

**Extended rotation groups (double groups):**
*Character tables and direct product tables*

$D_2^*$	$E$	$R$	$2C_2(z)$	$2C_2(y)$	$2C_2(x)$
$E_{1/2}$	2	-2	0	0	0

$D_3^*$	$E$	$R$	$2C_3$	$2C_3R$	$3C_2$	$3C_2R$
$E_{1/2}$	2	-2	1	-1	0	0
$E_{3/2}$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$	$\left\{ \begin{array}{l} -1 \\ -1 \end{array} \right.$	$\left\{ \begin{array}{l} -1 \\ -1 \end{array} \right.$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$	$\left\{ \begin{array}{l} i \\ -i \end{array} \right.$	$\left\{ \begin{array}{l} -i \\ i \end{array} \right.$

$D_4$	$E$	$R$	$2C_4$	$2C_4R$	$2C_2$	$4C_2'$	$4C_2''$
$E_{1/2}$	2	-2	$\sqrt{2}$	$-\sqrt{2}$	0	0	0
$E_{3/2}$	2	-2	$-\sqrt{2}$	$\sqrt{2}$	0	0	0

$D_6^*$	$E$	$R$	$2C_6$	$2C_6R$	$2C_3$	$2C_3R$	$2C_2$	$6C_2'$	$6C_2''$
$E_{1/2}$	2	-2	$\sqrt{3}$	$-\sqrt{3}$	1	-1	0	0	0
$E_{3/2}$	2	-2	$-\sqrt{3}$	$\sqrt{3}$	-1	1	0	0	0
$E_{5/2}$	2	-2	0	0	-2	2	0	0	0

$T_d^*$	$E$	$R$	$8C_3$	$8C_3R$	$6C_2$	$6S_4$	$6S_4R$	$12\sigma_d$
$O^*$	$E$	$R$	$8C_3$	$8C_3R$	$6C_2$	$6C_4$	$6S_4R$	$12C_2'$
$E_{1/2}$	2	-2	1	-1	0	$\sqrt{2}$	$-\sqrt{2}$	0
$E_{5/2}$	2	-2	1	-1	0	$-\sqrt{2}$	$\sqrt{2}$	0
$G_{3/2}$	4	-4	-1	1	0	0	0	0

$$E_{1/2} \times E_{1/2} = [A] + B_1 + B_2 + B_3$$

	$E_{1/2}$	$E_{3/2}$
$E_{1/2}$	$[A_1] + A_2 + E$	$2E$
$E_{3/2}$		$[A_1] + A_1 + 2A_2$

	$E_{1/2}$	$E_{3/2}$
$E_{1/2}$	$[A_1] + A_2 + E$	$B_1 + B_2 + E$
$E_{3/2}$		$[A_1] + A_2 + E$

	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
$E_{1/2}$	$[A_1] + A_2 + E_1$	$B_1 + B_2 + E_2$	$E_1 + E_2$
$E_{3/2}$		$[A_1] + A_2 + E_1$	$E_1 + E_2$
$E_{5/2}$			$[A_1] + A_2 + B_1 + B_2$

	$E_{1/2}$	$E_{5/2}$	$E_{3/2}$
$E_{1/2}$	$[A_1] + T_1$	$A_2 + T_2$	$E + T_1 + T_2$
$E_{5/2}$		$[A_1] + T_1$	$E + T_1 + T_2$
$G_{3/2}$			$[A_1 + E + T_2] + A_2 + 2T_1 + T_2$

*Direct products of ordinary and extended representations for  $T_d^*$  and  $O^*$*

	$A_1$	$A_2$	$E$	$T_1$	$T_2$
$E_{1/2}$	$E_{1/2}$	$E_{5/2}$	$G_{3/2}$	$E_{1/2} + G_{3/2}$	$E_{5/2} + G_{3/2}$
$E_{5/2}$	$E_{5/2}$	$E_{1/2}$	$G_{3/2}$	$E_{5/2} + G_{3/2}$	$E_{1/2} + G_{3/2}$
$G_{3/2}$	$G_{3/2}$	$G_{3/2}$	$E_{1/2} + E_{5/2} + G_{3/2}$	$E_{1/2} + E_{5/2} + 2G_{3/2}$	$E_{1/2} + E_{5/2} + 2G_{3/2}$