed, but such a name must not have, its edges becoming frayed . On IBM's own website you will find reference to a keypunch operator of their 024 80-column Card Punch recording data "onto 80-column punch cards."

7: [Sumerians yield to Akkadians around 2500 B. C.] K 4.

8: [Old Babylonian times now] K 4; D 11.

8: [Late form of 9] vdW 38.

8: [30 made with three hooks] M 167.

9: [Ambiguous and changing size of wedges] K 6-7; M 166-7.

9: [Punchcards] The cards may largely be gone but their name has only been partially forgotten. A letter from Jerome S. Shipman of Potomac, Md. tells me that the correct term is "punched cards", and refers me to pg. 12 of the 75th Anniversary edition of IBM's house journal Think, where the products of its predecessor company C-T-P are said to have "ranged from commercial scales... to tabulators and punched cards." A caption on pg. 28 of the same issue describes, he says, "an advanced machine for high-speed production of punched cards." These cards may have flicked through their processors with what at the time was lightning speed, but such a name must not have, its edges becoming frayed. On IBM's own website you will find reference to a keypunch operator of their 024 80-column Card Punch recording data "onto 80-column punch cards."

11: [E-Mach, Ninmah] C 221.

12: [Double wedge for 0] M 403; Mc 30; K 6; vdW 39; A 9; H i 29.

12: [Double wedge appears betw. 6th and 3rd c. B. C.] M 167 (6th c.); M 403 (2nd c.); N2 1113; K 6; N1 27 ("We feel that it did not exist, say, before 1500: and we find it in full use from the third century B. C. on.")

12: [Various uses of separator sign] I 376.

12: [Variety of zero wedge signs] vdW 39; A 9; N3 15; K 6; I 376, 379, 381.

12: [Bêl-bân-aplu] N3 73; N1 27; N4 34 (n. 45); text (according to N3) may be from as early as the time of Darius; in N4 Neugebauer writes: "A more precise dating to the end of the eighth century B. C. would be possible if, as seems likely, the person mentioned in the colophon turns out to be the same as the owner of the tablet published by Langdon VT plate IV."

12: [Zero made with three hooks, like a 30] N3 73: "Das als 'Null' fungierende Interpunktionszeichen ist in diesem Text von 30 nicht zu unterscheiden - es sieht fast aus, als hätte es der Abscheiber nicht verstanden und wirklich für 30 gelesen."

12: [Another scribe making his zero with one hook] N4 34-5.

12: [Double wedge only in medial column] K 6; M 403; A 9-10; vdW 56. Now Ifrah, writing in 1981, says (I 379-380): "No Babylonian mathematical document published so far has shown the zero sign at the beginning or end of a written number... In the past, some historians of science concluded from this that the Babylonians used the zero only in a medial position and that their zero was therefore not functionally identical with ours. But as we now know from the work of Otto Neugebauer, Babylonian astronomers

differed from Babylonian mathematicians in this respect: they used the zero at the beginning and end of written numbers, as well as in a medial position." In Neugebauer, however, I find (N1 20): "But even in the final phase of Babylonian writing we do not find any example of zero signs at the end of numbers. Though there are many instances of cases like •, 20 [Neugebauer represents a cuneiform zero-sign by a dot, • and separates places by commas], there is no safe example of a writing like 20, • known to me. In other words, in all periods the context alone decides the absolute value of a sexagesimally written number."

12: [Advantages of flexible notation] N1 27 (ad 13); N4 2: "It is perhaps not out of place to emphasize that the ambiguity in the Babylonian writing of numbers [caused by lack of a marker for 0 in the last place] is by no means a disadvantage. Just as we multiply 0.0325 by 73.20, or 3.25 by 0.732, or 325 by 732 in exactly the same way, a sexagesimal computation can be carried out regardless of the place-value, which can be determined at the end in the same way as we determine the place-value of the decimal-point.... The flexibility of the notation is one of the most significant features of Babylonian mathematics and perhaps constitutes the most important element for its further development."

13 (Airy nothing): A local habitation and a name have certainly been given to airy nothing itself in France. According to Sanche Gramont ([The French: Portrait of a People](#); Bantam Press, pg. 9),

the Alpine village of St. Cyrice was still listed in the directory of communes as of 1969 although it had no inhabitants. Since everything in France has a reason, the one given here is that the graves in its cemetery were still visited by relatives each November.

TWO: THE GREEKS HAD NO WORD FOR IT

14: ["I got plenty o' nuttin'..."] from Porgy and Bess (Act II), in Gershwin, George, & DuBose Heyward, Porgy and Bess: Libretto (New York: Hal Leonard Publishing Co., 1935).

14-15: [Passage from The Odyssey] Od. IX 355-7, 364-70, 403-8 (trans. Ellen Kaplan).

16: [Homeric letters for numbers] M 268-9.

16: [Homeric Greeks combined H and T for 100 talents] M 269.

16: [5th c. unexplained Athenian reform in number notation] K 132.

16: [24 old and 3 new letters for numbers] M 264-5; K 132.

17: [Bar over letters to indicate they are numbers] K 132.

17: [Order of Greek letters for numbers chaotic] M 270.

17: [Alexander invades Babylonian empire in 331 B. C.] EB

"Alexander"; I: 547.

17: [Symbol for 0 in astronomical papyri] These symbols for 0 occur for the Greeks, says Dilke (D 57), only in an astronomical context. *These* symbols, perhaps - but see below for other ways that Greek mathematicians used zero, with other signs.

18: [Sign for 0 from initial Greek omicron in ουδεν - or not] M 399, 404; A 104 (not); vdW 50, 56; H i 39, 45; N1 14.

18: [ουδεν to μηδεν] L-K 42.

18: [Sign like μ for 0 found in 15th c. Byzantine texts] SK 55; N1 26.

18: [Neugebauer dismisses omicron as origin of 0] N1 14.

18: [Date of Diophantus] He lived perhaps as late as 250 A. D., perhaps as early as 70 A. D., as M. Kline argues. Neugebauer fixes a lower bound at 150 B. C., the approximate date of Hypsicles, whom Diophantus quotes; and an upper bound of 364 A. D., when we know Theon, who quotes Diophantus, was alive. See the full discussion at N1 178-9.

18: [Diophantus' notation, and M with superposed o for monad] K 139-40; vdW 281; H ii 457; [I ii 522-3 ??].

18: [μ with superposed o for $\mu\omicron\rho\alpha$, degree] H i 45.

18: [Nabokov passage] from his "An Evening of Russian Poetry."

19: [Ptolemy's date] M 405.

19: [Always a barred 0 in Greek astronomical papyri] K 132; N1 14;
M 399; (0 without bar); D 57.

19: [0 in middle and at ends of numbers] T ii 445, N1 12.

19: [Earliest texts of Almagest and Diophantus long post-date their
authors] K 54; N2 836-8.

19: [Bar over 0 disappears by Byzantine times] N1 13-14.

20: [Egyptians had neither a 0 nor positional notation] K 16 (not
positional); M 396.

20: [Aristocratic Greek scorn of logistic] K 131.

21: [Backgammon board as descendant of counting-boards] S1 ii 166.

21: [Counting boards go back at least to 7th c. B. C.] M 301 (Solon)..

22: [Boards may go back a thousand years earlier] Nis passim. In
speaking of an "Old Babylonian" text Nissen remarks that at the

time of its compilation the sexagesimal system didn't have a zero, so that a scribe would have to keep in mind the fact that a place wasn't occupied. The scribes, he says, didn't mark the space but seldom made mistakes: "This suggests that such tablets contain, as a rule, only results of calculations which were performed earlier by means of a tallying board or some other calculation aid. It is, for example, possible that on such a tallying board the space for the zero would simply have been left free and thus the zero automatically accounted for." (Nis 150).

22: [Finger-arithmetic] M 201-220 in general; M 211 (in Greece, citing Aristophanes).

22: [Solon's poem] M 301; H i 48; it is from Diog. Laert. i. 59.

22: [Passage in Polybius] M 300; H i 48; it is in Polybius v. 26. 13.

22: [Illustration of finger-reckoning] After a copy of Bede's works in the Biblioteca Nacional at Madrid (c. 1140), as reproduced in S ii197.

22-3: [Bede's finger-reckoning] M 205.

23: [Bede's "On Calculating...", written c. 730 A. D.] M 202.

23: [Greek boards and values of their columns] M 299, 302. The American edition of Liddell & Scott's Greek Lexicon, 1855, gives

the obol's value as "about three cents".

23: [Illustration of counting board] After drawings in M 302.

23-4: [Darius vase] Naples 3253: Apulian red figure volute krater.

Illustration after detail in M 304.

23: [O Boeotian for obol] M 303.

24: [Coptic text about noon-time shadow] N2 742: In a Greek "shadow [of the sun] table", whose source is a Byzantine codex written between the 12th and 16th centuries, a shadow length of half a foot at noon in the VI month is given, and Neugebauer comments: "...1/2, obviously chosen only to avoid 'zero'..." P2 6 suggests that a digit other than 0 "shows that in this scheme it was *not* assumed that the situation in Ujjayini can be generalised for all of India."

25: [Playing impishly with counters] Such impudence on the part of the aristoi is well attested (if any attestation were needed) by Plutarch's anecdote (in his parallel lives) about the young Alcibiades casting his dice in the path of an oncoming cart, and when the carter would not rein in, throwing himself in the way. Alcibiades' companions fled, but the startled driver backed off.

25: [Goethe and constellations] M 367.

25: [Evidence from Plato that Greeks calculated in the sand] Meno

82B.

25: [One way of writing, another of reckoning] It is certainly possible that Greeks, Romans, Egyptians or others, here and there, then or even now, actually managed to do arithmetic directly with alphabetic or other awkward symbols as well as - or instead of - on a counting board, since something in us relishes having a hand tied behind the back. Heath (H i 38) says that the 19th century historian of mathematics Tannery, "holding that the merits of the alphabetic numerals could only be tested by using them, practiced himself in their use until, applying them to the whole of the calculations in Archimedes' Measurement of a Circle, he found that the alphabetic notation had practical advantages which he had hardly suspected before, and that the operations took little longer with Greek than with modern numerals."

25: [Aristophanes' passage from Wasps] M 211; (it is Wasps 656, and reads: λογισαι φαυλωσ μη ψηφοις αλλ' απο χειροσ.

26: [Zero as adjective or noun] Zero seems more adjectival than most numbers, marking as it does the divide between the adjectives "positive" and "negative", and being adjective-like through giving value in positional notation. This may go far to account for the enthusiasm I have heard expressed for rethinking numbers as vector sums of quantifiers (their usual role) and zero-bred "qualifiers", with "infinity" somehow on the diagonal between

these basis elements. My German informant tells me that all three meet at a point he calls "relation", whose character has yet to be discovered. Each man his fig-tree and his arcanum.

26: [Phaedrus passage] 274-5.

27: [Pythagorean secrecy] K 147-9. The Pythagoreans pre-date Alexander's invasion of Babylon by two centuries or so. Later Neo-Pythagoreans could have incorporated what mathematical knowledge Alexander brought back, but it is also possible that word of zero reached Greece long before him. There are (late and dubious) traditions of Pythagoras himself having travelled in the east (vdW 94-5; S1 i 71; K 28). More interestingly, Pingree (P2 10) says: "It is reasonable, then, or at least so I believe, to see the origins of mathematical astronomy in India as just one element in a general transmission of Mesopotamian-Iranian cultural forms to Northern India during the two centuries that antedated Alexander's conquest of the Achaemenid Empire." Could this transmission not therefore have been by way of, or included, Greece?

THREE: TRAVELLERS' TALES

29: [Portrait of Archimedes] Städtisches Kunstinstitut, Frankfurt am Main. vdW Plate 24: "Death of Archimedes. Mosaic, probably from the school of Rafael. Copy (or falsification), originally thought to be authentic Roman work. Various elements in the framework

(the waterhens - porphyryons - in the four corners and the tendrils) are very successful imitations of Roman examples." Details of the legend of Archimedes' death at the hands of a Roman centurion, and of the authenticity or not of this mosaic, in vdW 210.

29: [Biographical data on Archimedes] Ar xv.

29: [Archimedes to Gelon] Ar 221.

29-32: [Archimedes' Sand-reckoner calculations] Ar 227.

29: [Take it that there are 10,000 grains in a poppy seed] Can we really so take it? Even granting that Archimedes is purposely exaggerating (as I point out on pg. 30), this seems too extreme to be true. if you look at your next poppy-seed you will see that a sphere its size could only hold some hundred or so grains of the finest sand. Yet we have in his Assumption 5 (Ar 227): "Suppose a quantity of sand taken not greater than a poppy-seed, and suppose that it contains not more than 10,000 grains."

The "10,000" is indubitable: it is "myriad." The word for poppy-seed, *μηκων*, leaves us a little room to manoeuvre: In his Greek Mathematics, Thomas says (T ii 200 note a): "A poppy-head (for so *μηκων* is here to be understood, not 'poppy-seed', v. d'Arcy W. Thompson, The Classical Review, lvi (1942) p. 75)..." And this is what D'Arcy Thompson has to say [Tho 75]: "...Mr. Thomas,

following Heath and Heiberg, tells us how Archimedes began with a poppy-seed, containing, say, 10,000 grains of sand, and having a diameter not less than a finger's breadth. But a poppy-seed is one of the least of seeds, and is of the order of magnitude of one single grain of sand; the poppy-head, or capsule, is what Archimedes is thinking of. This is what *μηκων* means; and the older editors, Torelli and Rivalt, say nothing about a seed, but merely say *non magis papaveris*. Moreover, I have counted and weighed the seeds in a garden poppy-head, and found that 10,000 seeds comes very near the mark."

Well and good: such a poppy-head might well hold 10,000 grains of sand. But remember that 40 of these *μηκων* make up a finger's breadth, for so Assumption 5 continues: "Next suppose the diameter of the poppy-seed to be not less than 1/40th of a finger-breadth." If visual inspection won't convince you that poppy-heads are much too big to fit 40 into a finger's breadth, consult Culpeper's Complete Herbal (London: W. Foulsham & Co. Ltd.), where he says of the wild poppy (*papaver rhoeas*), pg. 282, that its small green head, when ripe, is "not much bigger than one's little finger's end, wherein is contained much black seed." Even allowing for the variety in poppies, the variation in the size of their heads and the finger-breadths of the most sausage-fingered adults, none will accommodate 40 such heads.

Recourse to the various meanings of *μηκων* won't help: Liddell & Scott give 1. the poppy (ll. viii. 306), 2. the head of a poppy

(Theophrastus), 3. poppy-seed (Thuc. 4.25). Attempts to tamper with 1/40th fail because Archimedes' subsequent calculations are consistent with it.

This may have to remain one of the greater (or much lesser) mysteries.

30: [Archimedes draws on Aristarchus] H ii 82.

30: [Archimedes juggles with Aristarchus' numbers] H ii 82.

30: [Archimedes' conclusion: 10^{63} grains of sand in universe] M 141.

31: [Two New Yorkers estimating grains of sand at Coney Island] We 202 (they were Edward Kasner and James Newman).

31: [Present estimate of number of particles in universe] We 205.

31: [Gauss on Archimedes missing 0] M 141.

32: [Archimedes: "And let the last number..."] T ii 201.

32: [Archimedes: "And let the process continue..."] M 140.

32: [1 followed by 800 million zeroes] M 140.

32: [Sand Reckoner a tour de force] H i 41.

32: [Plato on play] Laws vii 803.

32: [Archimedes' father Phidias and sun's diameter] H ii 82.

33: [Aristarchus' 71,755,875] H ii 15.

33: [Number of pages needed to write solution to his problem]
V 306. This is the problem of "the Cattle of the Sun."

33: [A mathematician on large numbers] David Pollack, at the time a
graduate student in the Math Department at Harvard.

33: [Archimedes to Gelon] Ar 221.

34: [St. Paul to Ephesians] Eph. i 21.

34: [Primo-vigesimo-centillion] We 202, who comments there: "One
of the least frequently used words in the English language."
Its frequency has hereby increased.

34-35: [Donne passage] Don 442 (= Sermon 26: First Sermon in Lent,
1626/7?).

FOUR: EASTWARD

36: [Passage from Iliad] II. 496-8 (trans. Ellen Kaplan).

36: [Parallel words for counting and recounting] How widespread this parallelism is testifies to how deep it must be in our natures.

Think for example of 'story' and 'count' in French (conte, compter), German (Erzählung, zählen) and the Hebrew li'spor (to count), le'saper (to recount) - as Curt Leviat kindly pointed out to me.

36-7: [Goethe on mathematicians and Frenchmen] Moritz, R. E., Memorabilia Mathematica entry 813 (Dover edition, pg. 123).

37: [Russellian definition of number] Russell, "Introduction to Mathematical Philosophy", in Benacerraf & Putnam, Philosophy of Mathematics (Prentice Hall, 1964) p. 123.

37: [Follow this dance to India] see note to pg. 27, on Pythagorean secrecy, for Pingree's point about this transmission.

37: [Story of Indra and the ants] Z2 4-7.

38: [Date of Surya Siddhanta] S 1 i 34, from whence the information comes that this early edition was published by The Swami Press in Meerut.

38: [Marlowe called atheist for saying Indian texts pre-Adamic] Nicholl, Charles, The Reckoning (Harcourt Brace, 1992) 57.

39: [Illustration of Buddha calculating] figure based on a shale sculpture, Gandhara School (Pakistan, 2nd or 3rd c. A. D.) of Maitreya, the Buddha of Future Times (in Hinayana Buddhism). In the Musée Guimet, Paris. Hand gesture (mudhra): modified to the Vitarka mudhra, sign of reasoning.

38-9: [Lalitavistara passages] Mi 171, Fo 133 ff.

38. [Evidence of date for Lalitavistara] M 138 dates it prior to 300 B. C., but from EB:

(i) v. 16 96-97 article "Lamaism" (T.W. R. D. = T. W. R. Davids):

1. About the commencement of A. D., when Huns conquered northern India, school of Buddhist teachers arose who called their doctrine 'The Great Vehicle' [= ferryboat = Mahayana], differing from the former 'Little Vehicle' [= little ferryboat = Hinayana] by advocating Bodhisatship rather than Arhatship.
2. Its earliest professors and greatest expounders were in N. W. India.
3. "Buddhism arose in countries where Sanskrit was never more than a learned tongue, and where the exclusive claims of the Brahmins had never been universally admitted. The great vehicle arose in the very stronghold of Brahminism, and among a people to whom Sanskrit... was the lingua franca. The new

literature, therefore, which the movement called forth, was written, and has been preserved, in Sanskrit."

4. Its principal books of Dharma, or doctrine, are nine - of which the eighth is the Lalitavistara.
5. "The date of none of these works is known with any certainty, but it is highly improbable that any of them is older than the 6th century after the death of Gautama." [i.e., 6th century after 543 to 488 B. C.]
6. "Of the authors of these nine works,... nothing has been ascertained. The founder of the system of the Great Vehicle is, however, often referred to under the name Nagarjuna, whose probable date is about A. D. 200."

(ii) EB v. 4 pg. 748, article "Buddhism" (Davids):

Dates Buddha's death to between 488 and 543 B. C.
also: pg. 737, footnote: Lalitavistara may be as old as the Christian era.

(iii) Z Appendix B (p. 617): Mahayana Buddhism begins c. 150 A.D., and Nagarjuna is c. 200 A. D.

38: [Yojana] spelling and length from CI 15. Clark quotes

Brahmagupta on a yojana being 8000 times a *nr* [= height of a

man], which if I take to be 5'3" at this time gives 7.95 miles.

Thomas Burke (in private conversation) gives a 'yodjana' as the length you could go with an ox before unyoking him. If this means ploughing, that is a furlong, or about 1/5 mile; but if a cart drawn by a team of oxen, about two or three miles, and a league - the conventional translation (Mo) - is about three miles (OED). Albiruni (Sa 167): "[a] yojana is equal to 8 miles or 32,000 yards." (sic! 8 of our miles = 14,080 yards!)

39: [Niyuta = 10^{11} etc.] M 137.

38-9: ['Grains', nit] M has 'track' ('grains de poussière' and 'lente') and 'poppy-seed' where Fo has 'grain' and 'nit' (Skt. 'liksa'). In conversation Thomas Burke agrees with 'nit' and 'grain', replaces M's 'ram' by 'sheep', M's 'ox' by 'cow', but finds all three sized denominations mysterious. Mo 901 has liksa = a nit, young louse, the egg of a louse, and remarks: "As a measure of weight, = 8 trasa-renus" (= the mote or atom of dust moving in a sun-beam, Mo 457; = 3 or 30 invisible atoms). Tur 640 has liksa = nit, young louse, and calls it a unit of length, where 7 liksa = 1 uka = louse. However Mo (English-Sanskrit) has, under 'poppy-seed', liksa. Du confirms Mo readings. Albiruni on liksa (Sa 162): "If the Hindus wish to be particularly painstaking in these things [metrology], they give the following scale, based on the measurements which Varahamihira prescribes for the construction of idols:-
1 renu or particle of dust = 1 raja

8 raja = 1 balagra, i.e., the end of a hair

8 balagra = 1 likhya, i.e., the egg of a louse

8 likhya = 1 yuka, i.e., a louse

8 yuka = 1 yava, i.e., a barley-corn...

"The following weights occur in the book Caraka. I give them here according to the Arabic translation, as I have not received them from the Hindus *viva voce*. The Arabic copy seems to be corrupt, like all other books of this kind which I know. Such corruption must of necessity occur in our Arabic writing, more particularly at a period like ours, when people care so little about the correctness of what they copy.

"Atreya says:

6 particles of dust = 1 marici

6 marici = 1 mustard-seed (rajika)

8 mustard-seeds = 1 red rice-corn..."

(Sa 166): "[Varahamihira] says [with regard to distances] -

8 barley-corns put together = 1 angula, i.e., finger

4 fingers = 1 rama (?), i.e., the fist

24 fingers = 1 hattha, i.e., yard, also called dasta...

The yard is equal to 2 mikyās or 24 fingers; for the Hindus determine the sanku, i.e., mikyās, by idol-fingers. They do not call the twelfth part of a mikyās a finger in general, as we do, but their mikyās is always a span."

40: [Different account of units] Mon 457 under trasa-renu, 3 (in Brahmavaivarta Purana) or 30 (in Bhagavata Purana) invisible

atoms making up this mote of dust.

40: [Rabbit-, ram- ox-grains] Wo 77, and note 3, says: "grain de poussière de lièvre: probablement il faut entendre: soulevée par le pied d'un lièvre."

40: [Buddha's remark] translated from the French of Fo 133; in conversation Thomas Burke finds no passage in the Sanskrit corresponding to "...living outside his house".

40: [Evidence of Greek influence on Indian astrology, astronomy and math] N1 165-167; 172-177; 186.

Evidence not listed: N1 173 (Varahamihira uses days incorrectly for degrees); N1 175 (Romaka and Paulisa Siddhantas' Greek origins); N1 166 (Mesopotamian influence on Hindus via Greeks); N1 164-177, esp. 166-7, 173-7, 186-90 (data and ideas on Babylonian and Greek influence on Hindus); N1 189, vdW 56-7, I 457, N2 1113 (Hindu decimal-place notation as modification of sexagesimal system they learned through Hellenistic astronomy); cf. style of word problems in Archimedes (Cattle-problem) and Lilavati; N1166-7 (Roman trade center and Tamil astronomy); N1 167 (Hindu astronomy dates from early A.D., not B. C.); N1 187 (Almagest and Hindu astronomical texts both at court of Shapur I in 250 A D.). See too M 398-9 for general remarks.

40: [Greek loan-words for zodiacal signs] N1 166.

40: [Greek loan-words for astronomical terms] vdW 56.

40: [Indians writing fractions as Greeks did] vdW 56; Ca2 75.

40: [Greek planetary theory in Hindu astronomy] N1 166.

40: [3:2 ratio] N1 162, 187. Pingree (P2 4) writes that this ratio is appropriate only to extreme Northwest India.

41: [Supposed date of Surya Siddhanta] N1 174.

41: [Instructions in Surya Siddhanta] N1 174.

41: [Varahamihira on flourishing Greek astronomy] N1 175.

41: [Gwalior] I 438.

41: [Indubitably first 0 in India] Ca1 89; M 396-7; SK 52; see I 437.

41: [Copperplate documents with 0 in them] I 437.

41: [Forgeries of copperplate documents] Ca2 48; SK 45-47.

41: [Wranglers over Greek priority] I 440. The case for a Greek source of the Indian zero is put widely; see, for example, N1 13-14, 26, 27; K 132; A 104; vdW 50-56; H 29, 39, 45. Cf.

also P2 10, referred to in note above to pg. 27. Counter-claims abound, anchoring the notion of zero in, for example, the Buddhist concept of sunyata. On this Ruegg (Ru 176-7) comments: "Our present state of knowledge lets us say that there appears to be no direct connection between the mathematical zero and the sunyata concept in... Buddhism... At the most one could perhaps speak only of a subjacent and remote resemblance or cognitive homology between independently developed concepts."

41: [List of types of fallacy] Fi 47 (negative), 48 (presumptive), 53 (possible), 82 (pragmatic), 87 (aesthetic).

42: [Making out dim figures in the haze] I am indebted to a correspondent, Franz Krojer, for calling my attention to a possibly significant figure in this haze: the Syrian neo-Platonist Iamblichus of Chalcis (d. circa 330 A.D.). A follower of Plotinus, he mixed Pythagorean and Oriental strains together in a brew that confused more than it clarified. It is undeniable, however, that he (and so, perhaps, those before and after him in this tradition) treated zero almost on a par with the other numbers: as gingerly, in fact, as we see it treated a millennium and a half later.

Here is the relevant passage, from his Introduction to the Arithmetic of Nicomachus (*In Nicomachi Arithmetica Introductionem*, Pistelli, 21 B 2-4 and 23 B 5 - 25 B3). My

translation depends not only on Herr Krojer's, but on those whom he in turn (as he writes me) relied on: Hans Freudenthal, and Messrs. Lucassen and Stolwijk.

Iamblichus laboriously explains what subtracting 5 from larger numbers will be ("...going back from 8, taking 5, we will add 2 to 3, for from 8 the fifth is 3; and from 7, going back in order the fifth is 2..."). When he works his way thus back to 5 itself, he says:

"Taking nothing from 5 (the fifth from 5 is nothing [ouden]), we shall add it to this nothing, and then will it be itself. Thus, since the monad [1] is indivisible, something conceptually yet smaller is nothing (rather than a half, as some say), and it is everywhere related to this monad by analogy: for this monad itself becomes the midpoint of the two numbers drawn together from opposite sides (i.e., the middle between 2 and 0 is 1).

"The very name of nothing (ouden) is the clearest signal to us that the monad is by nature the smallest [number], and is indivisible: for the word 'ouden', when split up [into oude and hen: 'not even one'] is the denial of any sort of being, which would not be thinkable were halves or thirds or any other such parts left over.

"Needless to say, the one, when multiplied by any other number, in no way exceeds it, and just so little changes when multiplied by itself - since it is at once the boundary between number as such and nothing.

"For usual numbers, when multiplied by themselves or by another number, never, in either case, retain their own value, but always bring some third number forward. Nothing, on the other hand, when seemingly* multiplied by itself or by another number, is never exceeded, but 0×0 or 0×9 are both 0, and not at all 9. And similarly [when 0 is multiplied by] other numbers.

"But the one, as the middle between [number and nothing], when multiplied by another number, yields that number, and when multiplied by itself, itself."

(*) A key word here is 'doxeie', from doxein, to seem. According to Herr Krojer, Hans Freudenthal, (5000 Jaren internationale Wetenschap, Groningen 1946), remarks: "Iamblichus uses the word 'doxein' because he considers that the non-existence of zero only allows one to speak of a seeming multiplication." This is a telling observation, showing how - for all the disorder of his thought -

Iamblichus is probing the boundaries of the then current understanding of numbers, and letting zero enter their ranks under the cloak of analogy and fiction.

Here then, three centuries before Brahmagupta, we find the first, hesitant steps being taken in doing arithmetic with zero. How startlingly these steps resound in the emptiness of our knowledge about this transitional time.

42: [Date of Aryabhata] Cl xi.

42: [City of Flowers, near ruined palace] S1 i 153-4. City of Flowers is Kasumapura; palace is in Pataliputra - ruined, when the Chinese traveler Fa-hien saw it a century before Aryabhata's birth; built by genii at Asoka's command.

42: [Even a shadowy third] Cl x.

42: [Spelling of Aryabhata/Aryabhata] Cl vii-viii.

42: [Two Aryabhatas; reputation] Cl ix-x; xv-xvi.

42: [Pearl shells and dates] S1 i 153 (Albiruni's comment on all the Hindu mathematics he found in 1000 A. D.).

42: [Is the text of Aryabhata authentic?] CI xii-xiii.

42: [Aryabhata's borrowing] S1 i 156 suggests that his incorrect formula for the volume of a sphere, which would give π a value of $16/9$, resulted from an error in copying Ahmes, i.e., the Rhind papyrus: where $(16/9)^2$ would have given Ahmes' value of 3.16...

43: [Details of Aryabhata's numbering system] CI 2-9.

43: [Note on Aryabhata's numbering system] Since there were 25 consonants in his first group (called *varga* letters), from k to m, and 8 in the second (*avarga*), from y to h; and since the *varga* consonants were in the odd, the *avarga* in the even places, Aryabhata could write the numbers from 1 to 20 with just the first twenty *varga* letters (k through n) as ka to na. That meant that the eight consonants of the second, *avarga*, group could stand for 2 through 9 in the tens place (which is why I speak of s as 'in effect' the eighth letter of the second group, while in fact it is the seventh). 26, for example, would be written 6 + 20: caya. Of course he would have had a choice of how to write the numbers from 21 to 25: 21 could be either pa (since p is the 21st *varga* consonant), or kaya: the first *varga* consonant standing for 1 unit, the first *avarga* consonant standing for 2 tens.

The system works beautifully. It is peculiar to find Clark (CI 4) thinking that the 8 *avarga* consonants stood for 3 through 10 - for then Aryabhata would have had no way of writing 26 through

29. It is also peculiar to find him saying (CI 8) that the digits may be given from left to right or right to left, "or in any order which will make the syllables fit the meter": for he makes it clear earlier (CI 4) that "no distinction is made between long and short vowels" - so that altering their quantity, not changing their order, would fit words to metric requirements. These are the mysteries of the Lower Math.

43: [Passage from Aryabhata] CI 2, 6.

43: [Aryabhata uses 'kha' for 'place'] CI 6-7.

44: [Date of Varahamihira] N1 229.

44: [Varahamihira uses 'sunya' for zero] SK 44.

44: [Date of Brahmagupta, in Ujjain] S1 i 157.

44: [Brahmagupta critic and admirer of Aryabhata] CI xv.

44: [Sunya, kha as empty] Hay 2; Dat 75; SK 44.

44: [Brahmagupta called zero 'kha', 'sunya', 'akasa'] Hay 3, noting 'kha' (in Dvivedin's edition of the Brahmasphutasiddhanta) at (inter alia) the following (locations in Colebrooke's translation referenced in parentheses):

18.2 a, c; 18.7 a, c (Col 8); 18.30 a, c (Col 31); 18.33 a, c

(Col 34);18.34 a, c (Col 34); 18.35 a, c (Col 36); 18.94 a, c
(Col 95); 18.95 a, c (Col 96).

'sunya' occurs at 18.30 a, c (Col 31); 18.32 a, c (Col 33);
18.93 a, c (Col 94).

44: [Date of Mahavira] S1 i 154.

44: [Mahavira in Mysore] S1 i 154.

44: [Mahavira a Jain] Ra x.

44: [Mahavira updating Brahmagupta] Ra xi.

44: [Mahavira's words for zero] Ra 6-7 (English), 6 (Sanskrit), 287-
295.

44: [Depth = vyama; firmament = gagana; the endless = ananta;
Vishnu's footstep = visnupada; middle air = antariksa] B 296-
304; Mo under each entry.

45: [Scansion of Mahavira's words for zero] in conversation Thomas
Burke says: Ananta: u u _; antariksa: u u _ u; ambara: _ u u; akasa:
_ _ u; kha: u; gagana: u u u; jaladharapatha: u u u u u; div: u;
nabhas: u u; viyat: u u; vishnupada: _ u _ u; vyama (or, he claims,
vioma): _ u.

45: ["...The sky becomes the same..."] Ra English 7.

45: [Salutation] Ra English 2.

45: [Explanation by translator of syadvada] Ra English 2.

45: [That Mahavira must have had a symbol for 0] Ra xiv-xv.

46: [Sebokht date, place, quotation] S1 i 166-7.

46: ["A hundred years ago people..."] passage paraphrased from Brockhaus in SK 43.

46: ["Was like making Nirvana dynamic"] vdW 56, paraphrasing G. B. Halsted, On the Foundation and Technique of Arithmetic (Chicago, 1912), pg. 20: "It is like coining the Nirvana into dynamos. No single mathematical creation has been more potent for the general on-go of intelligence and power." I like the Chicago demotic of that 'on-go'.

46: [First passage from Spengler] Sp I 66.

46: [Second passage from Spengler] Sp I 178. Spengler also points out (I 133) that the Indians had no clocks, and *therefore* no history, no life memories and no care. Who would have thought.

47: [Illegible inscriptions in Nanaghat caves; caves at Nasik] SK 23, 24; date of Nanaghat inscriptions: SK 22.

47: [Conjecture that 0 is 10 shorn of its spurs] SK 54; M 403.

47: [Support from conjecture in sequence of Arab numerals] SK 54
n. 3.

47: [0 for 20, 0 for 1] SK 23, 48-50.

47: [5 different guises for 10] SK 25 for first 4, Ca 2 50 for last: in
order: Saka, Nepal, Kalinga, Vakataka, Sanskrit 2nd c. A. D.

47: [Greeks used α for pictured unit] T i 86-7.

48: [Sumerian O for 10] vdW 40; M 164; L-K 64-65. Compare remark
of Nissen (Nis 119) cited above in note to pg. 7.

48: [Lang-Kirnberg's conjecture] L-K 65.

48: [Sanskrit sign for omission] WW §16 c pg. 9.

48: [Tartars enclose omission in an oval] SK 55.

48: [Bhaskara's o for subtraction] SK 55.

48. [Every ecological niche filled] I've omitted the arguments (as in
Sta 1 69) that since Panini, in the 5th or 4th c. B. C., puts lopa at
the end of a word for deletion, then substitutes, this is like a

zero and therefore a source for zero in India. I omit this because it is dismissed so thoroughly by Bronkhorst, Ba 1, passim, and Ba 2.

Efforts inspired by national pride haven't stopped here to claim the trophy of zero for Immemorial India. Pingala's binary representation system (which "clearly needed a symbol for zero") and Panini's lopa are trundled out again in "The Sign for Zero" by Subhash C. Kak (Mankind Quarterly, vol. XXX no. 3, Spring 1990, 199-204). He goes on to argue that the oval symbol for zero derives from the fish-like Brahmi sign for 10; that since this sign is found in first and second century A. D. inscriptions, its use must long have preceded those; that the Brahmi numerals, having been shown (by Kak) to derive from the Indus numerals, the zero sign is thus pushed back to the third millennium B. C. in India.

Kak's arguments are phrased in terms of what is "likely" and "not unlikely", what "must have been" the case, what "may have played a role", what is "conceivable" and what "one would expect". In personal correspondence Takao Hayashi (personal communication, e-mail of Dec. 19, 1999) writes to me that while he hasn't read this particular article of Kak's, he has read others, and "...I have the general impression that most of his arguments... are based on speculations which are in turn based on other speculations, such as 'the derivation of the Brahmi numerals from the Indus numerals, etc.'"

With regard to the argument here, he writes: "...I do not think that the shape of zero was derived from the Braahmii symbol

for ten. In Sanskrit... the sign for zero is called bindu... and the other nine numerals, 1 to 9, are called a.nka.... The word a.nka therefore always meant 9 (not 10)... The sign for zero must therefore have been originally a simple ``dot" (bindu) used as a sign for indicating a vacant place, that is, a place where there is no numerical sign (a.nka)..... If it were, like the other nine numerals, a sign derived from a Braahmii numeral, it would have been included in the concept of a.nka."

49: [Sand-covered Indian boards and dhuli-kharma] M 397.

FIVE: DUST

51: [Furrows blurred by moving hands and counters] M 397-8.

51: [Writing numerals instead of placing counters in sand] M 397-8;
H i 47.

51: [Gerbert's date and his apices] M 322-3; H i 47.

51: [Apices from Latin apex] M 324.

51: [Gerbert's numerals were West Arabic] H i 47; M 413; cf. SK 88.

51: [Our 2, 3 and 7 from rotated apices] M 327, 413, 418-9; SK 66 n.
4, 88.

51: [Gerbert's zero sign, and 'sipos' from psephos] M 325, 301.

52: [Gerbert accused of intercourse with evil spirits] Ca1 115.

52: [Pulvis eruditus and Cicero] Cassell's Latin Dictionary, revised Marchant and Charles (Funk & Wagnalls) art. 'pulvis'.

52: [Numquam eruditem illum pulverem attigitis] *ibid.*, art. 'pulvis'.

52: [Remigius of Auxerre] S1 ii 178.

52: [Horace's schoolboy's wax tablets] S1 ii 165 n. 2, quoting Hor. Sat. I, 6, 74.

52: [Roman counter-board sanded too] Ca1 64.

52: [Calculi] M 315.

52: ['Abacus' $\alpha\beta\alpha\xi$ from Semitic abq] S1 ii 156 n. 3; contrast M 301: legless table.

52-3: [Pseudo-Boethius and the Mensa Pythagorica] S1 ii 177.

53: [Pythagoras' arches] M 323.

53: [Esoteric traditions] S1 ii 60: Iamblichus (c. 325) says

Pythagoras introduced board into Greece from Babylon. Role of Adelard of Bath in this: S1 ii 177.

53: [Gobar numerals in Moorish Spain, c. 950] SK 65, M 416.

53: [How gobar dots worked] M 416.

54: [Diacritical marks of Semitic people] SK 67.

54: [Diacritical marks as zero letters] Hu 137.

54: [Role in Hebrew of superscripted dots] Tan 6/6/98, 6/13/98; Be 6/13/98; EJ vol. 16, "Masorah", 1.2.2.1.

55: [Dot for pledge to complete unfulfilled task] M 403.

55. [Dot to mark a lacuna] SK 53 n.5; cf. superscripted small circle in Skt. to signal omission: WW § 16c. Falk writes to me that the small superscripted zero "is known to me only from the mss. of the Padapatha of the Rgveda, where it denotes a series of at least 3 words which are not there. This circle is termed *galita* and it is used when a 3 word series inside a stanza is identical in shape to the same 3 words at the beginning of a stanza some place prior to the place in question." He writes too that there is no tradition of a date when the Padapatha was put into writing, "and nothing allows us to assume a date before the 10th century A. D." (Fal).

55: [Sunyabindu, the dot marking a blank] SK 53, 53 n. 5.

55: ['Bindu' dot connected to Siva] Mon art. 'bindu'.

55: [Ibn Abi Ya'qub al-Nadim] S1 ii 466.

55: [Indians said to have subscript dots for 0] SK 67, 68.

55: [Superscripted o in Bhaskara for subtraction] SK 58.

55: [Passage from commentator on Bhaskara] Commentator is Krishna; in Co 4 trans. of Lilavati (Ch. 2 sec. 1), apparently quoting Krishna; cf. Banerji on his pg. 4.

55: [Arabs used '0' for 5] SK 67. An ornamented hollow 0 was also used by the Arabs for zero. Neugebauer (N1 26) comments: "That the Arabic form for the zero symbol (a little circle with a bar over it and related forms) is simply taken from Greek astronomical manuscripts was recognized by F. Woepcke in 1863..."

56: [Bakhshali ms. on birch-bark] Ca 1 85.

56: [Bakhshali on India's northwestern frontier] Ca 2 77.

56: [Questionable date of Bakhshali ms.] SK 43 n.3; 52; S1 i 164;

Hay 2.

56: [Bhakhshali probably not early] Jo 241; Hay 2: "The Bakshali ms. certainly proves the existence of a symbol for zero in the time of its composition, but the date of the prose parts, where the symbol occurs, seems to be about the 7th century A. D., though some of the rules and examples written in verse may go back earlier..."

56: [Subandhu: Vasavadatta] SK 44-5; Dat 81, quoting Subandhu (in the translation of Louis Gray): "And at the time of the rising of the moon with its blackness of night, bowing low, as it were, with folded hands under the guise of closing blue lotuses, immediately the stars shone forth,... like zero-dots [sunya-bindu], because of the nullity of metempsychosis, scattered in the sky as if on the ink-blue skin rug of the creator who reckoneth the sum total with a bit of the moon for chalk."

56: [Yavanajataka and bindu, kha] P 406-7.

56: [Original of Yavanajataka is Greek; and details] P v.1 pg. 3.

SIX: INTO THE UNKNOWN

57: [Kite-catenary image] I'm indebted to Michael Kaplan for this image, which he found in Mario Alvadori, Why Buildings Stand Up (Norton, 1980), pg. 168: the architect Ellet offered a \$5 prize to

the boy who could fly his kite across Niagara Falls - string, then steel wires of ever greater thickness followed, until Ellet could hang the lovely catenaries of his bridge from it, which Roebling later completed.

57: [Sent howling back] S. N. Sen, "Survey and Studies in European Languages", in Sen 21-121, esp. 101-105, where Pingree's approach is criticised in general, but not the specifics of this passage. Hay 2: "I don't think that the Yavanajataka (ch. 79) gives very strong evidence, back to 149/50 A. D. of a Greek source..., since the Yavanajataka does not contain the words 'bindu' and 'kha', for zero according to K. S. Shukla, whose interpretation of the verses in question is mathematically better than Pingree's. We cannot of course disprove Greek influence on the Indian concept of zero (at least on its symbol), but, at the same time, there is no 'strong evidence' to show it. One can just speculate it."

57: [Making of Irish pots] Nee v. 3 p. 12 n. d. Do we hear an echo of this in the sailor's definition of a boat as a hole in the water you pour money into?

57: [Da Vinci's scribbled "Di mi se mai..."] Sir Kenneth Clark, Leonardo da Vinci (Penguin Books, 1959) 147.

57: ['Bindu' dot for expansion of secondary plot element] Mon art.
'bindu': "A detached particle, drop, globule, spot, dot; a drop of

water taken as a measure; a spot or mark of colored paint; the dot over a letter representing the anusvara (supposed to be connected with Siva and of great mystical importance); a zero or cypher; in mss. put over an erased word to show that it ought not to be erased = 'stet'; the sudden development of a secondary incident (which, like a drop of oil in water, expands and furnishes an important element in the plot)."

58: [Problem in Bakhshali ms.] Ca 2 78.

58: [Second problem cited from Bakhshali ms.] Ca 1 91.

58: [Brahmagupta's 'yavat tavat' and colors for variables] Ca 2 75,
Ca 1 93.

59: ['Nothing' a false substantive] Q 240. I wonder if Quine's insight came via the 1941 children's book by Wanda Ga'g, Nothing At All (for a copy of which I am grateful to Trudy Barash of Madison Wisconsin's great Canterbury Bookstore). This is the story of the invisible third brother of a doggy trio, whose other members are Curly and (not Larry but) Pointy. Since he looked like nothing at all, that was his name, and he had just as much fun as any other dog. After adventures in which no one was anywhere, the epiphany sees him transformed - through the agency of a Parmenidean jackdaw with a book of magic on Nothingness and Somethingness - into the equally false substantive, Something-after-all. This is one of those books which adults and children

read differently. I don't know how old Quine was when (and if) he read it.

There are of course more false substantives than 'something' and 'nothing'. As Sir Walter Scott points out, in the persona of Andrew Fairservice, "Ony thing is nae thing." (Rob Roy, Chapter 18). In fact, when you stop to consider it, which substantives *aren't* false? Applying Quine's criterion, that of a false substantive any property may with equal validity be asserted and denied, there is hardly a proper name of which this isn't so: "John is tall", "John is not tall" - depending on which John you mean; "London is large" (as is true in England), "London is small" (true in Canada). The only true substantives, by Quine's criterion, would be those rare nouns which have but a single instantiation (let us call them just this once hapax deiknoumena). Perhaps "Poughkeepsie" would be an example (but it would take some checking through the gazetteer; and even then, there is many an adjective whose appropriateness to that town in the Hudson Valley would be disputed by those who have scraped an acquaintance with it). A prefatory demonstrative pronoun - "this", "that" - would cut down considerably on the latitude of adjectival applicability to a noun, so that we might call such a pairing "a relatively true noun-phrase". What Quine may unintentionally have done, however, is to call attention to the variable nature not just of our pronouns but of nouns themselves. Haven't we learned from Plato that every object is variable? Don't we see that when we solve for the variable in an equation and discover that it is a particular number - 3, $\sqrt{2}$, π - under

wraps, isn't the number it hid an adjective? Aren't they alone never false?

59: [Fulness = brahman, which pervades the universe, and can no more change than the 'absolute element' in Buddhism, empty (sunya) only of the accidental] Ru 176.

59: [Etymology of sunya] Tur under sunya, suyate, svi; Ru 176; Mo under svi.

59: [Etymology of kha] Mo.

59: [Akasa and purna for brahman, with akasa as name for 0 in Varahamihira, purna in Bhaskara] Ru 176. Bhaskara uses purna for zero in saying he was born in rasa-guna-purna-mahi saka (Ka vol. 5 pt. 1 702). See too Ru 173 on the Buddhist theory of dharma as a receptive, neutral substrate..

60: [Empty place for Elijah] Be: "Elijah announces the Messiah... He wanders between two worlds, often as a beggar, to test people's virtue, or to keep the Rabbis posted on the latest in heaven." Tan: "Elijah isn't exactly an angel, but a substantial folklore exists in which mortals knowingly or unknowingly entertain him." Such a chair is left at the circumcision (bris) ceremony, since Elijah will announce the coming of the Messiah, who now might be this very child (and a correspondent, Curt Leviant, tells me he has seen such chairs hanging on the walls of shuls in Carpentras,

France, and in Bokharan rite shuls in Jerusalem; he refers the interested reader to the article "Elijah, Chair of", on pg. 643, vol. 6 of the Encyclopedia Judaica). Five cups of wine rather than four are placed on the Seder table because (Curt Leviant tells me) of a dispute two thousand years ago over whether four or five cups was the correct number (five for the promises God made to the Israelites, only four of which however have as yet been kept). The dispute was settled by including a fifth cup "for Elijah", who when he came would settle all disputes. Compare the empty cup which Saki turns down (at least in Fitzgerald's translation of Omar Khayyam's Rubaiyat) for the poet, when he dies; and the empty chair set, in some Christian traditions, for St. Anthony (perhaps through his connection with Elijah).

60: [The milkmaid's song] E. M. Forster, Passage to India, Ch. VII (pg. 80 of the Harcourt Brace paperback edition).

60: [Salutation from Vija-Ganita] Properly Introduction, Co 129.

60: [Broken into short dashes] H i 37.

60: [Curls over, dot or dots before and after] H i 36-7.

61: [Subscripted left dash to increase value by 1,000] H i 36.

61: [Strokes growing out of or above letters] H i 36.

- 61: [Penant for these strokes knew no bounds] Nor does it yet.
Superscripted apostrophe-like dashes rain down every page of modern mathematics, now standing for derivatives, now for constants or variables paired with unmarked ones, now for the writer's momentary needs. In his excellent Notes on Fermat's Last Theorem (Wiley, 1996), Alf van der Poorten remarks (pg. 73): "Typically, a ' on a summation sign tells not to include some term which obviously should not be included. I think of it as an instruction not to be silly."
- 61: [Superscripted accent or two for fractions] H i 42.
- 61: [Archimedes' way of writing 10/71] H i 42.
- 61: [Dates of Alexandrians] Pappus f. 284-305 (H ii 356); Diophantus f. 250 (H ii 448); Heron 150 B. C. - 250 A. D. (H ii 299), but probably closer to Pappus (H ii 356).
- 61: [Diophantus' superscripted M] He also thus separated, in equations, his units (constants) from his variables.
- 61: [Diophantus' and Pappus' dot for myriad] Diophantus: Ca 2 25;
Pappus: Ca 2 54.
- 61: [Heron on automata] H ii 308.
- 61: [Heron putting dots above to raise by 10,000] H i 39.

61: [Another tradition continues the series] Ca 2 54.

61: [Hebrew dots above for 100, superscripted accents to distinguish numbers from words] I 254-6; Tan 7/1/98.

62: [Unknowns in Babylonian texts] vdW 63-75.

62: [Unknown in little puzzles] vdW 280, K 138, H ii 441-2.

62: [Date of Thymaridas] H i 69.

62: [Flower of Thymaridas] H i 94.

62: [Diophantus' def. of 'arithmos'] "an indeterminate number of units": πληθος μοναδων αοριστον: H ii 456-7.

62: [Figure like 4 in some texts for unknown] H ii 456.

62: [Shapes of symbols for arithmos] H ii 457.

62: [Diophantus' ζ⁹] Heath (H ii 457) cleverly argues (with epigraphical support) that this curious symbol may arise from a contraction of the first two letters, αρ, of arithmos (αριθμος), putting it in the tradition of the symbols for monad and myriad. And Ellen Kaplan points out that this symbol bears a hereditary likeness to the 4-shaped figure described in this paragraph -

opening up a large field of conjecture.

62: [General use of 'arithmos' symbol for the unknown] H ii 457.

62: [Papyrus (Michigan) of 154 A. D.] H ii 457.

62: [Diophantus' use of false position anticipating Indian] H ii 461.

The method of false position was probably much earlier: it appears to be used in the "heap" problem to be found in the Egyptian Rhind Papyrus from about 1550 B. C. (Ne).

62: [Aristotle on void] So 140-1, citing Phys. IV, 7.213b3 ff.

62: [Aristotle foisted definition of void on predecessors] So 141.

63: [Possible = actual for eternal bodies for Aristotle] So 168, citing 203b30.

63: [Aristotle dismisses void] So 142.

63: [Timaeus outweighed Physics until middle 12th c.] J 16.

63: [Date of Timaeus] Cor 1.

63: [Timaeus a Pythagorean astronomer] Tay 1.

63: [Timaeus possibly a mathematician] Bu 3.

63: [Timaeus a figure invented by Plato] Cor 2-3.

63: [First passage from Tim.] 48E-49A Cor.

64: [Second passage from Tim.] 50B-C Cor.

64: [Never takes on any character] This understanding of the placeholder as without character, hence able to take on - hold - different values, undergoes curious transformations later in various versions of Zen Buddhism. D. T. Suzuki writes (Suz 187): "The mirror-nature of emptiness (sunyata) retains all the time its original brightness, and is never once beclouded by anything outside which is reflected in it."

Nishada Kitaro (according to Robert E. Carter) (Cart 84) speaks of the self as nothing: "and because it has no characteristics of its own, it is able to experience an indefinite number of forms as characteristics. Yet in doing so it reveals its own form: it is formless." This formlessness is apparently the Nothingness "which is prior to any dualism... which is as it is, and before it is sliced up by the dualistic logic of being and non-being... It is not simply the negation of Being, but is beyond any predication." (ibid., 83) Hence, apparently, it is best apprehended neither verbally nor intellectually but by feeling (ibid., 83).

In another version (that of the Gakudo-dojo), the formless self is

the absolute subject as nothingness (His 15), and "cannot be evaluated [sic] or devaluated, but, transcending value and non-value, still contains the value and non-value within itself" (ibid., 16). Or with a California accent: "And so all that side of life which you call unconscious, unknown, impenetrable *is* unconscious, unknown, impenetrable because it is really you. In other words, the deeper you is the nothing side." (Wat 57).

Descriptions such as these which deny their own descriptive powers hover, it seems to me, on the brink of Richard's Paradox without benefit of Gödel.

64: [Third passage from Tim.] 50E-51B Cor.

64: [Fourth passage from Tim.] 52A-D, after Cor ('reckoning' for 'judgment' my translation).

64: [Plato uses 'chora' for 'space'] Cor 200 n. 2.

65: ["The such-and-such"] 49D Cor; Cor 179 n. 2.

65: [Brahmagupta's possible debt to the Timaeus] In speaking of Hindu creation myths, Albiruni (Sa 322) says: "...they mean by creation only the working with a piece of clay, working out various combinations and figures in it, and making such arrangements with it as will lead to certain ends and aims which are potentially in it... So, for instance, they relate that

Visvamisra, the Rishi, created the buffaloes for this purpose, that mankind should enjoy all the good and useful things which they afford. All this reminds one of the words of Plato in the book Timaeus: "The θεοι, i.e., the gods, who, according to an order of their father, carried out the creation of man, took an immortal soul and made it the beginning; thereupon they fashioned like a turner a mortal body upon it."

65: [Logistic and psephos] 51D (τιθεμαι ψηφον), 52D (ψηφου λογισθεις); Cor 197 n. 2.

66: [Gerbert's symbol for 0] M 324.

66: [Theca as brand] M 403.

66: [Date of Petrus of Dacia] SK 59.

66: [Petrus of Dacia's description of brand] SK 61.

66: [Story of Adelard] SK 5 n. 2; Coc passim; S1 ii 177.

66: [Adelard playing cithara before Queen of France] Ha 492.

66: [Adelard in Córdoba] K 205.

66: [Adelard talking to philosopher in Salerno] Gib 11-12.

66: [Adelard in Cilicia] Ha 1 497.

66: [Adelard on light travelling faster than sound] Ha 2 333.

66: [Adelard's independence, and quote] K 207.

66: [N. O'Creat] Coc 62 (or his name might have been John, says Coc - but gives no grounds for this possibility).

67: [Three symbols for 0] Ca 2 51.

67: [Theca a receptacle, and significance of capital theta] SK 61 n. 3; see S1 i 204 n. 6: "Possibly from the medieval $\tau\zeta\iota\phi\rho\alpha$ from theca (teca), or from $\tau\eta\kappa\epsilon\iota\nu$ (to come to nought)." Liddell & Scott give this meaning of $\tau\eta\kappa\omega$ in Theocr; the primary meaning of the verb is "to melt away", "to reduce".

67: [Receptacle] Barry Mazur points out to me that Rafael Bombelli, the 16th century Italian algebraist, uses a symbol like a shallow cup for the variable in his L'Algebra of 1552. Was it his own invention? Could he have derived it perhaps as a rotation of the letter 'c', for 'cosa', Italian for variable? Or was he here echoing either $\theta\eta\kappa\eta$ or the Timaeus? In his introduction (pg. 8) he speaks of Diophantus, Indian and Arabic mathematicians, suggesting his reading was wide enough to have picked up what may have been a transmitted usage. See too Ca 2 128.

67: [Radulph] SK 60.

67: [Rabbi ben Ezra] SK 60 n. 4.

67: [Kha as hole in nave for axle] Mon.

SEVEN: A PARADIGM SHIFTS

68: [Bhaskara distinguishes 0 from the nine digits] SK 58.

68: [Null from *nulla figura*] M 422.

68: [Frenchman on zero putting on airs] M 422-3.

69: [Any five-year-old] There are notable exceptions, such as Benjamin Nuzzo of Newton and the five-year-olds in The Math Circle.

69. [The confusions caused by subtraction] Another nice puzzle is this. You can fit 13 guests singly into 12 hotel rooms, by having the 13th temporarily share Room One with a second guest, after which you put the third in Room Two, the fourth in Room Three and so on, until the 12th is in Room 11: at which point you escort the 13th guest to the empty Room 12.

69: [As we saw before] pg. 19.

69: [Mahavira saying zero becomes the same...] Ra English 7.

70: [Imaginaries as far back as Heron and Diophantus] S1 ii 261.

70: [Calculation with imaginaries in the Renaissance] S1 ii 262.

70: [Wallis in 1673, a cryptographer] S1 ii 262, Ca 1 184.

70: [Paraphrase from Wallis] S1 ii 263.

71: [Brahmagupta on subtraction of a number from itself] Col 359
(= Ch. 12, sec. 2 §19).

71: [Brahmagupta on addition of 0] Col 359 (= Ch. 2. sec. 2 §19).

71: [Brahmagupta on subtraction from 0] Col 359 (= Ch. 2 sec. 2 §20-
21).

71: [Bhaskara on operations with 0] Col 136 (= Ch. I sec. III §12).

71: [Date of Lilavati] Col: Introductory Dissertation to his
translation, iii.

71: [Passage from Lilavati] Ban 7.

72: [Mahavira on multiplying by and subtracting from 0] Ran 6.

72: [Brahmagupta agreed with Mahavira] Col 359 (=sec. II §22).

72: [Bhaskara agreed with Mahavira] Col Vija-Ganita 137 (= Ch. I sec. III §14).

72: [A number divided by zero is unchanged] Ran 6.

72: [Mahavira "obviously thinks..."] Ran 6.

72: [Passage from Brahmagupta] Col 340 (= Sec. II, §23, 24); Dat 241.

72: ['Khacheda' from 'kha'] Dat 243.

73: [Passage from Bhaskara] Col 137 (= Ch. I §III 3.3.0).

73: ['Khahara' synonymous with 'khacheda'] Dat 243; Col 138.

73: [Sundial analogy] This commentator is Krishna. For his date, see Col Introductory Dissertation.

74: [You can't legitimately divide by zero] While the question of division by zero is settled in the eternal mind of mathematics, it may not be equally secure in this and that mortal mind. Some, like the Indian genius Ramanujan, knew at age four that you couldn't divide by zero. Some kept their fingers crossed: "There is what may perhaps be called the method of optimism," wrote Maxime Bôcher in 1901 (Congress of Arts and Sciences, St. Louis,

vol. I, pg. 422), "which leads us either willfully or instinctively to shut our eyes to the possibility of evil. Thus the optimist, who treats a problem in algebra or analytic geometry will say, if he stops to reflect on what he is doing: 'I know that I have no right to divide by zero; but there are so many other values which the expression by which I am dividing might have, that I will assume that the Evil One has not thrown a zero in my denominator this time.'"

Even a Justice of the Supreme Court can think that division by zero yields infinity. "Supreme Court Chief Justice William Rehnquist" (according to an article in the Denver Post) "was quick to solve a riddle posed recently by the Washington Post. The Post asked which automobile was referred to by a licence plate that read 1 DIV 0. 'I believe it refers to an Infiniti, since when you divide 0 into 1 the result is infinity,' wrote the nation's top-ranking Justice, who was the first to reply with the correct answer." My thanks to the perceptive Cecil Munn, of Fort Worth, Texas, for bringing Chief Justice Rehnquist not only to my attention but, much more significantly, to the bar of Abstract Justice.

74: [a/0 doesn't mean anything] What does the following argument of the young Leibniz (see pg. 50 of J. M. Child's The Early Mathematical Manuscripts of Leibniz) tell us about the diabolic power of analogy? Leibniz knew that while the infinite series

$$1/1 + 1/2 + 1/3 + \dots \text{ diverges - or as we say, } = \infty,$$

he also saw that

$$1/1 + 1/3 + 1/6 + 1/10 + \dots = 2/1,$$

$$1/1 + 1/4 + 1/10 + 1/20 + \dots = 3/2$$

and so on. Reasoning backward, then, he concluded that

$$1/1 + 1/2 + 1/3 + \dots = 1/0,$$

so that this expression must be the same as ∞ .

On the subject of division by 0 and the diabolic, an Irish correspondent, Rory O'Brien, is ingeniously engaged in trying to extend the number system by the addition of a symbol for that illegitimate operation. Taken from the spreadsheet warning "#DIV/0!", and with a fine sense of the power of names, Mr. O'Brien calls this new element 'divil'. His question is this: if we follow the path that zero itself trod, from an outcast to a member of the numerical community, what would happen were we to invest this divil with an existence on a par with that of other numbers? Certainly not 0 itself, nor an unknown nor an indeterminate, divil combines, under multiplication, with any number n to yield divil again (and so resembles 0): for any such error magnified by n remains an error. But what is divil^2 , or $\text{divil}(\text{divil})$?

74: [Lack of proofs in India] K 198.

74: [Passage from Albiruni] Sa 177. Note that Albiruni's name is variously spelled: at times (as in Sachau's translation)

"Alberuni", at times Al-Biruni; and accented at times with long marks, at times with \hat{i} and \hat{u} .

74: [[Hindu comment on Albiruni] Sa 23-24.

74: [Bhaskara on 0^2 and $\sqrt{0}$] Col Ch. I sec. III §14, 15.

74: [Hindu extension of operations to irrationals] K 185-6.

74: [Problem from Bhaskara] Ca 2 83.

74: [Bhaskara shying away from imaginary numbers] Col 135 (= Ch. 1 sec. 5).

74: [Bhaskara on negative roots] Ca 1 93; B 43-4.

74: [Mahavira's equation] K 187.

76: [Passage from Homer] Od. IV.791-3 (trans. Ellen Kaplan).

77: [Passage from Virgil] paraphrased from Aenead VI.305-310.

77: ["What here we are..." "Was hier wir sind, kann dort ein Gott ergänzen". From a fragment written in his madness.

77: [Heraclitus] Fragment 93 (in Plutarch, de Pyth. or. 21, 404E).

77: ["A new threshold..."] Ru 181.

78: [Shift in center of gravity] We know that a paradigm shift has taken place when people think so easily in the new terms, and look so instinctively (it seems to them) from the new point of view, that the displaced viewpoint is hard to understand, and even when grasped seems absurdly primitive or unnatural.

EIGHT: A MAYAN INTERLUDE

80: [Zero symbol: tattooed man] after "Detail of a plaque from Palenque, showing a full-figure representation of '0 kins'" (fig. 28-23) in I 420.

80: [Dates and place of Maya culture] I 397; Ho 21.

81: [August 13, 3114] So in Sch 319. Ho 51 gives 13 August 3113, Coe 1 46: 11 August 3114 "in the Thompson correlation"; I 415-416: "It has been calculated (in what is known as the Thompson Correlation) at August 12, 3113 B. C." A remarkable correlation, then.

81: [Biographical details of Ussher] EB article 'Ussher'; Carr (passim).

81: [Ussher's dating] This is the date to be found in Ussher's Chronologia Sacra (Oxford 1660), pg. 45. I find in his Annales

Veteris Testamenti a Prima Mundi Origine Deducti (London 1650), pg. 1: "In principio creavit Deos coelum et terram quod temporis principium (juxta nostram chronologiam) incidit in noctis illius initium, quae XXIII. diem Octobris praecessit, in anno periodi Julianae 710." That is: "In the beginning God created heaven and earth, which beginning of time (according to our chronology) fell in the start of that night, which preceded the 23rd day of October in the year 710 of the first Julian period." (trans. Ellen Kaplan). This is confirmed (at the slight expense of a year's ambiguity) in Ussher's preface to this volume, on his unnumbered pg. 5: "...perplexis chronologorum difficultatibus tandem superatis, in hanc concessi sententiam: à vesperâ primum mundi Diem aperiente ad mediam noctem primum Christianae aerae diem inchoantem, annos fluxisse 4003. dies 70 & horas temporarias 6." Gould, however, gives Ussher's date as noon on the 23rd [Go 89], and Keller [Ke 414] as 9 a.m. on the 26th. A day, or four days, off in such matters is the difference between the creation of the universe and the dividing of the firmament from the waters on the one hand, the creation and the making of sun, moon and stars on the other: no small matter. Will the tangled chronologies never in fact be unbraided, and the precise time of the definitive beginning of things always be hidden from us by Eastern, Western and Southern gods to keep us in our (indefinite) place? Ussher's dating, Ellen Kaplan points out, might not have been all that eccentric, since what he may have been calculating was the date of Rosh Hashanah in 4004, and the time on that day when the first star could be seen.

81: [Maya recorded important dates in terms of zero day] Sch 317
ff.; I 405 ff.

81: [Units up to an alautun] I 411.

82: [Shells, etc. as symbols for zero] after Th 180 passim, and I 421
(fig. 28-24), 424 (fig. 28-25) and C 362. The proliferation of
Mayan symbols for zero is reminiscent of our own multitude of
names for it. In their case a playful tendency toward rebuses and
puns (cf. Ho 25) may have been involved; for them and us alike,
however, an accumulation of names around deep points, whose
"real names" are left unspoken out of dread (the 99 names of
God), is, I think, primarily responsible.

82: [Formal significance of dating] I 421, citing Guitel.

82: ["Phantom" days] I 409.

82: [People call the Haab "the vague year"] So, for example, Coe 1 44,
Sch 318.

83: [Pol Pot's Zero year] Mentioned in Tony Hillerman's Finding Moon
(Harper Paperbacks, 1995) pg. 240.

84: [No activities during the "phantom" days] I 409, quoting Diego de
Landa.

84: [A fresh sacred year commenced] It is as if we were to take our day- and month-names with greater mythological seriousness, and start a new year only when January 1 fell on a Sunday. Since this happens every six of our current years (thanks to leap year), these Mythological Years would each be 2,191 days long. We too, of course, have a variety of calendars, public and private (the fiscal, school, solar and religious years, for example); what is striking about the Maya is their need to record important dates in so many of their calendars at once.

85: [Gods of the underworld ruled by the Death God] Sch 51, 61 n. 61.

85: [Freud on repetition] In Beyond the Pleasure Principle.

86: [Light patina of conjecture] With the nature of light as an exemplar, my attempt is to make just a single path through averaging over the divergent conjectures of others. Such a quantum electrodynamical approach to history might make the best use of the partial quality of our knowledge and communication, the overlaps of ignorance and the multiplicity of motives, the false emphases of fashion and the feathering out of hypotheses to close the gaps in evidence.

86: [Fear that time might stop] Sch 320.

86: [Offering up essence of vitality] Implied in Sch 220.

86: [Inducing the gods to pick up again the burden of months] Sch 175-185, esp. 176.

87: [Venus and Haab] Coe 1 160.

87: [Lunations and Tzolkin] Coe 1 159.

87: [Synodic year of Mars] Coe 1 160.

87: [Conjecture about relevance of synodic year of Mercury] Coe 1 160. Mercury's synodic year is actually 115.88 days.

87: [Mutilations every five years] Sch 182.

87: [A pair of scholars] Linda Schele and Mary Ann Miller (Sch xiii, 175, 253).

87: [A system of dating] The Coba Cycle. See Sch 320.

87: [Time's beginning] By Schele's reckoning (Sch 321), 1.42×10^{38} years to beginning of time; by Houston (Ho 49), 1.42×10^{32} . The confusions are reminiscent not only of Bishop Ussher but of the different values of the same words for powers of ten in Buddhist counting. But what are a few powers of ten sub speciae eternitatis? Ellen Kaplan's reckoning, 1.42×10^{27} , is the one given in the text.

87: [Our own culture's 1.2×10^{10}] Kra 56.

87: [The Long Count itself thought of as cyclic] Sch 17; Coe 1 45-46

87: [Birth of ruler cyclically coinciding with that of a mythical
ancestress] Coe 1 168.

88: [The birth a rebirth] Sch 321.

88: [Pacal] Coe 1 103.

88: [Ritual ball-game] Sch 249, 251-254, 257.

88: [Zero as other contestant] Sch 251.

88: [Tearing off of lower jaw] Sch 54, 61 n. 61.

NINE: MUCH ADO

90: [Hindu numerals in Baghdad by 773] SK 92.

90: [Al-Khowarizmi's work on arithmetic] SK 97 on his writing it,
K 192 for his date. At about this time too the Jewish
astronomer Abu 'l-Teiyib, Samed ibn 'Ali, wrote in Baghdad a
work on Hindu arithmetic (SK 97).

90: [Interest in learning, translation into a common tongue, spread of learning and three centers of learning] M 408-9.

91: [Avicenna] story from N1 24; details from EB art. "Avicenna".
"A penny" = 3 dirhems. Those who cannot encompass this figure will find it corroborated in the solemn Chapter X of Tristram Shandy's fourth book.

91: [Passage from Amy Tan] Tan, Amy, The Hundred Secret Senses (Ivy Books, 1995) pg. 306.

91: [Indian source of Chinese 0] I 392.

91: [At times a dot for zero in China] Nee 12, where Needham says that in the Khai-Yuan Chan Ching (718-729), a compendium of astronomy and astrology, "Whenever there is an empty place in a column..., a dot is always placed [to signify it]."

91: [Circles within circles] in an eleventh century Cham inscription, from the southeast coast of what is now Vietnam: I 441-445; FI 69 (similar zeros in Cambodia, Sumatra and Java).

91: [Ling] Nee 16. Needham comments that the ancient meaning of 'ling' is the last small raindrops after a storm, or drops of rain remaining afterwards on objects. He says this was its sense in the Book of Odes, and remarks that he has never seen it in a

mathematical sense earlier than the Ming (1368 to 1644), and that the earliest book in which it is prominent is the Suan Hsüeh Pi Li Hui Thung ("Rules of Proportional Exchange") by Lo Shi-Lin (1818). cf. Li 69: "it is in the Shu-shu Chiu-ching [c. 1647] that we find, for the first time in China, the zero in print..."

91: [Taoist emptiness once conjectured as source of 0] Nee 16; and Nee 11-12 says "It may be..." and "We are free to consider the possibility...". Michio Yano [Y] says: "Needham's book is out of date. There is no evidence to support [the statement that there was a Taoist component in the concept of sunya, or that the hollow circle travelled from China to India]."

92: [Illustration of mancala board] After "A Mancala Board from the Maldive islands in the Indian Ocean", pg. 110 in A. B. and E. F. Provenzo, Favorite Board Games You Can Make and Play (reprinted Dover 1981).

92: [Names for mancala] *ibid.*, 115.

92: [Conjecture that zero's origin would be buttressed had mancala been imported by Arabs into Africa] This beautiful idea is due to Ellen.

92: [Mancala boards at Karnak and in rock ledges along caravan routes] Provenzo (*op. cit.*) 110.

92: [Cowrie shell and Mayan zero] Less mad, and just barely probable, is that the shell-form of the Mayan zero came from the counters (not their absence) that Mayan merchants may have used in "computations performed on the ground or on a flat surface" by means of counters, presumably cacao beans or maize grains (so Coe 1 185, citing Landa as an "irreproachable source").

92: [Zero came West by 970, perhaps even 850] 970: With Gerbert (SK 112), or as early as 850: SK 101, who instance the travel of merchants and give a lively account from Ibn Khordadbeh's Book of Roads and Provinces from 850: "The Jewish merchants speak Persian, Roman (Greek and Latin), Arabic, French, Spanish, and Slavic. They travel from the West to the East, and from the East to the West, sometimes by land, sometimes by sea. They take ship from France on the Western Sea, and they voyage to Farama (near the ruins of the ancient Pelusium); there they transfer their goods to caravans and go by land to Colzom (on the Red Sea). They there reëmbark on the Oriental (Red) Sea and go to Hejaz and to Jiddah, and thence to the Sind, India, and China. Returning, they bring back the products of the oriental lands.... These journeys are also made by land. The merchants, leaving France and Spain, cross to Tangier and thence pass through the African provinces and Egypt. They go to Ramleh, visit Damascus, Kufa, Baghdad, and Basra, penetrate into Ahwaz, Fars, Kerman, Sind, and thus reach India and China."

93: [Words for zero]

θηκη SK 61 n. 3

theca SK 58 n. 6; S3 76 (quoting Johann Huswirt's Enchiridion,
Cologne 1501)

teca SK 59 n. 1

tecca (Italian) SK 65 n. 2

sipos SK 57 n. 4, 60 n. 4

omicron (French) SK 61

chilerae (Latin) SK 62 n. 6

psephos (Greek = stone)

τζιφρα SK 57 n. 9

τζυφρα SK 62 n. 6

τζυμφρα SK 62 n. 6

tziphra SK 57

tsiphra SK 57 n. 4, 62

as-sifr M 401

sifr M 401

zefirum (Fibonacci) SK 57; M 401

zefiro (Italian) M 401

zephyr SK 59

ζεουero (Italian 14th c.) SK 58

ζεφiro (Italian) SK 59 n. 5

zefro SK 58 n. 4

zepiro SK 58 n. 3

zepro SK 58 n. 4

zevro SK 58 n. 4

zeviro SK 58 n. 4
zevero SK 58 n. 4; M 401
zeuero SK 58 n. 1
zero (Italian, English, German) M 401; SK 59
ciphra SK 57 n. 4
cifra (Latin, 16th c.) SK 57 n. 4
cyfra Sk 57 n. 4, 62 n. 4
zyphra(s) SK 62 n. 6
ziphra (English) SK 62
ziphera SK 62
ziphyra SK 62
zypher (German 16th c.) SK 58 n. 5
ziffer (German 15th c.) M 401
sifra (Hebrew) SK 60 n. 6
siphra Sk 62
syphra SK 62 n. 6
cifra SK 60 n. 2; S3 76 (op. cit.)
cifrus SK 62 n. 6
chifras SK 62 n. 6
cyphar (English 16th c.) SK 58 n. 5
cipher (English) SK 58, 59, 62
sypher (Thomas Usk)
chiffre (French 14th c.) M 401
chifre SK 120 n. 3
cyffres SK 120 n. 3
giffre SK 120 n. 3
galgal (= wheel: Hebrew) SK 57 n. 4, 60 n. 5

circulus (Italian) SK 58, 60 n. 6
circulos SK 61 n. 1
circul SK 58 n. 6; S3 76 (op. cit.)
circolo (Italian) SK 61 n. 2
circularis nota SK 61 n. 1
circular note (German 16th c.) SK 61
figura circularis SK 61 n. 1
figura privationis SK 61 n. 2
rota SK 57 n. 4
rotula SK 60 n. 4
das Ringlein (German) SK 60, 60 n. 7
nihil SK 62 n. 1
figura nihili (English 13th, German 16th c.) SK 58, 58 n. 6,
62 n. 1; S3 76
figure de nulle valeur (French 16th c.) SK 61 n. 5
nulla (Italian) SK 59 n. 6, 62 n. 2
nulle (French) SK 60 n. 6
nullo (Dutch) SK 62 n. 2
null SK 61
rien (French) SK 62, 62 n. 3
niente (Italian) SK 62 n. 2
naught (English) SK 56
aught (English) Pa art. "naught"
O (English) SK 50

93: [Sifr or as-sifr as ancestor] Sab; SK 57; M 401; S1 ii 71.

93: ['Psephos' influencing such zero-names as 'sipos'] M 325.

93: ['Siphra', 'cifra' allied to counting] SK 57 n. 4.

93: ['Siphra' allied to 'tsiphra', crown] SK 57 n. 4.

93: [Connection of 'sifra' to 'crown' and 'counting'] "Samekhs and tsades tend not to interchange [as they would have to have done here, between 'sifra' and 'tsiphra']. Tsade-pa-resh means many things and does include roundness in its lineage. The derived word 'tsphirah' appears in Isaiah 28:5, where it is translated by 'diadem' or 'crown', and in Ezekiel 7:7, 10, where - possibly on the basis of an Aramaic sibling, the Authorised Version translates "morning" (other translations have "doom" or "time of year"). Scholars strain to relate these, as e.g. Luzzatto, who explains that the word refers to the round of the year. The word appears again in the Mishnah (Kelim 16:3), where it again apparently signifies a cycle or twist. The Lexicon of Old Testament Hebrew links 'tsephirah' to the Arabic 'dafara', meaning 'plait' or 'braid', and also gives the meaning 'rim'. In modern Hebrew, the principal meaning is 'siren' or 'alarm', deriving, I would guess, from another offshoot of the root tsade-pa-resh, meaning 'bird'" Tan Dec. 16 1998.

93: [Descent in Italy of terms to 'zero'] M 401, SK 58 n. 4.

93: ['Cipher' for secret writing] Pa art. 'cipher'. Commynes, 1486,

was the first to use the word so, according to Partridge.

96: [Manichaeism most hated and feared] Bro 46.

96: [Void identified with evil] Wi 239, citing Valentinus: το κενωμα.

96: [Evoking forces into being] Run 13.

96: [Manichaeism] For good summaries of the doctrine and history, see Run 12-18 and Bro 46-60.

96: [calling 0° 1°] N 2 600, where Neugebauer says that Aries 0° is usually called 'first degree' or 'day 1' of Aries, "a terminology which upset ancient and modern arithmetic alike." In his footnote 23 to this sentence he cites Pliny's Natural History II 184, where it is stated that at Meroe the sun reaches the zenith twice, at Taurus 18 and at Leo 14, "i.e., 43° distant from Cancer 1°, instead of Cancer 0°." In the text N. continues: "Since this norm has been used consistently by Hipparchus and Ptolemy it remained the basis of reckoning in mathematical astronomy, Aries 0° being the vernal point, i.e., the intersection of equator and ecliptic. We know from Hipparchus that the majority of the 'old' mathematicians divided the ecliptic in this form. This statement agrees with sources still available to us; Euctemon (about -430) placed all four cardinal points on the 'first day' of their respective signs. The same norm holds for Callippus (about -330) and is underlying the era of Dionysius (beginning

-284/3). As far as we know this norm is attested nowhere in Babylonian astronomy."

Other instances of 1 for 0 mentioned by Neugebauer in N2: (pg. 159): "Out of six observations cited by Ptolemy [3rd c. B. C.], four use an otherwise unknown astronomical "Era Dionysius"... [this] counts from a year 1 which corresponds to 285/4 B.C." (pg. 279) on Hipparchus's terminology: "...30° (or 0°) in our notation is called 'end of the sign'... 'beginning'... must be interpreted as 0°..."

(pg. 582 n 22): on Geminus in his Isagoge: "Also reminiscent of Hipparchus in calling 0° of a sign its 'first degree'."

(pg. 596 n 19): "...Rhetoricus (≈ A D. 500) who says 'at the 30th degree of Cancer, that is at the 1st degree of Leo' ... the traditional mixup between 'first degree' and 0° or 1° mars ancient as well as modern interpretations..."

The peculiarity of this usage may be softened by considering our own willingness to speak equally of "going back to square one" and "returning to zero".

96: [Roman counting] Gra 91.

96: [Prima materia] Ju 45.

97: [Alchemical transformation] Ju 364-5.

97: [Mandala as circle] Ju 42 n. 2.

97: [Zero the number of the Tarot's Wise Fool] Wai 152-5.

97: [Arabic numerals as symbols for initiates] M 423.

97: [Noviomagus' numerals] Ca 2 69. Illustration after figures selected from the plate published in Heilbronner's Historia matheseos (1742), as reproduced in Ca 2 69.

97: [Source of Noviomagus' numerals] Ca 2 68.

97: ["Un nombre donnant..."] vdW 59; M 422.

97: [Aquinas' objection] Aq 435: Q. 45 art. 2 obj. 4.

98: [Aquinas' reply] Aq 436: reply to objection 4.

98: [Crafte of Numbrynge] SK 11, 149 (where the last paragraph has no sentence break despite a full stop after '10'. I have begun a new sentence with 'here'); SK 87 for it being the earliest English text on the subject; date: S 1 ii 32.

99: [Knuth's notation] We 209-210.

99: [Hierarchy in Middle Ages] Huy 46 ff; and see Lovejoy's Great Chain of Being passim.

99: [Vanishing point] On the vanishing point as a visual zero,

see R 19.

99: [Donne] Don 471 (= Sermon 25, The Spital, April 22 1622). Lest you think that Donne somehow distinguished between nothing and zero, observe that he also said: "...If there be nothing of the next world at the end, so much peace of conscience, so much joy, so much glory, still all is by *nothing* multiplied, and that is still nothing at all." (446 = Sermon 31).

99: [Thomas Usk] S1 ii 9 n. 2; SK 121 (Testament of Love is dated 1387).

100: [Cologne] By implication: for Blakney takes this sermon to be late (BI 328 n. 1), and Eckhart preaches in Cologne on Feb. 13, 1327; on Feb. 22 of that year he is notified that his appeal to Rome (against charges of heresy) had been denied - and this is the last notice we have of him as alive; by March 17, 1329, the bull of John XXII speaks of him as dead (BI xxiv).

100: [Eckhart father of German idealism] BI xiii.

100: [That all creatures are nothing] BI 294 (= Defense IX, 30; IX, 13).

100: [To be empty of things is to be full of God] BI 85.

100: [Why God is nothing] Wi 335, 337. Huy 204 suggests an analogy of God as spirit to spirit as invisible breath.

100: [God immovably disinterested in his creation] BI 85.

100: [Nihte] Blakney stresses translating Nihte by a mathematical zero: BI 83 and 316 n. 5.

100: [God rarefied enough to go into zero] BI 83.

100: [Beyond humility to God] BI 83, 85.

100: [That his vision was that moment discovered to him] BI 232, 328 n. 8.

100: ["God and I are one"] BI 232.

100: [Multiplication methods] M 441-2: *multiplicare per crocetta*, *per ripiego* (by factors), *per colonna* (as we do), *per gelosia*.

100: [Passage from Adam Riese] M 442 "a lot of brains" = "Sie nimmt viel kopffs."

100: [Date and title of Riese] M 430.

100: [Nuremberg father to his son] M 428.

101: [Difficulty of iron division] M 331.

101: ["Faire par algorithme"] M 423.

101: [Scarcity of pencils and paper] SK 136-7, M 427-8: paper-making introduced into Europe in the 12th c., cheap paper not until the 19th c.; pencils of modern type date from the 16th c.
M 224: paper appeared in Germany in the 14th c. Cr 219: paper in Italy in the 13th c.

101: [Problem from 1489 textbook] M 429-30, its answer: M 430, verified by Ellen Kaplan.

101: [Prosdocimo de' Beldomandi] paraphrased from SK 137 n. 1.

102: [Availability of books in the vernacular] SK 150.

102: [Printing of textbooks] M 431: Rechenbücher (books of reckoning) were among the earliest printed books.

102: [Dates on monuments] SK 137.

102: [Dates on coins] SK 139, M 439.

102: [Zero too easily turned into a 6 or 9] We 1 24, vdW 60-61.

102: [Ordinance of 1299 in Florence] M 426.

102: [Venetian text on bookkeeping] M 426-7.

102: [Stationers required to write prices in letters] SK 133.

102: [Frankfort calculators to abstain from digits] vdW 61, M 427.

102: [1594: canon in Antwerp] vdW 61.

102: ["A horned beast..."] Gautier de Coincy (1177-1236), quoted in SK 120.

103: [Numerous calendrical reforms] L1 241 on Bede; L1 146 cites Dionysius Exiguus, Abbo's corrections, Ademard. G 141: Caesar; 143: Clavius; 153: Meton. 103: You will find a brilliant explanation of the reasoning behind the calendar invented by Dionysius Exiguus in the sixth century A. D. (the calendar by which we now reckon our years) in Christian Marinus Taisbak's "Dionysius, Zero and the Millennium: the Real Story" (in Focus: the Newsletter of the Mathematical Association of America, vol 20 no. 6, August/September 2000, pp. 14-15). He shows that Dionysius cleverly devised the dating he did for convenience of reckoning rather than for historical accuracy.

103: [Cassini] N2 1062 n. 4. On his life: EB vol. 5 p. 459.

103: [Hesitant appearance of year 0] N2 (as above) says that in his Élemens d'astronomie, published in the same year, Cassini speaks ambiguously of "...l'année 146 avant l'époque de Jesus-

Christ dans la forme Julienne, et de l'année 145 avant Jesus-
Christ, suivant notre manière de compter..."

103: [Landes' observation about the significance of the end] L1 291.

103-4: [Disappointed peasants in mountain meadows] S1 i 327-8 on
the followers of Michael Stifel, the mathematician-
theologian.

104: [Time before and after Judgment days which have passed] L2 :
"The sense of linear time extending into infinity, which gives
us our incredibly accurate system of measuring, is the result of
accurate countdowns which 'just kept on ticking'."

104: [Landes: "longer temporal horizons"] L1 288.

104: [The more distant the day, the more it could be dwelt on in
hope] L1 290-291.

104: [Temperate zone] L2, where Landes cites St. Augustine's "Three
Opinions of the End" for 50 to 300 years as the length of this
temperate zone.

104: [A particular day at the end of time] L1 194: the End would come
when Good Friday fell on March 25, for computists in
Lotharingia.

104: ["A measure of wheat for a penny..."] Rev. 6:6.

104: [Heavens shutting up like a scroll] Rev. 6:14.

104: [Stars falling like ripe figs] Rev. 6:13.

104: [The four horsemen] Rev. 6:8.

104: [1000, 1260, 1533, 1843] L2.

105: [Matthew and Mark] Matt 24:36; Mark 13:32.

105: [St. Augustine and others putting off dreaded day] L1 292, esp.
n. 12; L2, on St. Augustine's "Three Opinions of the End".

105: [Bede on the end] L1 94-5.

105: [Different ways of dating] anno domini; anno mundi;
see L1 passim but especially 289-293.

105: [Errors in calculations] L1 94 on Abbo of Fleury, in 983, finding
errors in Bede's reckoning.

105: [Ademar] L1 passim.

105: [Ademar's substitution] L1 146.

105: [The anti-apocalyptic strong in writing] L1 288-9.

105: [The moon in Christian symbolism] Fe 45.

105: [Midwinter fires of Beltane (as mistakenly appears in the book's first printing) I am grateful to Snoo Willson for pointing out that the midwinter solstice was - and often still is - marked by Yule fires, while the fires of Beltane burn at the spring equinox.

106: [Subordination of cyclic to linear in Western music] Landes (L2) suggests looking in particular at the Notre Dame school of music in the 11th-12th centuries.

106: [Faust] So in the "Prologue in Heaven" to Part I of Goethe's Faust, 328-9:

Ein guter Mensch in seinem dunkeln Drange
Ist sich des rechten Weges wohl bewusst.

106: [History walks on human feet] I am indebted to Michael Kaplan for this image.

106: [Good ol' boy] EB art. "Leonardo of Pisa" translates 'Bonaccio' as "good, stupid fellow"; M 425: "Man of good spirit".

106: [Villa Dei] SK 11.

106: [Sacrobosco and his book] SK 133-4 on Sacrobosco, Ca 2 51 on the name of his book.

106: [Lecture notes] SK 134 n. 6.

107: [Algor] SK 135.

107: ['Teca' in Sacrobosco] Ca 2 51.

107: [Difficulty of some over unwieldy passages] S ii 15 and 15 n. 2.

107: ["Prima significat..."] S ii 80 n. 2.

107 [Filius Bonacci]: How much of a good ol' boy could Fibonacci's father have been? He was a terdjiman: interpreter to traders with the Muslims, a position which must have required the delicacy and tact with people that his son showed for numbers.

107: ["Blockhead"; "traveler"] SK 130.

107: [Illustration: Fibonacci] From a modern engraving, in S i 215, where Smith notes: "The portrait is not based on authentic sources."

107: [Fibonacci didn't just transcribe] Ca 1 120.

108: [A historian of mathematics] S i 217-8.

108: [Little merchant from Pisa] S i 217 n. 1, quoting from Libri's Histoire, I, xvi.

108: [Fibonacci speaks only of 9 Indian figures but the sign for 0] M 425. Sacrobosco also had only 9 figures; see the illustration in S i 223.

108: [Crafte of Nombrynge] SK 87 n. 5.

108: [Passage from Chuquet] Ca 2 100-101; my translation.

108: [Tally-sticks used as late as the 18th c. in England] M 237.

108: [Use of tally-sticks] M 223-25; S ii 193.

109: [Millers' knots] M 255-6.

109: [Illiterates could reckon with their fingers] M 201.

109: [Finger-reckoning up to a million] M 208 on Bede's system.

109: [Passage from Luther] M 366.

109: ["Les courtisans..."] M 366, for date as well.

109: [Plutarch's variation] I 263, citing Plutarch on Orontes.

110: [Invention in Italy of double-entry bookkeeping] Ori 109.

110: [Double-entry invented before 1340] Cr 210.

110: [Mattäus Schwartz] Bax 136.

110: [Schwartz worked for Fugger] Bax 133.

110: [Passage from Schwartz] Bax 136 and 231-2 n. 35.

110: [New vocabulary, and especially double-entry, leading to conservation laws] This elegant idea is due to Ellen Kaplan.

111: [Luca Pacioli] S i 252.

111: [Value the leveller] So, obviously, in Marx; but typically in Mach too: "The sciences most highly developed economically are those whose facts are reducible to a few numerable elements of like nature." From The Science of Mechanics (Open Court, 1960) pg. 582.

111: [Merchant's accountant using a counting board] so Datini. Or 77.

111: [Ledgers kept in Roman numerals] M 426.

111: [Ledgers kept in words] Cr 205.

111: [New symbols creeping into Datini's accounts] Cr 205.

111: [New symbols carried entries from invoices to account books]
M 427.

111: [Pacioli and Roman numerals] Cr 222.

112: [13th century German ballad] M 432.

112: [Gerbert's counters] M 325.

112: [Gerbertistas] M 331.

112: [French boards replace lines by unmarked counters] M 367.

112: [Counters on English boards] M 368.

112: [Passage from and date of Ulrich Wagner] vdW 61.

112: ["On the lines with quills"] M 340.

112: [Introduction for to Lerne...] M 337.

113: [Illustration in Riese] M 340 fig. 175.

113: [Passage from Riese] M 431.

113: [Woodcut of 1535] M 435.

113: [John Palegrave] Cr 112.

113: [Zero overcomes the abacists] M 331.

113: [The woodcut in the Margarita Philosophica and Dürer's
Melencholia]

1. Panofsky (Pan 161) says that the "Typus Geometriae" in Reisch's book - he cites editions of 1504 and 1508 - "...includes nearly all the devices appearing in Dürer's Melencholia I."
2. We find these resemblances between the Typus Arithmeticae (M 350, vdW 60, S1 i 325) and the paired (Pan 156) St. Jerome and Melencholia of Dürer:
 - (a) The composition of the two engravings, taken as one picture, resembles that of Reisch's woodcut, especially
 - (b) The placing of the vanishing point, near the right-hand margin of the St. Jerome (cf. Pan 158), where the figure of Arithmetic would be; and the unusually shallow depth of the two;
 - (c) The outstretched hands of Arithmetic, and the scrolled "Typus Arithmeticae", resemble the bat-like imp in the Melencholia with its held title;
 - (d) St. Jerome's table resembles that of Reisch's Boethius;

- (e) The shadow of this table is similarly cast, and the light left, dark right walls in the Reisch are imitated in the dark left wall of the St. Jerome, the light right side of the Melancholia.
 - (f) The windows are similar;
 - (g) Much more significantly, the expression of puzzled disappointment on the face of Dürer's winged figure resembles that on Reisch's Pythagoras; while an expression of content characterises both Reisch's Boethius and Dürer's St. Jerome.
 - (h) St. Jerome's and Boethius' right hands are similarly placed.
 - (i) Accidentally, perhaps, the second and third cells of the top row in the magic square in the Melancholia contain the numbers 3 and 2 - which are paired on Arithmetic's gown.
3. More far-fetched, perhaps, is this: the geometric sequences displayed on Arithmetic's gown: 1-2-4-8 and 1-3-9-27, Reisch certainly found in Plato's Timaeus (35 B-C): see Bury's excellent note 2 (Bu 66) and its continuation in his note 1 to 36A (Bu 68-71). Now in this latter note, Bury points out that the complete series of terms (in the Timaeus), from 1 to 27, has 34 terms in all. 34 is the sum of each row, column and diagonal of Dürer's magic square. This may be coincidence. It may be a coincidence Dürer relished. If you want a magic square made up of the first 16 integers you have no leeway. But just this meeting

- of necessity with significance may have appealed to him.
4. As for the numbers on Pythagoras' abacus, 1241 and 82 (that the numbers are such and not reversed follows from the way line-boards were set up at the time, with the units line nearest the reckoner - see M 350): Barry Mazur points out that 82, being twice 41, may be meant to show the abacist in the midst of multiplying 1241 by 2, in the time that the algorist has swept through a much longer calculation. I don't see the significance, however, of 1241 and its double, 2482, except that between them we have the 1-2-4-8 pattern again.
 5. Might Dürer have chosen St. Jerome to replace Boethius not only because he translated from middle eastern languages into Latin, as Boethius had from Greek; but because there was a link through calculation between them: St. Jerome's careful description of finger-arithmetic in his commentary on Matt. 13:8 [M 209]?

114: [Reisch's Margarita Philosophica] M 350; vdW 60; S i 325.

Illustration after reproduction in M 350.

TEN: ENTERTAINING ANGELS

116: [Baucis and Philemon] Ovid, Metamorphosis viii. 624-724.

116: [Abraham and Sarah] Gen. 18. 2-16.

116: [Entertaining angels unawares] Epistle to the Hebrews 12.2.

118: [Diophantus inventing exponential notation] Ca 2 71-2, 342-3.

119: [Oresme] Ca 2 91-93.

119: [Chuquet] Ca 2 100-103.

119: [Chuquet invents a^0] It has been suggested (as by Alf van der Poorten, Notes on Fermat's Last Theorem (Wiley, 1996), pg. 56, at the apparent instance of Don Zagier (personal e-mail correspondence with van der Poorten, 4/18/99 and Zagier, 4/20/99) that Diophantus "copes with zero" in the form of x^0 , since he writes expressions with descending powers of x , the power indicated by a letter with superscript prefixed to his variable ς , and a final constant, prefixed by M with a superimposed o , *as if this prefix stood for x^0* . But recall from pg. 18 that Diophantus used that symbol for "monad", to separate units after it from the myriads before - or in this case, from expressions with variables. There is no evidence that he thought of units (constants) as coefficients of $x^0 = 1$. In fact, he had ample opportunity to say he did, at i.2.3 - 6.21 (T ii 523), where he describes his exponential notation - but he doesn't take it. Instead, he says:

"The number which...has in it an undetermined multitude of

units, is called arithmos, and its sign is ζ. There is also another sign denoting the invariable element in determinate numbers, the unit, and its sign is M with the index o."

Zagier tells me (e-mail of 4/23/99) that in speaking of Diophantus he had only meant to describe his work in modern terms, not to attribute such a use of zero to him. Clearly the restless imp of zero has been up to its old mischief, living as it does in the interstices of language. The laurel crown for x^0 still rests on the head of Dr. Chuquet.

119: [Michael Stifel] S i 327-8; Ca 2 139-147; S ii 520-21, where this translation is given of a Latin passage ending "...clausis oculis abea." Ellen Kaplan suggests that the long a of 'abea' may have been a misreading of a superscripted 'm', giving 'abeam', the jussive subjunctive "let me depart".

120: [Robin Goodfellow's transformation] See Jules Michelet, Satanism and Witchcraft (Citadel Press, 1939) 31.

120: [Character is fate] So Heraclitus, frag. 119, ηθος ανθρωπω δαιμων - a theme especially prominent, in characteristically different ways, in Hardy and Henry James (in whose Golden Bowl, for example, the figures are left alone together to behave as they must).

121: [17th century French mathematicians] such as Fermat (1608-

1665); Mersenne (1588-1645); Descartes (1596-1650); and later Lagrange (fl. 1770).

122: [Biological clocks and circadian rhythm] Melbin, Murray, Night as Frontier (New York: The Free Press, 1987) 63, quoting Gay Gaer Luce, Biological Rhythms in Psychiatry and Medicine, pg. 140: "The free-running rhythm is 24.8 to 25 hours on the average, but there are considerable individual differences." These estimates, from work in the 1960s, have been recently revised (see "The Tick-Tock of the Biological Clock" by Michael W. Young, Scientific American, March 2000, pp. 64-71) to a figure much closer to 24 hours - a rare instance of scientific and common sense converging.

122: [German and Dutch wood-carvers] the pieces referred to are in the Kenneth Thompson European Collection in the Art Gallery of Toronto, namely:

#293: Medallion of Lot and His Daughters, German (Nuremberg), mid 1500s: boxwood, diameter 5.1 cm., thickness 1.3 cm. (AL 427);

#304: Sander: Hunting Scene with Stag, Boars, Rabbits and Dogs, German, circa 1680: boxwood, silver, 12.1 x 3.8 x 3.8 cm. (AL 7-7-97);

#427: Prayer Bead: The Queen of Sheba Visiting King Solomon, and Adoration of the Magi, Netherlandish, circa 1520: boxwood, diameter: 6.1 cm. (AL Nov. 26 97).

123: [That 1,111,111,111,111,111,111,111 is prime] We 1 200.

This number, known as R_{19} , the 19th repunit, was shown to be prime in 1918. R_{23} (23 1s in a row) and R_{317} are the only other known prime repunits. Space prevented me from using the latter here, and affection for the relatively small, the former.

123: [Portrait of Fermat] After the engraving by F. Pouilly.

123: [Date of Fermat's "Little Theorem"] K 276.

123: [Fermat's Little Theorem] If you have a nodding acquaintance with congruences (a way of talking about numbers that turn out to be the same if you cycle them around - so that 7 and 4, for example, are the same on a clock with only the three numbers 0, 1 and 2, and we say $7 \equiv 4 \pmod{3}$), then you will enjoy this gem of a proof by Ivory, in 1806, of Fermat's Little Theorem. If p is a prime and you work with a cycle of length p - so we'll write " \pmod{p} " - and if x isn't $0 \pmod{p}$, then each of the numbers

$$x, 2x, 3x, \dots, (p-1)x$$

is congruent (in some order) to the numbers

$$1, 2, 3, \dots, (p-1)$$

Each set, in fact, is a "complete set of residues \pmod{p} " - i.e., all possible remainders on division by p - except that 0 has been left out of each. This must mean that

$$x \cdot 2x \cdot 3x \cdot \dots \cdot (p-1)x \equiv 1 \cdot 2 \cdot 3 \cdot \dots \cdot (p-1) \pmod{p}.$$

Now cancel 2, 3, ..., $(p-1)$ from each side and you're left with

$p-1$ x's on the left and 1 on the right, i.e.,

$$x^{p-1} \equiv 1 \pmod{p},$$

as desired.

124: [In the world of p ...] I am indebted to Yori Oda of Harvard for finding Professor Katsumi Nomizu of Brown to undertake the peculiar task of translating my imitation Japanese into Japanese itself; to him for his striking translation; and to Professor Takeshi Kokubo of Boston University for the elegant simplicity of his rendering it in brush-writing. The form is a senryuu rather than a haiku, lacking the 5-7-5 syllabic structure (although with an effort we could read:

In the world of p
you can't tell a to p less
one less one from naught)

and a seasonal reference (yet to me the world of p is inherently spring-like). In syllables, Professor Nomizu's translation reads:

p no kuni de wa
 p hiku 1jyou shitara
1 dato sa.

It is intriguing that not even nothing appears in the Japanese.

124: [Elizabethan "empty dishes"] Fu 133, and take (with seasoning)

her ch. 4 passim.

124ff: [Details of the "Public Key Algorithm" (called RSA, after its inventors, R. Rivest, A. Shamir and L. Adleman): Note: in the following, 'number' will always mean natural number, and brackets contain a condensed version in standard notation of the explanation they accompany.

1. Choose two primes p and q and form their product, $n = pq$.
2. Count how many numbers less than n have no factor in common with it. This turns out to be $(p-1)(q-1)$. Call that product k . [p, q prime $\Rightarrow \phi(n) = (p-1)(q-1) = k$]
3. Choose any number e (for 'encode') which has no factor in common with k . [$(e, k) = 1$]
4. Because e and k have no common factor, it turns out that there will be another number d (for 'decode') such that de will leave a remainder of 1 on division by k . [$de \equiv 1 \pmod{k}$]
5. Your public key is the pair $\langle e, n \rangle$. $\langle d, n \rangle$ is your private key.
6. To encode a message, first turn it in some way previously agreed on by sender and receiver into a single number m (which must be less than n).
7. Compute m^e , then divide this by n and take the remainder c . c is
the message you publicly send. [$c \equiv m^e \pmod{n}$]
8. Since de plays the role of 1 on division by k , m^{de} will be the same as m^1 , i.e., simply m , on division by n . [$m^{de} \equiv m^1 \equiv m \pmod{n}$]
9. Hence raising c , the message received, to the power d , decodes it. [$c^d \equiv (m^e)^d \equiv m^{ed} \equiv m^1 \equiv m \pmod{n}$]

For further details and discussion, see Kaufman, C., Radia Perlman & Mike Speciner, Network Security (Prentice Hall, 1995) 134-147.

125: [First catch your rabbit] Apparently the original recipe for hasenpfeffer stew began "First case your hare", i.e., skin it - but sound as always followed the urge to make sense.

125: [Very large primes] As of December 1998, it took about a second to find out whether a 50-digit number was prime but a month to test a 2000 digit number (Ne). Choose the length of your primes in terms of your travel plans.

126: [Ann and Anne] I am indebted for the twins example - though not for their names - to Henry Cohn in the Math Department at Harvard. This example is allegorical since the twins represent complex mathematical schema, such as tangled graphs, where we wouldn't have to worry about them tipping a wink to the people probing them.

126: [Odds better than a billion to one] $2^{30} = 1,073,741,824 > 10^9$.

126: [Rewriting proof so that an inconsistency will show up everywhere] This is called making the proof "transparent". See Ba passim.

127: [An expert, and his calculation of the odds of mistaking an

erroneous proof for a valid one] The expert is Laszlo Babai; the calculation is in Ba 2.

127: [Quotation about time of calculation] Ba 2.

127: [Cramming infinitely many points into a closed box] This is the three-dimensional version of the Bolzano-Weierstrass Theorem.

127: [Some mathematicians object...] The seminal critique of non-constructive proofs is L. E. J. Brouwer's. See his "On the Significance of the Principle of Excluded Middle in Mathematics, Especially in Function Theory", in vH 334-345, especially §2 (337-9).

128: [Austrian logician Gödel] Czechoslovakia claimed him, Austria reared him. In America he found what peace he could.

128: [Emblematic story of Gödel] This anecdote probably derives from Gödel's reputation, via his brother, of never having made a mistake in high school Latin. I am assured by Georg Moser, Secretary of the Kurt Gödel Society in Vienna [Mos], that the anecdote as it stands is apocryphal; that in fact Gödel was known for never interrupting himself when he started a sentence, and did in fact publish results later shown to be erroneous (though this doesn't affect what he may have thought of the validity of those results at the time he uttered them). In

tenuous support of this story, however, Moser relays Georg Kreisel's remark that "if someone showed to G. that one of his ideas was faulty, G. reinterpreted his first presentation so that the new interpretation was correct." Let us conclude: Si non è vero è ben trovato.

128: [Sufficiently rich] Rich enough, that is, to do classical mathematics - or simply arithmetic - in.

128: [Natural extension] This "natural extension" consists in upgrading the formal system by adding to it an axiom stating that the system is consistent.

130: [Title of Al-Khowarizmi's book] S ii 446-7.

130: [Translations of al-jabr...] I have found the following translations:

restoration and reduction Ca 1 103

completion and comparison SK 5

restoration and cancellation M 410

restoration and equalization K 411

reduction and cancellation S i 170

restoration and opposition S ii 388

redintegration and equation S ii 388

Aamir Rehman, via Razeeb Haque [Haq], relays this from Prof.

William Granara at Harvard, citing Edward William Lane's An Arabic-English Lexicon (Librarie du Liban, Beirut 1968), part 2,

pg. 374: "Perfective addition and compensative subtraction; or restoration and compensation; because of the frequency of these operations in the reduction of equations."

130: [Algebrista y sangrador] S ii 389, K 192.

130: [How al-jabr works] S ii 388.

130: [In the words of one historian] Louis Karpinski, the K of SK, but quoted in Ca 1 103.

130: [Omar Khayyam's method] S ii 446-7; Smith mentions that Al-Khowarizmi had two methods of his own.

130: [Napier first to set equations equal to zero] S ii 431, citing his De Arte Logistica, earlier than 1594. In n. 5 to this page Smith writes: "The credit is often claimed for Stifel (c. 1525), but he, like Harriot, made no general practice of equating to zero." The Dictionary of National Biography, art. "Harriot" (p. 1322), gives him priority, but adds that he procrastinated, and never published his work. He was known as one of the "three magi" of the Earl of Northumberland, with no small reputation for dealings in the occult. Marlowe said of him "that Moyses was but a Juggler, and that one Heriots can do more that hee."

131: [Portrait of Napier] After the engraving presented by the Baroness Napier to the College of Edinburgh, reproduced as the

frontispiece of Mark Napier's Memoirs of John Napier of Merchiston.

131: [Napier's castle besieged] She 45-50.

131: [Napier fighting off land-raiders] She 51.

131: [Napier fighting neighbors in court] She 51.

131: [Napier dabbling in the occult] She 70.

131: [Napier designed submarines] She 85.

131: [Napier designed tanks] She 85.

131: [Napier bewitched pigeons] She 71.

131: [Napier searching for treasure by sorcery] She 73-78.

131: [Napier's apocalyptic deductions] She 63.

131: [Napier rumored to be in league with the Devil] She 70, 72.

131: [Napier wearing black; his black cock] She 70, 72.

131: [Bacon on mathematics and subtlety] quoted in She, v.

131-2: [Story of catching thief] She 71.

132: [Equations to nothing] Na Appendix, Ch. 10, ¶4, pg. 517.

133: [Quotation from Lord Napier] She 72.

136: [Other examples of factoring by the astute addition of 0]:

1. In another part of the forest, called number theory, you ask such questions as: which whole numbers can be written as the sum of two squares? These are very difficult questions to answer, but an important insight along the way to this one came from Fibonacci in 1202. He saw that you could always write the product of the sum of two squares,

$$(a^2 + b^2)(c^2 + d^2)$$

itself as the sum of two squares - and this he did by cleverly adding 0 to

$$a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2$$

in the form of $2abcd - 2abcd$. He did it this way:

$$(a^2c^2 + 2abcd + b^2d^2) + (a^2d^2 - 2abcd + b^2c^2):$$

for this, in disguise, is

$$(ac + bd)^2 + (ad - bc)^2$$

which is a sum of two squares.

2. As you would by now expect, this insight too generalises, and after long struggles allowed the French mathematician Lagrange, in 1770, to prove that every natural number can be written as the sum of four squares (sometimes you have to allow one or more of these to be 0). His proof depended on showing that the product of two sums of *four* squares,

$$(a^2 + b^2 + c^2 + d^2)(w^2 + x^2 + y^2 + z^2)$$

could itself be written as a sum of four squares. This time 0 was added in the mind-boggling form of

$$(abwx + abyx + acwy + acxz + adwz + adxy + bcwz + bcxy + bdwy + bdxz + cdwx + cdyz)$$

minus the same expression!

It looks a bit more human if you say that

$$(a^2 + b^2 + c^2 + d^2)(w^2 + x^2 + y^2 + z^2) = (aw + bx + cy + dz)^2 + (ax - bw - cz + dy)^2 + (ay + bz - cw - dx)^2 + (az - by + cx - dw)^2$$

But still you marvel at the vision and persistence.

3. If you have had a year or so of calculus - a third part of what is really all one forest - then you will enjoy the proof of a somewhat surprising truth. The derivative of the sum of two functions is the sum of those functions' derivatives: $(f + g)' = f' + g'$. The same neat pattern doesn't hold, however, for products. In fact,

$$(fg)' = f'g + fg'.$$

The proof, via the "difference quotient", begins like this:

$$(fg)'(x) \text{ by definition is: } \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

The trouble is that the numerator doesn't factor, so the clarion call for zero rings out.

We want to add and subtract some term in the numerator to make the quotient more manageable. Perhaps someone recognised what the answer had to be by thinking of the problem in another (probably geometric) way, and so realized that what you had to subtract and add - in that order - was the expression

$$f(x)g(x+h):$$

for then you have $(fg)'(x) =$

$$\lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$
$$= \lim_{h \rightarrow 0} \frac{g(x+h)[f(x+h) - f(x)] + f(x)[g(x+h) - g(x)]}{h}.$$

Some fancy footwork is still ahead, guided by knowing what result you want. Break up the fraction into two parts:

$$= \lim_{h \rightarrow 0} \frac{g(x+h)[f(x+h) - f(x)]}{h} + \lim_{h \rightarrow 0} \frac{f(x)[g(x+h) - g(x)]}{h}.$$

And rethink each part with the denominator under the term in square brackets:

$$= \lim_{h \rightarrow 0} g(x+h) \cdot \frac{[f(x+h) - f(x)]}{h} + \lim_{h \rightarrow 0} f(x) \cdot \frac{[g(x+h) - g(x)]}{h}.$$

That's the same as

$$\lim_{h \rightarrow 0} g(x+h) \cdot \lim_{h \rightarrow 0} \frac{[f(x+h) - f(x)]}{h} +$$

$$\lim_{h \rightarrow 0} f(x) \cdot \lim_{h \rightarrow 0} \frac{[g(x+h) - g(x)]}{h}.$$

Now $\lim_{h \rightarrow 0} g(x+h)$ is just $g(x)$, and $\lim_{h \rightarrow 0} f(x)$ is just $f(x)$; and

those other two limits are the definition of the derivative!

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = f' \quad \text{and} \quad \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} = g',$$

so we are home:

$$(fg)' = f'g + fg'.$$

In every one of these examples, a truth has evolved and been confirmed through the agency of zero: a process Lorenz Oken would have relished. Yet more outrageous ways of factoring by the deft addition of zero perch in the ever-ramifying tree of mathematics.

136: [Blake passage] quoted by P. J. Kavanagh in "Bywords", Times Literary Supplement Sept. 5, 1997.

137: [Quotation from Dr. Johnson] quoted without source in David Hackett Fischer, The Great Wave (Oxford 1996) pg. xvi.

137: [On $ab = 0$ implying a or $b = 0$] This isn't true, you'll find, back in some of our bubble worlds: in a cycle of length 12, for example (where 12 is therefore the same as 0), $4 \times 3 = 12$ and $2 \times 6 = 12$, yet none of these factors is zero there. In this context, 3, 4, 2 and 6 are called "zero divisors". You might have expected that

the mirror these spheres held up to reality would somewhat distort it. In a cycle of prime length, however, it is again true that if $ab = 0$ then one or both of a and b must be zero: another glimpse of the important role primes play.

139: ["Un pur..."] Cited by Richard Cobb in one of his New College lectures, as noted by Michael Kaplan. Possibly said in reference to Robespierre and his fate. In any case, the aphorism is based (as Jon Tannenhauser astutely points out) on lines from the end of the first Canto in Boileau's Art Poetique: "Un sot trouve toujours un plus sot qui l'admire."

142: [Passage from Churchill] We 2 230.

143: [An elusive last] This last axiom emerged from the 19th century in different guises, one of which, the Least Upper Bound Axiom, sounds harmless enough: if you have some number which is greater than any of the numbers in a non-empty set, then there is a least number with this property - a lowest ceiling on the heights in the set, if you will, or a least upper bound on them. Yet this axiom is immensely powerful, calling into being irrational numbers whose existence we could not otherwise guarantee. Controversy continues over this axiom's status.

ELEVEN: ALMOST NOTHING

144: [Functions] Spengler (Sp I 86 n. 2) quotes Goethe as saying:

"Function, rightly understood, is existence considered as activity."

144: [Fermat and Descartes invent the coordinate plane] K 302.

146: [Schopenhauer's joke] Scho ii 271. By contrast, Descartes said that the angle between two curves was "...the most useful, and the most general problem, not only that I know, but even that I have any desire to know in geometry." (K 345).

147: [Rootlets in the middle ages] See for example S ii 684-5 on Jehudah Barzilai, 13th c.

147: [17th century toying] What follows is seen first in Fermat, 1629, published in 1637 (K 344-5); but there are many tentative predecessors, such as Oresme in the 13th century and much earlier, Tabit ibn Qorra (c. 870): see S ii 685.

151: [Fermat: "Remove it"] K 345.

151: [Lore and language of the Almost Nothing] Not only are there Western tales of will o' the wisps, pocket Undines, fairies and familiars, but these minuscule apparitions seem universal. Consider, for example, the Menehune in Hawai'i (Pu passim).

151: [Why shouldn't least be most?] Compare the similar mid-19th century American criticism of homeopathy's "less is more"

principle.

151: [Emerson's advice] Essays, vii: "Prudence", where the exact wording is: "In skating over thin ice, our safety is in our speed."

152: [Barrow's differential triangle] S ii 690.

152: [Barrow's argument] K 346.

152: ["Freed from the loathsome burden..."] K 346.

152: ["Sufficiently small"] Pascal (K 346).

152: ["As small as you like"] Leibniz (K 385).

152: ["Indefinitely small"] Leibniz (K 385).

152: ["Infinitely small"] Leibniz (K 376).

152: [Endless divisibility] Roberval (K 350).

152: [Error] Leibniz (K 384).

152: ["Less than any assignable quantity"] Leibniz (K 386).

152: [Ignore the relatively small] Stevin (K 351), Leibniz (K 374, 384).

152: [Bernoulli passage] K 387-8; its date: K 387.

153: [Passage from O'Brien] The Third Policeman (New York: Lancer Books, 1967) pg. 87.

153: [Meister Eckhart] BI 316 n. 5 to pg. 82.

153: [Frank Brimsek] Fischler, Stan, Bobby Orr and the Big, Bad Bruins (New York: Dodd, Mead, 1969) pg. 159. The importance of zero to Boston sports fans has not diminished. The great Celtics' center, Robert Parrish, wore the number 00. Frank Brimsek was voted in November 1999 one of the top hundred New England sports figures of the century (Boston Globe, Nov. 15, 1999, pg. D2). And the Red Sox have managed to preserve this most important of numbers in their record of World Series titles since trading away Babe Ruth to a team in New York.

153: [Preserving the ratio of a vanished triangle's sides] K 387.

153 [Leibniz dropping negligibly small terms] K 374-5, 384.

153: [dx as minute distance] K 376, 385.

153: ["Incipient quantity"] K 386.

153: ["Infinitely near"] K 385.

153: [Relative zeros] K 387.

153: [Zeroes a formal device, like imaginary numbers] K 385.

154: [Monads] vP 28.

154: [Monad a conceptual point] vP 31.

154: [Monad as pure interiority] vP 28.

154: [Monads "windowless"] vP 66.

154: [Monads active] vP 56.

154: [dx active] K 374.

154: [Monads more verbs than nouns] vP 45, 32: "Perhaps this might be expressed... by calling the world of spatial and visible phenomena the 'translation' of the monads' immaterial structure." vP 33: "These symbolic languages however are translations of the monads, conceived of as metaphysical symbols... the most fundamental symbols have a real content or value: the monads are not just imaginary 'bricks' but the real elements of the world as also of scientifically disciplined thinking about it."

154: [Mathematical points as viewpoints] vP 68.

154: [Leibniz' "Pre-established harmony"] vP 62.

155: [Portrait of Newton] After Seeman's painting.

155: [Newton as the last of the magicians] Key 277.

155: [God's starry cryptogram] Key 279.

155: [Newton's two most fruitful years] K 357.

155: ['o' borrowed from Gregory] K 359.

155: [Newton dividing by 'o'] Ca 1 194, 195.

155: [Newton transmuting his terms] So in his Methodus of 1671
(K 362).

155: ["Expunging" negligibly small terms] Ca 1 195.

156: [Newton's "moments"] So in his De Analysi of 1669
(K 359-362).

156: [Twinkling of an eye] 1 Corinth. 15:52.

156: [Not neglecting the minutest errors] K 363.

156: [Mathematical quantities described by a continuous motion]
K 363.

156: ["These geneses..."] K 339.

156: [Newton's change in viewpoint] Note that while Newton's basis for mathematical space and activity is dynamic, physical space, light, ether and matter remain corpuscular for him. See Ko 172 and 207.

156: [Barrow's triangle becomes "evanescent"] K 364.

156: [Standards of clarity in Greek mathematics] A rigor found in proofs by contradiction. Cf. Archimedes' "method of exhaustion". See K 352.

156: [Fighting among themselves] See for example the exchange of letters between Leibniz and Newton's hitman, Samuel Clarke (Lei 216-280).

156-7: [Nieuwentijdt] K 385.

157: [Passage from Berkeley] Newman, James (ed.) World of Mathematics (Simon & Schuster, 1956) i. 292.

157: [Berkeley on scientific method] *ibid.*, 293.

157: [End justifying means] Cavalieri (K 383).

157: [Means just pragmatic devices] Cavalieri (K 383).

157: [Rigor not the concern of mathematicians] Cavalieri (K 383).

157: [Appeals to the heart] Pascal (K 384).

157: [Paradoxes here as useful as in Christianity] Pascal (K 384).

157: [Quotation from d'Alembert] Ca 1 243.

157: [Pascal and finesse] K 384.

157: [Leibniz on over-scrupulousness] K 385.

157: [Areas made up of indivisible lines] K 349. There was an immense variety of interacting views about the nature of space that lay in the background of these images in particular and the rise of calculus in general. They are well presented in Gr passim, but especially in his Chapter Eight. He does not, however, suggest what seems to me likely, that these views come trailing not only clouds of Aristotelian glory but more significantly tatters and shreds from the Timaeus: how impenetrable body could coexist with indivisible space (Gr 235) was a puzzle at this time because it was a too literal

understanding of Plato's metaphor meant to describe the relation of number to place-holder (or variable).

157: [Forms remaining after their magnitudes removed] So Leibniz (K 387).

157: [Newton on little smatterers] K 358.

157: [19th century understanding] Major figures in this understanding are Bolzano (1817), Cauchy (1821) and Weierstrass (1841-59) (K 951-2). For a survey of the current revisionist views of Cauchy-Weierstrass, see Me and La, passim.

158: [The necessary gap] Observe that the gap must nevertheless be there: the tolerance must not be zero but as close to zero as your challenger chooses: as they say in the trade, $\varepsilon > 0$, where ε is the measure of tolerance.

158: [Tolerance demanded and achieved] So thoroughly has this notion of almost nothing become, in our time, identified with nothing at all, that it is not surprising to find a company making precision components boasting (like the Custom Products Corporation of Menomonee Falls, Wisconsin, whose flyer I picked up in Detroit) that it is "committed to zero defects", while explaining that its products are manufactured "to a tolerance of .000022 inches".

158: [Taking a limit] This description of taking a limit applies to the situation we have had before us. Restrictions and generalizations readily follow.

158: [Attending to the technicalities] At such times as you are tempted to think of mathematics as aspiring to the rigor and clarity of the law, it is salutary to remember that important legal principle: *de minimis non curat lex*: law doesn't sweat the small stuff. But it is precisely the small stuff - not only the epsilons and deltas that grease the derivative's skids, but detail, always revelatory detail - that mathematicians most sweat over, and that so often provides them with insights the whole conceals.

159: [Mathematical procedures used before being formalized] This practice goes back at least as far as Archimedes, who, in his Method, describes a procedure which, he says, "is, I am persuaded, no less useful for the proof of the theorems [previously published] themselves; for certain things first became clear to me by a mechanical method, although they had to be demonstrated by geometry afterwards because the investigation by the said method did not furnish an actual demonstration." (Ar pg. 13 of "Supplement").

159: [Representation in many essentially different ways] This is a paraphrase of the Löwenheim-Skolem Theorem.

162: [Bernoulli and le Marquis de l'Hôpital] Bra 239, citing the letter of l'Hôpital to Bernoulli of March 17, 1694, asking him "...to communicate to me your discoveries... it would not please me if they were made public."

162: [Division by zero impossible] The stress falls on "in our full-fledged world", since in those bubble-worlds we spoke of before, peculiar things can happen. See note to 137.

162: [Zero valueless in Centre County Pennsylvania] Bec.

163: [Teresi and M.I.T.] Ter 1 93, Ter 2.

163: [Two conjectures about 0^0] These conjectures were in the first part of Chapter 10, on pg. 119.

164: [Cleverly applying "Bernoulli's Rule"] The argument that

$\lim_{x \rightarrow 0^+} x^x = 1$ runs as follows:

$$\lim_{x \rightarrow 0^+} x^x = \lim_{x \rightarrow 0^+} e^{\ln(x)} = \lim_{x \rightarrow 0^+} e^{x \ln x} = \lim_{x \rightarrow 0^+} e^{\frac{\ln x}{\frac{1}{x}}} = \lim_{x \rightarrow 0^+} e^{\frac{x-1}{-x^2}}$$

$= \lim_{x \rightarrow 0^+} e^{-x} = 1$ (where the fourth step follows from the Rule).

166: [Monstrously strange] It isn't just that, like all functions on the

complex plane, this one is multi-valued: it has a whopping "logarithmic singularity" at the origin. We owe much of our understanding of such to the 19th century French mathematician Emile Picard, who studied a large class of functions with wildly wandering values near points which are particularly singular (less of an oxymoron than it sounds - certainly less than 'very unique' is).

167: [The x^x spindle] For details see Mey passim (on whose article the two spindle diagrams are based).

168: [The tower of x^x] As, passim.

169: [$0^0 = 1$] That $0! = 1$ is a shade less contrived, if we rethink factorials as integral outputs of the gamma function.

169: [When we extended exponents] See pp. 133-4.

169: [The hope that new linguistic structures will yield new insights] See above, pg. 178.

169: [Quotation from Euler] Quoted in Bra 219.

169: [Euler's dates] 1707-1783 (S i 521 n. 1).

172: [Devoted to eliciting zero's signals] Much devotion is often required, since it is sometimes anything but easy to determine

whether a number is zero in disguise. Is $e^{i\pi} + 1$? Is $\tan^4(7\pi/15) - 6\sqrt{3}\tan^3(7\pi/15) + 2\sqrt{3}\tan(7\pi/15) - 1$? The zero-determination problem has yet to be fully solved: the problem, that is, of coming up with a mechanical way of determining whether a complicated expression involving the algebraic, exponential and logarithmic operations is or isn't zero (the full solution awaits the proof of Schanuel's Conjecture). (Cho 446).

172: [Growing neglect of infinitesimals] On Euler's rejection of them, see K 429; on Lagrange's, K 430.

172: [Triumph of the view that made change fundamental] This triumph is strikingly shown in a passage from G. Chrystal's Algebra: An Elementary Textbook for the Higher Classes of Secondary Schools and Colleges, first published in Edinburgh in 1886. Very early on the author introduces "a special symbol, namely 0" to denote the case $+a - a = -a + a = 0$. He then makes the telling and curious remark (pg. 11 of vol. I in the Chelsea 1964 reprint): "It will be observed that 0, as operationally defined, is to this extent indefinite that the a used in the above definition may have any value whatever. It remains to justify the use of the 0 of the ordinary numerical notation in the new meaning. This is at once done when we notice that in a purely quantitative sense 0 stands for the limit of the difference of two quantities that have been made to differ by as little as we please."

172: [Robinson's car] Da 342.

173: [Ghosts of what aren't even quantities] This note can be read as a jeu d'esprit - a brief essay belonging perhaps to the category once called 'curious'. The Greeks originated a style of proof which came to be called "reductio ad absurdum" and "proof by contradiction": showing a conjecture true by assuming it false and deriving a contradiction from this assumption. Harmless enough, surely, and more than that: brilliant, and at times the only proof known or available. It becomes problematic when the conjecture is that something actually exists - some mathematical object (such as the point where a continuous graph, which has both positive and negative outputs, crosses the x-axis). To prove that if it didn't, something would go haywire elsewhere in the known universe of mathematics, is to skate on thin ice indeed - for such a proof yields not a single property of the object whose existence it establishes - save that very existence. This is reminiscent of ontological proofs (such as Anselm's) of God's existence. This mode of proof, when applied to existence theorems, has come under attack in our time by those who want us to know *how* mathematical objects exist, not just *that* they do: who want us to be able to construct them. I have heard a constructivist compare what we know, after studying a proof of existence "by contradiction", to what we know after stopping a stranger and asking if he could give us directions in an unfamiliar town, and his replying "Yes, I could", and walking away.

Is it too far fetched to compare such proofs that establish existence by dwelling on the consequences of non-existence, to the role zero plays in positional notation? There it gives value while having none itself; here too value is established by the absence of value. That is: no value is given to the object whose existence this proof by contradiction establishes, other than the vacuous value *that* it exists. Yet now that it does - however ghostly its presence - it begins to combine with the other objects that also exist (some through direct constructions, some from an equally shadowy birth), and their combinings now give it traits, properties, values. So a contrived character injected into the real world (as by a hoax or an impersonation - think of Ferdinand Demara) cannot but acquire values from his surroundings, they and he enhancing reality together. Perhaps we need to acclimate ourselves to what I might call such "positional proofs", recognizing that, just as "100" gradually takes on meaning in the context of its neighbors on the number line and its role in various propositions and their instances, so objects whose existence is only guaranteed in this zero-like way become meaningful in their interplay with other objects given in a variety of ways, directly and indirectly.

173: [Well-founded fictions] Robinson thought that Leibniz' characterization of infinite concepts as *actiones bene fundatae* seemed right (Da 333).

173: [That Robinson thought Leibniz shared his view] Da 349-50.

173: [New deductive procedures] Da 354-5.

174: The excellent translation by Max Knight (Kn 17) is of "Der Lattenzaun", whose first three stanzas in German are:

Es war einmal ein Lattenzaun,
mit Zwischenraum, hindurchzuschauen.

Ein Architekt, der dieses sah,
stand eines Abends plötzlich da -

und nahm den Zwischenraum heraus
und baute draus ein grosses Haus.

The conclusion is too disturbing to repeat. It is of some interest, however, that (as I learned only after citing these lines) Robinson was fond of Morgenstern's verses, to the point of writing imitations - or should I say, making different models - of them (Da 41).

TWELVE: IS IT OUT THERE?

175: [Cosmologists on theory and measurements of virtual particles]
Kra 55.

176: [Reynolds] I am indebted to Martin Gardner (Ga 2) for raising the

ghost of Reynolds.

176: [Reynolds on the highest philosophical interest] Re 1 44.

176: [Reynolds on the likelihood of his theory] Re 1 43-4.

176: [Size and velocity of grains] Re 1 14.

176: ["Could anything be more simple?"] Re 1 15.

176-7: [Accounting for gravitation] Re 1 37.

177: [We are all waves] Re 1 33.

177: [Reynolds' model of the universe] Re 1 28, photograph facing
pg. 28.

177: [One physicist, and quotation from him] The physicist is
David Griffiths, the quotation is from Gri 19.

177: [Positive energy states of a different particle] Gri 21, Wei
i. 12-14.

177: [Temperature of outer space] Corn 41, Tan.

177: [Casimir effect and "zero-point energy"] Ya passim, Ren passim.

177-8: [Confirmed at Los Alamos] By Stefan Lamoreaux.

178: [The Invisible College] EB v. 4 354-5, art. "Robert Boyle".

178: [Date and details of Wright's painting] Nic 12-13.

178: [The world well lost for one more zero] So the story told me by Tomás Guillermo (Gu) of the M.I.T. nuclear physicist, devoted to adding a zero each year to the decimal expansion of the neutron's charge - which now - after his death - may, it seems, turn out to be positive after all.

178: [Fairbank's open window] Fa 10-11.

178: [Details of apparatus, principles and conclusion] Corn 40-45.
This is the Bose-Einstein condensate. Cornell remarks on pg. 44 that non-zero energy at "zero temperature" follows from Heisenberg's Uncertainty Principle.

179: [It isn't anything they can prove] Geo.

179: [John Collier's story] In Fancies and Goodnights (Doubleday, 1952), pg. 139. Collier actually has his devil say "twenty million billion light years", but his devil must have been drinking himself, since that would be a measure of distance, not time. I have changed it because it doesn't do to embarrass a devil.

179-80: [Differing theories of time] (Buc 68). Cyclic: J. Richard Gott III and Li-Xin Li (who "recently proposed that the universe is trapped in a cyclic state...[but] our bubble broke off from the cyclic proto-universe; it is no longer cyclic but instead is always expanding and cooling."); without a beginning: Andrei D. Linde; perceived differently within and outside a bubble: Bucher and Spergel; directionless; James B. Hartle and Stephen Hawking. Georgi (Geo) comments: "No sane, living cosmologist makes time cyclic any more. If the CMBR didn't cure them, the idea of inflation and measurements of omega should have."

180: [Knockers at the gate] Kra 55: "Like it or not, empty space is not empty after all."

180: [Nothing is unstable] Frank Wilczyk, quoted in Od. Has he those apparitions of particles in mind, or is he offering us a paradox to strengthen our muscles which yet may lame us: what Mallarmé called "tutelary poison, always to be inhaled although we die of it" ("toujours à respirer si nous en perissons")?

180: [Questions outpace answers] SA xii. 1.

180: [Extract many senses from a text] SA xii. 42.

180: [Prope nihil] SA xii. 3.

180: [Current views on matter and anti-matter] My thanks to Dick Teresi (Ter 2) for this elegant summary.

181: [How Zeus found the world's center] EB v. 7 p. 975; Oxford Classical Dictionary, art. "Omphalos".

181: [Aristotle taking Pythagoreans to task] De Caelo ii.13 (293a18). Barry Mazur calls my attention to this passage from Peter Kingsley's Ancient Philosophy, Mysticism and Magic: Empedocles and the Pythagorean Tradition: "In his 'On the Heavens' [Aristotle] raises the issue of the earth's location in the universe, and cites the view of the 'Italians' or 'Pythagoreans' to the effect that the midpoint of the cosmos is occupied not by the earth but by a central fire. He then goes on to offer the following line of reasoning in support of this view... This logic has seriously been accepted by a number of modern writers as expressing Pythagorean ideas... However, from Aristotle's own words, one point is perfectly clear: he is ascribing this belief [that he ridicules] not to Pythagoreans at all, but to 'others' whom he specifically distinguishes from them."

181: [For Aristotle, earth at center] De Caelo ii.14 (296b8-16).

181: [Cupola of the earth] Sa 306.

181: [Coordinates of Lanka] Sa 308 for longitude, 306 for having no latitude.

181: [Yavana-Kotij] Sa 306-7. This is a late opportunity to remark on how unusual it is for a man to test his comprehension of the text (in this case, in Arabic) he has edited by translating it into German; and then to translate it all over again, a year later, into English. That is however exactly what Edward Sachau did with Albiruni's India. He was supported in his long efforts by having given his word to complete them to teachers and mentors with antique names: old MacGuck de Slane and Osmond de Beauvoir Priaulx. From his preface Sachau sounds a modest, humorous man one would have liked to have known. When he says of Arabic (Sa xlvi-xlix) that "...this language exhibits sentences perfectly clear as to the meaning of every single word and to the syntactic connection, and nevertheless admitting of entirely different interpretations", he reminds me of Collingwood's experience on first reading Kant, and mine of almost everything.

181: [Plan of Yavana-Kotij] After diagram in Sa 307.

181: [The palace in Atlantis] Plato, Critias 114-117; the plan is in Bury's edition, facing pp. 286 and 287.

181: [Atlantis sank for its sins] Critias 121 B-C.

181: [Lanka is Langa, the cannibal island] Sa 309, 310.

182: [Stoic view of universe] J 22. See for example Lucretius,
i. 918 ff.

182: [On Nicolaus Cusanus] Wi 315.

182: [Universe imprecise] Ko 8.

182: [Universe without center] Ko 12-13.

182: [Our views of the universe lacking in objectivity] Ko 16-17.

182: [Passage from Cusanus] Ko 17, quoting De Docta Ignorantia, 1. II.
cap. 12, p. 103.

182: [Passage from Book of the XXIV Philosophers] Ko 279 n. 19
to ch. 1.

182-3: [Passage from Cusanus] Ko 17, quoting as before. My
translation slightly modifies his.

183: [Passage from Pascal] Pa #72. The French reads: "Tout ce que
nous voyons du monde n'est qu'un trait imperceptible dans
l'ample sein de la nature... C'est le Sphere infinie, dont le
Centre est par tout, la circonference nulle part." See too Ma 25,
and cf. Rabelais, Pantagruel to Panurge, advising him to gauge
the luck of his marriage by dreams (iii. 13): "...in contemplation

of that infinite and intellectual sphere, whereof the center is everywhere, and the circumference in no place of the universal world, to wit, God (according to the doctrine of Hermes Trismegistus) to whom no new thing happeneth, whom nothing that is past escapeth, and unto whom all things are alike present..." For a discussion of this "extraordinary metaphor", as he rightly calls it, in the work of Bradwardine and others, see Gr 138-141.

183: [Kepler's heliocentric universe] Ko 63, 67, ch. 3 passim.

183: [Newton's heliocentric universe] J 101.

183: [Quotation from Galileo] Ko 96.

183: [Galileo's motives] Ko 98-99.

183: [Newton's absolute space] Ko 252.

183: [Leibniz' view of space] Ko 245, J 115.

183: [Newton's and Leibniz' premises] Ko ch. XI, 235-272 passim. In particular, pg. 235 for Leibniz' criticisms of Newton's "low and untrustworthy opinions of God"; and pg. 260.

183: [Leibniz's criticism of Newton] Lei 246, Leibniz' fifth letter to Clarke, §29; cf. Koyré's discussion (Ko 260-61 and especially

163-4 with a passage from Newton, and Koyré's note 18).

184: [Moving uniformly with respect to one another] i.e, inertial reference frames.

184: [Scientific hypotheses resonating with science fiction] It may not be wholly lacking in significance that the cosmologist Fred Hoyle wrote science fiction novels.

185: [William Whewell] Med 166.

185: [Descartes] in his Principia Philosophiae (1644), ii 36, and see the discussion in Mach (Mach 361).

185: [Conservation of conservation laws] It is interesting to compare Mach, citing Newton's laws of conservation of center of gravity (Mach 377) and conservation of areas (Mach 382) with Feynman, who mentions neither, but has conservation of baryons (Fey 4-7) and of leptons (Fey 4-8).

186: [d'Alembert in 1743] This is the date of his Traité de dynamique.

186: [d'Alembert's rewriting of Newton's laws] My treatment follows Lan 88-93, who remarks (pg. 89): "Apparently nothing is gained, [by giving] merely a new name to the negative product of mass times acceleration. It is exactly this

apparent triviality which makes d'Alembert's Principle such an ingenious invention."

186: [Understood in the same way] While the equations of statics are algebraic, however, those of dynamics are differential.

187: [Mathematical metaphors] This point is Howard Georgi's (Geo).

187: [Leibniz' claim that only individuals are real] See above, pg. 170, and vP 27-29, Ko 262.

187: [Fahrenheit] EB v. 10 126, art. "Fahrenheit".

187: [Réaumur] EB v. 22 947 art. "Réaumur".

187: [Celsius] EB v. 5 609 art. "Celsius".

187: [Kelvin] EB v. 15 721-3 art. "Kelvin".

187: [Plimsoll and his line]: EB v. 21 841 art. "Plimsoll".

188: [Look at what scales you will]: Not all invented scales have made good use of zero, which has even been made to play a part in some of the Higher Foolishness. Gerry Harant, from Blackburn, Australia, writes to me that "When the metric system was first adopted in France at the time of the French Revolution, its

proponents made it clear that commonly used measuring units should preferably not be more than an order away from the last significant digit of the measured quantity. They therefore devised a system of prefixes spaced by factors of ten as multipliers and dividers. This resulted in 'incoherent' derived units for engineers and scientists, who therefore chose, for their own use, two subsets of the system, mks and cgs. Traders and the people at large used binary fractions - also recommended by the system's originators. You therefore bought wine in a glass known as 'an eighth' (litre), cooked using dekagrams (Germany) or hectograms (Italy) or fractional kilograms. This was a user-friendly system and hardly ever led to confusion. It was so friendly that older systems such as the Imperial became laughing-stocks amongst users of metrics.

"In 1962 a conference called to straighten out some of the system's anomalies instead developed into a nitpicking battle between proponents of cgs and mks, and the mks won. The resulting disaster was called *Système Internationale*, presumably because it was neither a system nor international. In addition to forcing people to use units like the Pascal (neither a pressure nor a vacuum) and insisting on the Newton instead of the Kgf, the SI barred prefixes which were not divisible by 10^3 . The binary divisions were also barred. This means that, for instance, you drink your beer by the ml and weigh your newborn by the gram. Beyond settling the argument about cgs and mks, the warring parties never appear to have considered the negative impacts of their decisions on ordinary

folk, trades-people and householders. One result of this is that the Imperial system is still in use and, indeed, young people educated in SI have adopted much of it and people's height is still quoted in feet and inches, land is still sold by the acre and houses are still quoted by the 'square' (100 ft²), forty years after the official change.

"One of the worst aspects is the havoc caused to zero. Understandably, ordinary people don't want to use decimal points, so zero is once again used as a place-holder when it occurs at the end of a measurement. This has bizarre consequences. I recently got a drawing in which a drainage trench for disposal of waste water was specified to be 30,000 mm long. You can see the evolution: under the Imperial system it was 100 feet, which quite properly should have been expressed as 30 m. However, because the rule is that all measurements must be in millimetres, we are given this meaningless string of zeroes.

"In other words, the concept of precision which defined the use of significant figures has been lost. A zero at the end of a measurement cannot be taken as a real integer. Conversely, the significance of the units of measurement has been lost. Fluid measurement on the content of wine bottles, for instance, is given in ml unless the content is one litre. If it is 1 3/4 litres, it reverts to being '1250 ml', thereby suddenly acquiring three additional significant places. Clearly the notion of a 0.1% accuracy in measuring soft drink with commercial machinery is almost as absurd as using a backhoe to dig a trench 30 m long

accurate to the nearest millimetre.

"People brought up under the Imperial system simply believe that metric measurements are by nature cumbersome for trade and commerce, although they are willing to accept them for scientific purposes. People educated under SI, the other half of the world, are not aware just how much more friendly metrics could be if we reverted to the original concepts with some modernization of units. Both groups misunderstand the dual role played by the humble zero."

188: [Loss of sense of position] Sac 42-52.

189: [Without a tighter breathing] These are the last two lines of Emily Dickinson's "A narrow Fellow in the Grass", from around 1865. See The Poems of Emily Dickinson (ed. Bianchi & Hampson) (Boston: Little, Brown, 1932), Part 2, #24.

THIRTEEN: BATH-HOUSE WITH SPIDERS

190 [Zero as psychologically negative]: More evidence - if more is needed - that some sort of psychological entropy carries us toward interpreting zero negatively: William Houston, writing in the Canadian paper The Globe and Mail, on Nov. 14, 2001 (pg. 53), quotes Jack Todd, of the Montreal Gazette, on the former Montreal Expo's president Claude Brochu: "In the Museum of Zeros, Brochu gets his own wing. Face it, people - this guy is a waste of DNA. A scheming, hypocritical... baseball hating,

grasping... nonentity who destroyed the Expos and made himself rich in the process. And those are the good things."

190: [Good-for-nothing] For a detailed study of good-for-nothings, see Jaf passim. Our friend Louise Rice heard a fascinating Mexican version of this in Cuernavaca: "Solo es un zero a la izquierda": He's just a zero to the left. Why "to the left"? Surely this shows a thoughtful appreciation of positional notation, since a zero to the right of a number elevates its value, but to its left - after a decimal point - lowers it. Such people, therefore, are not only of no value themselves, but reduce the value of their neighbors.

190: [Hamid the Second] I am grateful to Eric Simonoff for this anecdote.

190: [The disappearance of the singular and the memory of the singular] A central conservation law of physics declares that matter can never be lost, only changed in form. Our dearest hopes, our greatest sorrows, revolve around the missing dual to this law: that structure can never be lost, only changed in matter. Such a law would assure us that personality, the nature of a time and a place, feelings too subtle to express, the whiff of the particular, the grain of the accidental, persevere: remain timelessly as they momentarily were. There isn't an iota of religious fervor, of waking desire or aching dream, that doesn't amount to this.

190: [Anonymity] The most peculiar of zero embodiments must be that author of so much, Anon. Those whose names have been lost in time and those who preferred to conceal themselves here conjoin their identities, so that their motto should be a parody of America's: E Pluribus Nullum. How many more belong to this fellowship, for all that their works bear an authorial title? I mean not only those whose proper names have disappeared behind pseudonyms (Stendhal, George Eliot, Orwell, B. Traven), but any whose name alone was common to the authorial voice and their own - and that must mean all. For even the frankest autobiographer, assuring you (as Baudelaire did) that here we have his *coeur mis à nu*, has selected, reworked and given a literary rather than lived unity to his published persona.

Wouldst thou find my ashes? look
In the pages of my book,

said that forgotten American poet, Adelaide Crapsey - and she was right: ashes only. Like her near contemporary Emily Dickinson, what she wrote were letters to the world that never wrote to her, and the exchange was just: contrived personalities awake no echo in a conglomerate one.

190: ["...to pass beyond and leave no lasting trace"] From the prefatory poem by McFee to his Casuals of the Sea. The shimmers in the bevelled glass are seen by his young Hannibal

Gooderich in Chapter XV (pp. 95-97 of the Modern Library edition).

191: [To live as a zero] I would like to thank the immensely well-read German TV journalist Alexander Weil for reminding me that Melville's *Bartleby* is an archetypal incarnation of nothing (the word itself appears 25 times in the story). Like Poe's Raven, he comes to roost in the narrator's room, but here not as an emblem of loss but as a projection of the narrator's emptiness. Sub-titles unriddle allegories: a century before Camus' Le Mur, Melville's tale was "A Story of Wall-Street", and walls as impasses are everywhere here, and screens, and partitions, blocking off life but unable to conceal the nothingness within. "I would prefer not": *Bartleby* stands between necessity and free will (books on which the narrator studies while trying to dismiss him), but we feel the accent slipping, as the story progresses, from "I would *prefer* not" to "I would prefer *not*". Did life at sea help to engender this species of existential melancholia? Compare Conrad's *Secret Sharer*. Much more significantly, compare Moby Dick, where the active metaphysician Ahab would strike through the mask of appearances to find if behind it the universe is malignant or indifferent. In the present variation, it seems that Melville explores the possibility that the void lies within the narrator, who - like his airshaft - is "deficient in what landscape painters call life," finds his will dissipated to mere preference and this at last to indifference. *Bartleby*'s pallor recalls the

"appalling" whiteness of the whale.

There is so much more to say about Bartleby and "Bartleby", from the ginger-cakes he eats - so like the obol placed under the tongue of the dead - to his reiterated "I am not particular", the malaise he represents being general; from anticipations of him in Melville's own writing (as in the poor devil of a sub-sub-librarian who assembles the prefatory quotations for Moby-Dick) to his reappearance both in Melville ("I and my Chimney") and in others (such as Twain's "Mysterious Stranger") - but not here, not here.

191: [The superfluous man] A notion prominent in Turgenev and Chechov.

191: [The man without qualities] Robert Musil's great unfinished (perhaps by its nature unfinishable) novel, Der Mann ohne Eigenschaften (see note to pg. 193 below).

191: [Henry James' John Marcher] In his story, "The Beast in the Jungle".

191 [Zero men who never actually lived]: It is a remarkable truth that non-being can exert considerable power. Some forty years ago the Polish aphorist Stanislas Lec wrote that you can easily make a chain out of a sequence of zeroes ("Aus einer Reihe von Nullen macht man leicht eine Kette", in the German translation) - referring, of course, to the banded, oppressive strength of

faceless Commissars. But don't nullities naturally collude,
drawn together by their mutual repulsiveness?

191: [A digit in front of a string of zeroes creates value where none
had been] Shakespeare expresses this beautifully in the
Prologue to Henry V:

...But pardon, Gentles all,
The flat unrayesd Spirits, that hath dar'd,
On this unworthy Scaffold, to bring forth
So great an Object. Can this Cock-Pit hold
Within this Wooden O, the very Caskes
That did affright the Ayre at Agincourt?
O pardon! Since a crooked Figure may
Attest in little place a Million,
And let us, Cyphers to this great Accompt,
On your imaginarie Forces worke.

In a letter to the Times Literary Supplement of March 10, 2000,
Ernst Gombrich suggests that "Shakespeare wanted the
prologue... to describe the Globe as a 'wooden *naught*', rather
than a 'wooden *O*'... The upright oval on the page of the First
Folio could, of course, be pronounced either way, but to my mind
the suggested alternative would greatly enhance the appearance
of that marvelous piece of rhetoric: the escalation from
unworthy scaffold to *cock-pit* to *wooden O*, leading cunningly to
the metaphors of *figures* and *ciphers*..."

I would add that we then hear an echo of this naught in "O
pardon", since it is these ciphers of actors who crave pardon. In
a letter to the TLS of April 14, 2000, Humphrey Tonkin points
out that Shakespeare would have had the o signifying zero

pronounced "o", "naught" having begun to take its place in the seventeenth century.

191: [Naughtiness from naught] Pa art. "Naught", where he connects the merely misbehaving of 'naughty' to its obsolete senses of 'worthless' and 'wicked'.

191: [Crawling between earth and heaven] Hamlet iii.1.130.

191-2: [Donne on man as Nothing] Don 457: Sermon XI (3).

192: [Bunyan] From Bunyan's Autobiography, quoted in Ja 2 155.

192: [Cyberspace cryptography] Way 10.

192: [Animula vagula blandula] This is what is now called Hadrian's dying prayer to his soul:

Animula vagula blandula
hospes comusque corporis,
quae nunc abibis in loca -
pallida, rigida, nudula,
nec, ut soles, dabis jocos?

I have seen this translated as:

Ah gentle fleeting wandering sprite,

Friend and associate of this clay,
To what unknown region borne
Wilt thou now wing thy distant flight?
No more with wonted humor gay,
But pallid, cheerless and forlorn.

193: [Passage from Plath] PI 271.

193: [The philosophy building at Harvard] My thanks to Brian A. Sullivan, Assistant Reference Archivist at the Harvard University Archives, for uncovering at last some details of this story in a note of Mason Hammond's, dated Dec. 3, 1990 [HuB 1361.36 Box 9], which reads, in part:

"'What is man that thou art mindful of him' (Psalm 8 verse 4) is the inscription over the north door to Emerson Hall... A long standing oral tradition asserts that when the Hall was nearing completion in 1905 the Department of Philosophy, to be its occupant, proposed, perhaps at the suggestion of Prof. George Herbert Palmer, 1864, 'Man is the measure of all things,' a quotation preserved from the fifth century B. C. Greek thinker Protagoras. When, however, the Hall was completed, it was found that President Eliot, 1853, had substituted the present inscription without consulting the Department."

Hammond then describes two slender threads of evidence to substantiate this story, concluding with a third: "Mr. Michael

Raines of the H. U. Archives... found... a letter from Boylston Professor Adams Sherman Hill, 1853, to President Eliot... In the second page of his letter, Hill proposes three quotations from different works by Emerson... on the back of the second page are three possible quotations in what seems to be Eliot's own hand... the third... is the quotation from the eighth Psalm that now appears on the building. There is no indication of what had brought it to his attention, but at least by May 7, 1905, Eliot had in mind to substitute it for the Department's proposed inscription."

Hammond's note concludes: "It might therefore appear as though President Eliot in changing the inscription to the quotation from Psalm 8 wanted to refute what he may have regarded as an arrogant view that man, not God, determines reality. Against this interpretation might be the following verses of the Psalm, which state that God made man little lower than the angels and had given him dominion over God's works. Perhaps President Eliot hoped that readers of his inscription would not be familiar with the rest of the Psalm and would read the inscription as a warning to man to be humble."

193: [Closing off of inner life, leaving the world monochrome] I find it striking, in this regard, that when the brilliant mathematician John Nash was in the depths of his mental illness he kept a notebook entitled "ABSOLUTE ZERO", in which

he pasted brilliantly colored scraps, such as a map of Princeton printed in primary colors (see Sylvia Nasar's A Beautiful Mind, pp. 285-6). Does this give us the glimpse that William McFee despaired of into "the mute soul's agony, the visions of the blind"?

193: [From disenchantment to disengagement] Literature, biography and case histories abound with examples. A striking one (Mu i. 13): "And since the possession of qualities assumes a certain pleasure in their reality, we can see how a man who cannot summon up a sense of reality even in relation to himself may suddenly, one day, come to see himself as a man without qualities." See too Freud's study of the mad Dr. Schreber (Fr 387-470), who thought that after the creation God had withdrawn to an immense distance (pg. 402), and that the people Dr. Schreber saw around him were "miracled up cursory contraptions." (pg. 456).

193: [Passage from Crime and Punishment] Part IV, ch. 1.

194: [Weyl] Wey 75.

194: [Passage from Ford] Ford Madox Ford, The Good Soldier (Oxford: World Classics, 1990) 82.

194: [Distinctions have made for meaning] That serious student of human folly, Mrs. Elizabeth Schweppe Whistler, reminded me of

this beautiful passage from Montaigne's "Apology for Raymond Sebond" (in the Frame translation): "Protagoras says that there is nothing in nature but doubt... Nausiphanes, that of things that seem to be, nothing is, any more than it is not... Zeno, that even one is not, and that there is nothing.... According to these doctrines, the nature of things is only a false or empty shadow."

194: [When distinctions rupture, negation floods in] You hear the sounds of this flood on your shortwave radio, where scattered through all those stations speaking their lonely sense are bursts and gurgles of man-made noise, and siren voices reciting strings of random numbers - or worse, a single letter, over and over again. Lurking among the frequencies are the null messages ending with strings of zeroes: the work, says Donald W. Schimmel in his Underground Frequency Guide, of the KGB and its successors (although I have received null e-mails from innocent correspondents just out to verify my address, and knew someone who once tried sending the empty telegram from France - but the keepers of that rational country would have none of it).

194: [The hollow within the ring] Contrast to this view that expressed by Clarisse, in Musil's The Man Without Qualities (Part II, Ch. 84, Wilkins translation): "She had... pulled off her wedding ring, and now she peered through it at the lamplit wall. 'There's nothing inside, and yet it looks as though that were

precisely what matters most."

194: [Zero as pure holding apart] This is true of zero not only in its holding negatives apart from positives, and in its resemblance to the variable, that holds an empty place for a value, and in its role in setting columns apart, but as { }, the pregnant empty set.

FOURTEEN: A LAND WHERE IT WAS ALWAYS AFTERNOON

195: [Passage in Beckett] There is something Beckett-like in the difficulties I have had in tracking this passage down, both on my own, through friends and through some of the world's leading Beckett scholars. Still, having the passage even without its attribution is, I suppose, better than nothing.

195: [Swinburne] The allusion is to "The Garden of Prosperine".

195: [A different pleasure] Numerous changes have been rung on the voluptuousness of annihilation and the embracing of nothing as the ultimate erotic pleasure. In speaking of Wagner's seduction by the Nirvana of Nothingness, Robert Martin Adams [Ad 177-8] writes that the Wagnerian void, "in fulfillment of a tradition as ancient in Europe as in India, is feminine, seductive, sexual - in a word, orgiastic. Voluptuous abandonment of self, whether to love or death or both at once, has a history as old as folklore, instances of it are familiar from Novalis and Keats (to look no

further back), and it achieves classic expression... in the ecstatic rites of Tristan und Isolde." For a surfeit of examples throughout the whole overwrought spectrum, see Ad passim.

195: [Melville] Moby Dick, in the prefatory "Extracts".

195: [Grow convivial upon tears] Some sub-sub-librarian has probably compiled a bibliography of works dedicated to tearful cheer. I hope he has not overlooked Hugo Arnot's Essay on Nothing (Arn), subtitled "A Quaint Jeu d'Esprit" and dedicated - or rather consigned - to Oblivion (from which it is here only temporarily rescued).

196: ["How sweet it were..."] Tennyson, "The Lotos-Eaters", Choric Song v: 1-3. 196: Another line of this is paraphrased in the chapter's title.

196: [Conspicuous leisure] See Veblen, Theory of the Leisure Classes, passim.

196: [Emaciated saints] Santa Fina of San Gimignano is an example.

197: [Flaubert] The reference is to his story "Un coeur simple".

197: [Father Antonio Margil de Jesús] Mar ix.

197: [Need for total abnegation] Mar ix. Worshipping your god with

gigantic numbers, or by reducing yourself to zero, may be no more than different routes down a circular drive.

197: [Those for whom humility is a strategy for salvation] A secular variant on this strategy is the parading of ignorance in order to let others see and praise you for how much you must really know. A nicely ironic subspecies of this was practiced by the mid-nineteenth century Order of the Star Spangled Banner, whose members were sworn to secrecy so that their organization became known as the Knows Nothing Party. The irony lies in this, that the party's aim was to preserve America for Americans, that is, for Protestants, by ridding it of foreigners in general and Catholics in particular, whom they knew to be dead ignorant. Their certainties - like that of so many American extremists - were founded on an ignorance so deep that no critical judgment could flourish in its canyons, so that they were not only Know Nothing in name but indeed in nature.

197: [The noise becomes non-being] Compare the position of Plotinus, for whom matter was non-being.

198: [Passage from Woolf] Woo 70-71.

198: [Some wash their faces to wash away their sins] See, for example, the palindromic inscription in the Mosque of Santa Sophia in Constantinople: Νιψον ανομηματα μη μοναν οψιν:: "Wash

not only your face but your sins." (Bombaugh, C. C., Oddities and Curiosities of Words and Literature (ed. Gardner) (Dover, 1961) 60).

198: [Those for whom mathematics is a play of pure forms] This engenders another peculiarly positive form of nothing, for those who think of beauty as abstract; the abstract as immeasurably remote from our pedestrian lives; and therefore Nothing as the swish of its garments as it brushes us by. So in her sonnet praising Euclidean geometry as such beauty incarnate, Edna St. Vincent Millay writes:

Let all who prate of Beauty hold their peace,
And lay them prone upon the earth and cease
To ponder on themselves, the while they stare
At nothing, intricately drawn nowhere
In shapes of shifting lineage...

This reminds me of Aristotle's remark that a geometric diagram is no more than an *aide memoire*, since the theorem and what it refers to aren't physical. Even more to the point is the position of mathematics in the third part of Plato's divided line. Keep in mind too that in German 'rein' means both 'mere' and 'pure'.

199: [Their motives differ] There is too the ecstatic blending of one's lower-case i in the greater, collective I whose features are those of Everyman. In some eras it appears with the simplified traits of The Worker, in others as the memorial

Unknown Soldier. Jules Romains' novella, Death of a Nobody, reflects back the reader as surely as does the Vietnam War Memorial in Washington. In all of these we sense once again why a blank can stand both for zero and for the variable.

199: [Cap of darkness] Surely it is no accident that the fictional French master criminal, Arsène Lupin, lived in l'aguille creuse, the hollow needle.

199: [Casual anonymity] It is worth recalling here the brilliant invention of the "centralizer" in Alan Harrington's '60s cult novel, The Revelations of Dr. Modesto. Centralizers were always to be found in the midst of any group photograph, and managed to slip through life unnoticed. Not so Harrington's heroes, Joan Vigoro and Mirko, the Human Fly. Compare to these centralizers Woody Allen's Zelig and contrast Lamont Cranston, who as The Shadow "had the power to cloud men's minds."

199: [The spy's studied banality] The face as mask opens the door to all the paranoia of invisible watchers and invisible events. Which most tellingly now encapsulates this syndrome, Antonioni's "Blow-Up" or photographs of the smiling Lee Harvey Oswald?

199: [Henry James' spider-web] In his preface "The Art of Fiction", in The Future of the Novel (ed. Edel) (Vintage, 1956), 12.

199: ["The Shadow and the Flash"] Jack London, "The Shadow and the Flash", in Moon Face and Other Stories (Macmillan, 1906).

199: [Passage from Emerson] Emerson, Ralph Waldo, "Nature" (published anonymously in 1836, republished in 1849 in Nature, Addresses and Lectures).

200: [Markers testifying to the sexlessness of office] On the distinction between 'man' as superordinate, standing for human being, and 'man' as a hyponym for males, see the letter of Hugo F. Reading, "Superordinate Man", in The Times Literary Supplement, June 27, 1997.

200: ['E' as sexless pronoun] I understand from Mira Bernstein that the omnimarker 'E' originated with Donald Knuth in the preface to his Joy of Tex. It declines (always capitalised): E, Er, Eir, Em.

200: ['Ha' as sexless pronoun] I am grateful to Peter Ginna for telling me about this serious contribution to the future of English.

200: [Invisible writers, anonymous readers] There is yet another way in which a zero presence adds value. Textual critics are especially alert to misprints, which may be betrayed by the subtlest of signs. Among these are passages, as in plays, that introduce a *vox nihili*, a voice from nowhere. Such phantom voices, then, betray not a character's but a typesetter's

presence. And beyond literature, may not a voice from nowhere signal someone rather than no one? So a trained listener in Australia, hearing faint, repetitive radio signals, recognized by their slight fluctuations that they weren't mechanically generated but came from a human operating by hand a desalinization pump - leading to the rescue of a capsized sailor.

Henry Yorke, the mid-twentieth century novelist who wrote under the pen-name Henry Green, is a perplexing example of the invisible writer. In a review of Jeremy Treglown's biography of Green, Mick Imlah (Times Literary Supplement, Oct. 16, 2000, pg. 36) speaks of the "instinctive cancellation of the author's ego" which he finds reminiscent of Keats' 'negative capability'. In this context he quotes Terry Southern: "The reader does not simply forget that there is an author behind the work, but... must, in fact, remind himself that there is one... Thus, in the spell of his own imagination the characters come alive in an almost incredible way..." But he also cites what Mary Keene, one of Green's girl friends, wrote of him: "He didn't really exist... There was a hole there. He only really existed in other people. He was living off the fat of other people and once the fat was gone he would go."

200: [Henry James and the Gentle Reader] The gentle reader was also Ada Levenson (Su 311-12).

200-1: [Details of Oken's life] EB vol. 20 56-7, art. "Oken, Lorenz".

201: [Swathed in scarves] At the time he walked there,
Bahnhofstrasse was probably still called Froschengraben: frog-
gutter. It seemed somehow wrong, however, to march so
positive a man as Oken along so negative a ditch.

201: [Zero the primary act] O 13 (§55).

201: [Zero eternal] O 13 (§57).

201: [Zero endlessly positing itself] O 15 (§60).

201: [God is Zero] O 17 (§67), 18 (§68).

201: [God is infinite intensity, man extensity] O 7 (§37).

201: [Man is arithmetic] O 25 (§98).

201: [Man is mathematics] O 26 (§100).

201: [Sea-mucus] O 186 (§901).

201: [Love arose from the foam] O 186 (§909).

201: [Mercilessly clean Zürich] The point is made in Walter Baumann
and Andreas Wolfensberger's Zürich (Zürich: Orell Füssli, 1987),
pg. 175, that Rilke admired the city for its wide variety of

soaps.

201: [Man is God manifested] O 25 (§98).

201: [Man is God conscious of Himself] O 25 (§99).

201: [God = + 0 -...] O 25 (§99).

201: [Man = + ∞ 0 - ∞] It would probably be endlessly gratifying to Lorenz Oken to know that under certain circumstances, zero is infinite. The circumstances are those of "Tropical Algebra", devised in the 1970s and widely used today in understanding, for example, how to optimise paths in a transportation network. In tropical algebra addition and multiplication behave pretty much as they do ordinarily, except that there is no subtraction. This is because addition is peculiarly defined: if a and b are any positive integers, $a '+' b$ is defined as the smaller of the two: $a '+' b = \min(a, b)$. So $2 '+' 3 = 2$ and $3 '+' 7 = 3$. Multiplication, on the other hand, is defined as what used to be addition: $a '·' b = a + b$ (so $2 '·' 3 = 5$ and $3 '·' 7 = 10$). Now if this system has a zero, it must behave as zero would: as the additive identity. So $a '+' 0 = a$. But $a '+' 0 = \min(a, 0)$. If this is to be a , then 0 must be greater than any a - infinitely large, in fact. I am grateful to Mira Bernstein for having this insight in the context of a brilliant Math Circle lecture given by Andrei Zelevinsky.

201: [Lolita on the changing odometer] Vladimir Nabokov, Lolita
(Putnam's, 1955) 251.

201: ["Successive terminal moraines..."] Tur 201.

202: [Traces listed by Turner] Tur 227, 221.

202: [Passage from Wolfe] Wolfe, Thomas, Look Homeward,
Angel (Scribner's, 1934) 36.

202: [Turner lamented closing of frontier] Tur 227.

202: [Passage from Needleman] In Burns, Ken & Amy Stechler Burns,
Hands to Labor, Hearts to God (Television program: Florentine
Films, 1984).

202: [Rimbaud] "Arrivée de toujours, tu t'en iras partout". ("A une
raison", in Illuminations).

FIFTEEN: WAS LEAR RIGHT?

203: ["Nothing will come of nothing"] Lear l.i.89. Lear's nothing will
come of nothing itself comes from Lucretius's "Nothing will
come of nothing" by way of Augustine's "Nothing will come of
nothing", and these in turn - if what they claim is correct -
from sources and sources and sources earlier yet, until
imagination's triple dots terminate in scholarship, which

notoriously has no end.

203: [Napier inventing binary] Nap 115, 129 f., and especially Cap. VIII (133 ff). Cf. She 122-3 and 125. Napier's invention, which he called "local arithmetic" (*arithmeticae localis quae in scacchiae abaco exercitur*), is a very clever method of converting a chess-board into a means of multiplication, by labelling its rows and columns with powers of 2. "It is", says R. A. Sampson (in Kno 191), "the expression of any number in the scale of radix 2, in which consequently every digit is either 0 or 1, and a mechanical method of multiplication based upon this, by means of an areal abacus or enlarged chess board, the multiplier being represented by points upon one side and the multiplicand by points upon another. The method is applied further to the extraction of square roots."

In the matter of binary Napier's priority is again contested (see note to pg. 130) by the advocates of Harriot, with the same difficulty of establishing what Harriot did and when, since his work was published posthumously.

206: [When will $4/7$ come up?] As the 28th item.

206: [Invention of Farey sequence] Har 36, Di 156.

207-212: [Deriving all numbers from 0] Even more interesting - and proportionately more difficult to answer (a book would hardly

suffice) - is the question of what the source of mathematical *invention* is. It can't lie in the world of experience: Kant rightly showed that experience follows from what mathematics studies. For all the work we do in familiarizing ourselves with and exploring a problem, we never know when or even if an answer will come - and when it does, it rushes up suddenly from nowhere - unexpected, unannounced - when stepping onto a bus, or waking from a dream. For all the impression its completed works give of being analytic - each predicate neatly tucked into its subject - any worker in the field will tell you that mathematics is synthetic: made from the gauziest whole cloth. Perhaps Janós Bolyai gave us our best answer, in a letter he wrote to his father Wolfgang in 1823 (as quoted in R. L. Faber's Foundations of Euclidean and Non-Euclidean Geometry, New York: Marcel Dekker, 1983):

I have made such wonderful discoveries [in hyperbolic geometry] that I have been almost overwhelmed by them, and it would be the cause of continual regret if they were lost.... I have created a new universe from nothing.

I am grateful to the unparalleled Jim Tanton for telling me about Galileo's patron and friend, Guidobaldo del Monte (1545-1607), who thought that, in proving $0 = 1$, he had witnessed the creation of something out of nothing, and so had established the existence of God. For by using, it seemed, only the Associative Law of Addition, he argued that

$$0 = (1 - 1) + (1 - 1) + \dots = 1 + (-1 + 1) + 1 + (-1 + 1) + \dots = 1.$$

Everyone knows that anyone who lived before us was twice as ignorant and half as bright as we are. In this spirit J. W. N. Sullivan (Aspects of Science, Second Series, Knopf 1926) wrote that "... in the time of Leibniz it was believed that the sum of an infinite number of zeros was equal to 1/2; and it was attempted to make this idiocy plausible by saying that it was the mathematical analogue of the creation of the world out of nothing."

Never mind that Sullivan has the date and the series wrong (the same kind of reasoning would give:

$$0 = 0 + 0 + \dots = (1/2 - 1/2) + (1/2 - 1/2) + \dots = 1/2 - (1/2 - 1/2) - (1/2 - 1/2) - \dots = 1/2).$$

What matters is that an arrogant assumption of the world's simplicity blinds us to its subtle constitution.

It took a lot of time and hard thought, in the nineteenth century, to realize that certain infinite series cannot be regrouped just anyhow, as finite sums can. Mathematics and mystery ever curtsy to each other.

207: [The Salem Codex] Date: M 411; content: M 423.

208: [O'Creat] S1 i 204 n. 5 says that he is "probably the same as Joh. Ocreatus", perhaps a student of Adelard's (Bub 174 n. 7), perhaps someone who did no more than write his name in a copy of Adelard's translation of Euclid from the Arabic (so

Wüstenfeld, cited in Bub 174 n. 7). It would be pleasing if his first name had been - or had been made up, along with him, to be - Neil or Niall, completing the pun. A M. Jourdain, cited in (He 9), says of him: "...un certain O'Creat, écrivain inconnu à tous les biographes anglais que nous avons consultés..."; and Henry himself (He 9) supplies the perfect epitaph: "Malheureusement les renseignements sur O'Creat sont nuls." He is, I take it, the man upon the stair.

209: [von Neumann on understanding mathematics] We 2 259.

210: [A man who wasn't there] This image is from a verse that is almost not there itself. It appears (under the rubric "Rootless Rhymes") in Roger Lancelyn Green's A Century of Humorous Verse (London: Dent & Sons, 1959) in this form:

As I was coming down the stair
I met a man who wasn't there.
He wasn't there again to-day:
I *wish* that man would go away!

In fact, however (if 'fact' is the apposite word), the verse isn't anonymous and its sense is reversed. It is by Hughes Mearns (1875-1965), is called "The Psychoed", and reads:

As I was walking up the stair
I met a man who wasn't there.
He wasn't there again today -
I wish, I wish he'd go away.

The psychotic element referred to in the title is, of course, the

wish to avoid this man, since it is widely believed unlucky to meet someone on the stairs (see the article 'Stairs' in Opie and Tatem's A Dictionary of Superstitions); hence Mr. Mearns should have thanked his (missing) stars.

210 [The empty set]: For the mathematician L. E. J. Brouwer the principle of mathematical induction rests on a 'before-after' relation abstracted from every emotional content and divested of all quality. This relation Brouwer called "the empty form of the common substratum of all two-ities". See W. P. van Stigt, Brouwer's Intuitionism (North-Holland 1990), pg. 153. The empty set has at least the form of a set, with its initial and terminal brackets. Brouwer's 'empty form' might therefore be even emptier.

210: [The empty set an exception] The mathematician Mike Artin tells me that in a course on set theory once given at M. I. T., the instructor informed his students that "Some people say there is a set which has no members - but we won't talk about that."

210: [Passage from Zermelo] vH 202.

211: [Wesley Salmon's proof] Ga1 32-33.

211: [Symbols for the empty set] Can anything stand for itself without precipitating us into hopeless logical conundrums? Among the various symbols for the empty set and its kin is that

used by Milnor and Stasheff in their book, Characteristic Classes, pg. 183, for the vacuous partition: namely, (my thanks to a reader, Joe Thrash, for calling this to my attention). Yet how can an absence stand for and not *be* an absence? If it stands for it, must it not fill it and so be its own worst possible instantiation? As you will see in the note to pg. 212, on making the abstract palpable, zero cannot be its own referent. You could argue, however, that since something always stands for something, by rights only nothing could stand for nothing. This may all be made clear in an elusive Elizabethan work I know of but (appropriately enough) cannot find, called Q Per Se Q.

211: [von Neumann's proof of 1923] In his "Zur Einführung der transfiniten Ordnungszahlen" (Mac 94).

212: [Quotation from Kronecker] Dan 101.

212: [Rilke] In a letter to his Polish translator Witold von Hulewicz, quoted in Duino Elegies (trans. Leishman and Spender), (New York: Norton, 1939) 128.

212: [Is our task to make the abstract palpable?] It would be worth thinking through at some greater length than here the relation between zero and holes. I have suggested that our symbol for zero came from the hole left by a counter removed from a sanded board, so that the natures of both seem twinned. Yet here

we have just seen everything generated from nothing, as if zero were all, in the end, that there was. Holes, however, cannot be the only things around: so reads Theorem 1.4 in Cas 193. For Casati and Varzi, "a hole is an immaterial body located at the surface (or at some surface) of a material object." (ibid., 192). They also deduce from their axioms that "If the universal individual exists, it surely isn't a hole" (T2.15, pg. 197), and "The null individual (if it exists) cannot be a hole" (T2.16, pg. 197). Zero, then, isn't the same as a Casati and Varzi hole. The fear (or hope) that zero might be that rarity, a sign which is its own referent, is thus dispelled. Casati and Varzi even boldly suggest (pg. 186), in a Leibnizean spirit, that since holes have spatial and temporal qualities and nothingness hasn't, we may suspect that nothingness might therefore not be real. In any event, I find it elating that zero isn't a depression.

212: [von Neumann's proof disquieting] We should acknowledge that our logical qualms are probably paralleled by moral ones, since it is unsavory at least to a Protestant work ethic to get something for - or even from - nothing. Perhaps we can see in this microcosm of nothing the distinction between the serious and the playful, since it is a quintessentially playful spirit that animates the reincarnation of von Neumann's approach in John Conway's games, where worlds are unfolded from the null set.

213: [Omar Khayyam] Rubáiyát, quatrain L.

213: [Pierce's paper] Sty 259-60.

213: ["My damned brain has a kink in it..."} Bel 556.

213: [Pierce's and Sheffer's notations] Sty 259-60.

213: [Sheffer's calligraphic zeal] L&L 306 n.

213ff: [The derivation of all logical connectives from "neither-nor"]

How to derive the logical structure of sentences from
"neither-nor"

1. A sentence can only have truth-value 0 or 1.
2. The possible combinations of these values for two sentences, A and B, are:

| A | B |
|---|---|
| 1 | 1 |
| 1 | 0 |
| 0 | 1 |
| 0 | 0 |

3. There are 16 possible ways of assigning 0 and 1 to these four rows:

| A | B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|
| 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 4 | | | | | 6 | | | | | 4 | | | 1 | | | | |

(Devotees of Pascal's Triangle will enjoy the number of columns that have 4, 3, 2, 1 or 0 ones in them).

4. Each column is the truth-table of a connective; we recognize at once the meaning of some of them. Column 1, for example, is pure tautology (true regardless of the truth-value of its parts), 16 pure contradiction (false regardless of its parts). 12 is the familiar "A and B", 2 the 'inclusive or' of mathematics (true so long as at least one part is true), 11 the 'exclusive or' of common speech (true only if exactly one part is true). 4 is "If A then B", 15 is $A \downarrow B$ ("neither A nor B"), and so on.

5. We will show how to derive the columns from column 15 alone. Note that whatever their meaning, two ways of connecting A and B, no matter how complicated, are equivalent if they have the same truth-table.

6. The key step is observing that "not A" (true when A is false, false when A is true) is $A \downarrow A$, since they have the same truth-tables:

| | | |
|---|-------|------------------|
| A | not A | $A \downarrow A$ |
| 1 | 0 | 0 |
| 0 | 1 | 1 |

For $A \downarrow A$ has value 1 only when both its parts have value 0.

Here both parts are the same: A.

7. To get column 2, for example (the 'inclusive or', which I'll just call "or" from now on), think through the fact that it is the negation of $A \downarrow B$, which you can check by comparing their truth-tables:

| | | | |
|---|---|--------|--------------------------|
| A | B | A or B | not ($A \downarrow B$) |
| 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 |

But as we have just seen, the negation of any sentence C is equivalent to $C \downarrow C$. So if C is the sentence $A \downarrow B$, its negation, "not($A \downarrow B$)", is $(A \downarrow B) \downarrow (A \downarrow B)$. This last formula, using only the connective \downarrow , is thus equivalent to "A or B".

8. All the rest of the columns will be drawn from $A \downarrow B$ in the same way. "If C then B", for example, is only false, mathematically, when C is true and B is false (i.e., you can't draw false conclusions from true premises, but anything

else goes). This is column 4. Think through that this amounts to "(not C) or B". Since we have, from step 7, the formula $(A \downarrow B) \downarrow (A \downarrow B)$ for "A or B", we will simply replace A in it by "not C" - that is, by $C \downarrow C$, giving us $[(C \downarrow C) \downarrow B] \downarrow [(C \downarrow C) \downarrow B]$, and this is the formula for "If C then B", expressed wholly with the connective \downarrow (you might check that this expression has the same truth-table as column 4).

9. These strings in which the only connective is ' \downarrow ' can be cumbrously long. The straight-forward "C and D", for example, is equivalent to "not (not C or not D)" (each has value 1 only when C and D each has value 1). Well, "not C" is $C \downarrow C$, "not D" is $D \downarrow D$; and as you know from step 7, we get "A or B" from $(A \downarrow B) \downarrow (A \downarrow B)$. Replacing A by $(C \downarrow C)$, B by $(D \downarrow D)$, "not C or not D" is

$$[(C \downarrow C) \downarrow (D \downarrow D)] \downarrow [(C \downarrow C) \downarrow (D \downarrow D)].$$

But we want the negation of this, which means writing the string twice with \downarrow between; so that "C and D" is the monstrous

$$\{[(C \downarrow C) \downarrow (D \downarrow D)] \downarrow [(C \downarrow C) \downarrow (D \downarrow D)]\} \downarrow \{[(C \downarrow C) \downarrow (D \downarrow D)] \downarrow [(C \downarrow C) \downarrow (D \downarrow D)]\}.$$

But then, our aim wasn't beauty but showing that all the

connectives could be derived from the solitary "neither-nor".
The rest follow, with more or less labor.

10. You could also derive all columns from column 5, the negation of "A and B" (called by some "the incompatibility of A and B"). This is what Sheffer did in 1913, representing it by "A|B". He used the same symbol Pierce had for "neither A nor B", and now universally called "The Sheffer Stroke".
'↓' replaced '|' for "neither-nor" some time later.

214: [Technicality or deep insight] See the discussion in Wa 401, which contains the curious remark that "One might speak here of the discovery of a new aspect. Could one seek for such an aspect? No; that something can be seen in a certain way we can see only when it is seen in that way. That an aspect is possible we see only when it is already before us."

215: [Whereof one cannot speak...] We know that in the very temple of Delight veil'd Melancholy hath her sovran shrine, and likewise that where God hath a temple, the Devil will have a chapel - but had any save readers of T. S. Eliot suspected that there is a void at the center of our verbal exchanges? So I now understand from the ingenious Lucien Preuss, who tells me that the measure of information delivered, for example, by one letter of a message depends on the number of letters that might have been there but aren't (since if there could with equal probability have been any of 26 letters, the amount of

information delivered by this one is $\ln(26)$). What might have been there but isn't: a void, says Preuss, the size of an occasionally ill-defined theca; and he refers the interested reader to E. T. Jaynes, Papers on Probability, Statistics and Statistical Physics, ed. Rosenkrantz (D. Riedel, 1983, pp. 211-314: "Where Do We Stand on Maximum Entropy?", especially pp. 233-4).

What, by the way, of ventriloquists, who speak while remaining silent? Wittgenstein once remarked that his voice came from somewhere near his ear - but the voice of the ventriloquist comes from somewhere near our ear as well, and this auditory dislocation puts in question the self as expressed by the body. Is it nowhere, or no thing? Is it this turning of persona into a void that gives the ventriloquist's dummy his confident insolence and the Delphic Oracle her prophetic aura? Steven Connor's Dumbstruck: A Cultural History of Ventriloquism (Oxford 2001) is suggestive of much speculation.

215: [Empty expressiveness of paper] A variation on something from nothing is something *through* nothing: the abhorrent vacuum inspiring us to fill it. Some of the most vivid examples are from literature, such as the blank page that aroused Mallarmé to scatter over it his little torn squares of paper with their crabbed linguistic notes, taken from his big wooden tea-chest. Then words would form almost invisible ligatures among themselves and the most diaphanous grammar would fall on them, until a poem whose subject was virtually nothing

released its virtual relations into the surrounding emptiness,
to mean for an Orphic moment.

SIXTEEN: THE UNTHINKABLE

216: [Julian Huxley] "Science and Society", a BBC broadcast of 1937.

This figure of thought, which I fondly call the Lichtenberg Gambit (after Georg Christof Lichtenberg's gently ironic remark about how wonderful it is that a cat's pelt has holes in it just where the eyes happen to be) is played out still in many a serious-seeming game. There is an instance of it in a Scientific American note on "Science and Religion" (August 1998, pp. 20-21), where what is dubbed the Anthropic Principle - namely, that many of the constants of nature have to be exactly as they are for life to exist - is taken as teleological evidence for God's existence (but if the constants were different, the universe would be playing some other melody perfectly). As Bacon remarked, where are the memorials in chapels to God's goodness from the sailors who did drown?

217: [Anything asserted about what doesn't exist is true] In his Proofs and Refutations (Cambridge, 1976) Imre Lakatos remarks (44 n. 3) that statements that hold for the empty set - vacuously true statements - "were a major innovation of the nineteenth century. Its problem-background has not yet been unfolded." He adds (104 n. 2): "...turning the empty set from a monster into an ordinary *bourgeois* set was an important event

- connected not only with the Boolean set-theoretical re-interpretation of Aristotelian logic, but also with the emergence of the concept of vacuous satisfaction in mathematical discussion."

217: [Schopenhauer passage] Scho apx. 17, quoted in Ja 1 28.

217: [Grübelsucht] Ja 1 41.

218: [A garden whose every leaf...] Lei paragraph 67.

218: [Salt dissolved in water] Z1 336.

218 [Nagarjuna and identity of opposites] Z1 520ff.; see Ru 175 ff.
on the Buddhist theory of sunyata.

There is much more to say about nothing, both in the outer language of experience and the inner language of mathematics; and numberless pages to write about zero, from sets of measure zero, zero-sum games, the location of zeroes in approximating functions and reconstructing a function from its zeroes, to zero-dimensional objects and nilpotent elements, not to mention (although in the fecund spirit of zero I now will) categories with no or many zero objects, polygons with zero area and line segments chock-full of points but with no length whatever (so great is the franchise of mathematics). The answer to a great mystery, the precise position of the primes in the set of natural numbers, lies coded in the zeroes of the Riemann zeta function - but no one has yet lifted that veil (a veil in fact thickened when what zeroes we knew of this function were

erased with the blowing up of the University of Wisconsin's computer, during the Greening of America; yet knowledge is never destroyed, only changed in form, and these zeroes, with many more, have since been recovered). What, indeed, in mathematics doesn't come down (as Mike Artin once remarked) to showing that some equation is equal to zero, or that some set is empty?

But I must take to heart Michael Stifel's advice, *clausis oculis abeam*, and leave with eyes closed.