3.d. On oscillating ecological models and time step convergence

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Introduction

Ecological models often comprise descriptions of biological, physical and chemical processes by means of differential equations (d.e.), relating the rates of change of the states to, often, themselves, to other states, and to environmental conditions: *dState1/dt* = f{*State1, State2, . . ., parameters, external conditions, etc.*}. The *State* as a function of time can be obtained from these equations by numerical integration, using Euler's scheme: $State_{t+\Delta t} = State_t + \Delta t\,(dState/dt)_t$. It will be demonstrated that Δt can not be choosen freely, but that its correct choice depends on the so-called characteristic time (τ) of the d.e. The characteristic time is the time needed to reach an equilibrium if the rate would be kept constant.

Demonstration

If G_E represents exponential growth, *W* water in a water tank, and G_L logistic growth, their d.e.'s could be: $dG_E/dt = G_E/\tau$; $dW/dt = (W_m - W)/\tau$ and $dG_L/dt = (1/\tau)\,G_L\,(1.0 - G_L/G_{L\max})$. The maximum value of *W* and G_L to be reached is W_m and $G_{L\max}$, respectively. The inverse of tau, $1/\tau$, may be called relative growth rate, relative inflow rate, and relative growth rate, respectively. Figure 1 shows the results of the numerical integrations of these equations. The time steps Δt used for the integrations and relative to τ are indicated in the figure caption for all three models. The solutions for an infinitesimally small Δt, i.e. the analytical solutions, are also plotted.

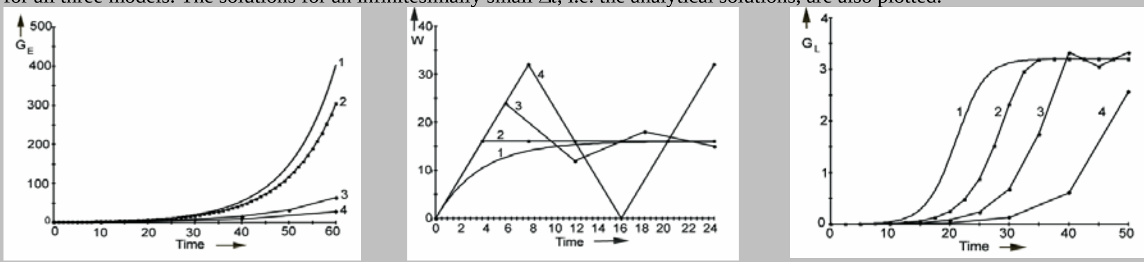

Figure 1. Numerical solutions to the d.e.'s for Exponential growth (a), a Water tank (b), and Logistic growth (c). Numbers 1, 2, 3, 4, mean for (a) $\Delta t = 0.01\tau, 0.1\tau, \tau$, and 2τ; for (b): $\Delta t = 0.1\tau, \tau, 1.5\tau$, and 2τ; and for (c): $\Delta t = 0.1\tau, \tau, 2\tau$, and 4τ, respectively.

What can we learn from Figure 1?

- Too large a time step may lead to oscillating solutions: nr. 3 and 4 in both the case of the Water tank and the Logistic growth.
- If $\Delta t = \tau$, the equilibrium is directly reached: nr. 2 in the Water tank. This is, in fact, the definition of τ. In the cases of Exponential and Logistic growth this can not be observed because of the (initial) positive feedback.
- A nice looking curve does not guarantee that a correct time step is choosen: nr. 2, 3, 4 in Exponential growth, and nr. 2 in Logistic growth.
- A good first estimate of the time step is: $\Delta t = \tau/10$: nr. 2 in Exponential growth and nr. 1 in both the Water tank and the Logistic growth.
- The final Δt can only be found by trial-and-error by diminishing the stepsize Δt until subsequent results do hardly change anymore. Solutions should in principle always converge to the analytical solution if existing. This principle is called time step convergence. In Exponential growth a $\Delta t = 0.01\tau$ gave a better result than with 0.1τ (nr. 1), but for the Water tank and the Logistic growth the $\Delta t = \tau/10$ rule is very good already: in the figures one can hardly see the difference with the analytical solutions. This is due to the negative feedback of the latter two models in contrast with the positive feedback of the Exponential growth where errors keep on accumulating.
- The time step in ecological models comprising more than one d.e. will be determined by the smallest τ.

Application of the principle of time step convergence

One of my MSc students was to work with a complex ecological model that, among other things, included runoff and erosion. The model originated from a renowned institute and was used in different projects in the Netherlands and abroad to advise policy makers. Since we did not know the model, we first wanted to get some insight in its time step and characteristic time. Therefore, we applied the trial-and-error method and hoped to assess the time step convergence of the model. Figure 2 shows the result of our efforts.

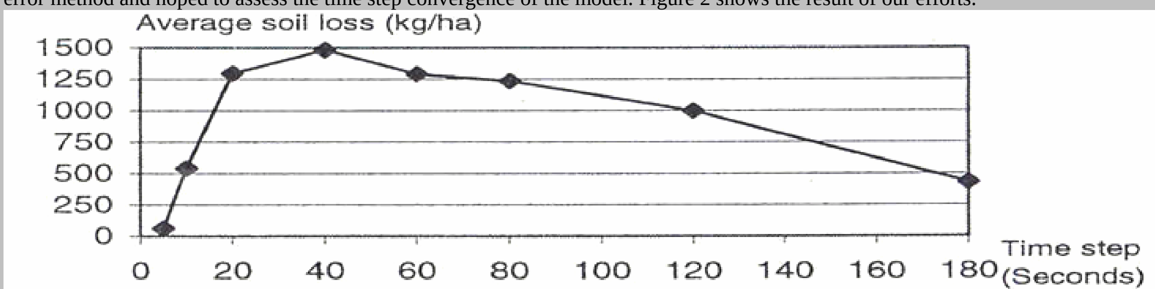

Figure 2. Change in average soil loss at different time steps.

Astonishingly there was no time step convergence at all. We could get any value of average erosion between 0 and 1500 kg ha^{-1}, depending on the time step choosen. The renowed institute panicked, could not find the error and the model was reprogrammed later on. My student could graduate, but he will never just believe models and programs to be true a priori anymore.

This is a very unfortunate example showing that time step convergence is a simple, yet powerful method of investigating complex programs.

Further reading

Leffelaar, P.A. 1999 (Ed.). On systems analysis and simulation of ecological processes; with examples in CSMP, FST and FORTRAN. Second Edition. Series title: Current Issues in Production Ecology, Vol 4. Kluwer academic publishers, 318 pp.

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