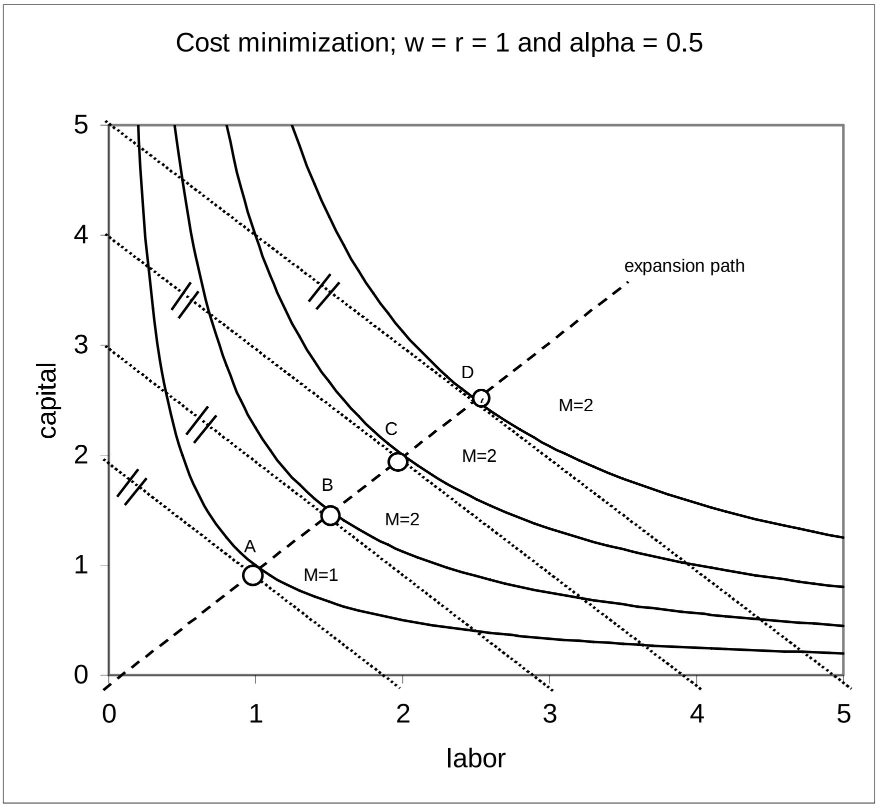
Chapter 4 Production Structure

Question 4.4

4.4A The graph below has four isoquants and four isocosts lines. Which isoquant for $M = 2$ corresponds to constant returns to scale relative to the given $M = 1$ isoquant?

Figure: Production isoquants



4.4B Suppose the wage rate increases, what happens to the slope of the expansion path? Explain.

4.4C Suppose the production process becomes more capital intensive, what happens to the slope of the expansion path?

4.4D Can we derive, by looking at the production structure such as in the graph above, how much manufactures will be produced? Explain.

Question 4.5

In the question 4-5 Excel file on the website you can find the capital stock per worker and real GDP per capita for a set of countries, taken from the Penn World Tables dataset version 5.6. Unfortunately, capital stock per country data is only available for 1992. Therefore, all other data are for 1992 as well.

4.5A Is there a relationship between GDP per capita and the capital stock per worker? Make a graph to illustrate your findings. (Hint: make a scatter plot)

4.5B Can you explain this relationship?

4.5C Why is the there no perfect correlation between GDP per worker and capital per worker?

4.5D Suppose your country is at the bottom of the Real GDP per worker list. What does it need to do to climb up?

4.5E Can you give examples of countries that are climbing the ranks? Can you give practical examples of the capital stock in a country?

Question 4.6

Consider a Dutch manufacturer who employs capital and labour in a neo-classical world to produce bicycles under constant returns to scale. The question 4-6 Excel file on the website is designed to assist you in determining the cost-minimising input combination of capital and labour to produce one bicycle. The file allows you to change the isocost line by changing the wage rate, rental rate, and total cost; the isoquant by changing capital intensity and total production; and a point of production by changing the capital and labour input. The starting values are: $r = 1$, $w = 1$, total cost = 2, $\alpha = 0.5$, $X = 1$, $K = 1$ and $L = 1$. Note that some of the values have to be changed before you can answer the questions below.

As a result of the booming Dutch economy, the manager of the firm is confronted with a rising wage rate for his workers, who now receive a wage rate of 2 instead of 1. The increased wage rate forces him to reconsider the current production plan.

4.6A Change the simulation parameters in the file in accordance with the information above to arrive at a new cost-minimising equilibrium for the production of o*ne* bicycle. How many units of capital and labour does the entrepreneur use? (Hint: you can either use equation 4.A5 in the Technical Notes, or trial and error to arrive at the correct combination; your answer does not have to be exact).

Now that he knows the optimal capital and labour inputs for the production of one bicycle the entrepreneur wants to determine what is needed to produce two bicycles.

4.6B Find the cost-minimising input combination for the production of two bicycles in the simulation. Is the capital-labour ratio the same as in question 4.6A? Why?

The entrepreneur is not amused by the new market conditions. He decides to change the type of bicycles he produces to become less dependent on fluctuations in labour costs.

4.6C Change the simulation parameter values so that we have a more capital-intensive production process.

4.6D Find the new optimal capital-labour ratio. Did the capital-labour ratio change relative to your answer in question 4.6B? Why?

4.6C Did the change you made in question 4.6C affect the cost share of labour? Explain.

Question 4.7

Technical Note 4.1 solves the unit cost minimisation problem for a Cobb-Douglas production function. This functional form is easy to work with and gives simple solutions. There are, however, many other possible production functions. Consider, for example, the following Constant Elasticity of Substitution (CES) production function for Food:

$$F = \left(\alpha_f K_f^{-\rho} + (1-\alpha_f) L_f^{-\rho}\right)^{\frac{-1}{\rho}}$$

4.7A Show that this production function also exhibits constant returns to scale by using the same procedure as in equation 4.4 of the book.

4.7B Define the Langrangean, as in Technical Note 4.1, and derive the first order conditions for unit cost minimisation using the CES production function above.

4.7C Using your answer to 4.7B, determine the optimal capital-labour ratio for the firm.

4.7D Show that the optimal choice of labourers and capital for the production of one unit of food is:

$$K = \left(\frac{\alpha_f}{r}\right)^{\frac{1}{\rho+1}} \left[\alpha_f^{\frac{1}{\rho+1}} r^{\frac{\rho}{\rho+1}} + (1-\alpha_j)^{\frac{1}{\rho+1}} w^{\frac{\rho}{\rho+1}} \right]^{\frac{1}{\rho}}$$

$$L = \left(\frac{1-\alpha_f}{w}\right)^{\frac{1}{\rho+1}} \left[\alpha_f^{\frac{1}{\rho+1}} r^{\frac{\rho}{\rho+1}} + (1-\alpha_j)^{\frac{1}{\rho+1}} w^{\frac{\rho}{\rho+1}} \right]^{\frac{1}{\rho}}$$

4.7E What is the minimum cost of producing one unit of food?

Question 4.8

Calculate the slope of the isoquant of food for the CES production function given in the previous exercise. You can use the "implicit function rule" which states that if you have a certain function " $F(x, y) = \text{constant}$ " and you want to take the first order derivative of y with respect to x, you can calculate:

$$\frac{dy}{dx} = -\frac{\partial F/\partial x}{\partial F/\partial y}$$