12 Random effects

Random effects are analysed using the general linear model commands, with a few additions:

- Those explanatory variables which are random must be declared as such.
- Additional output may be requested from a new command, VARCOMP, which relates to the new concepts of expected mean squares and variance components introduced in the main text.

It is in this context that you most commonly come across 'nested' variables.

12.3 A one-way ANOVA with a random factor

This first most simple example does not involve nesting, but introduces the extra output that may be produced with a random factor. We choose sstype(1) so as to base the variance components on the Sequential Sums of Squares. This is consistent with the methods used in the book and in the other languages.

SPSS COMMANDS FOR BOX 12.1 **One-way ANOVA with a random factor**

Syntax

```
glm CACONC by LEAF
  /random LEAF
  /design LEAF.

varcomp CACONC by LEAF
  /random LEAF
  /method sstype(1)
  /design leaf.
```

Menu route

Analyze > General Linear Model > Univariate

CACONC → Dependent Variable

LEAF → Random Factor(s)

Analyze > General Linear Model > Variance Components

CACONC → Dependent Variable

LEAF → Random Factor(s)

Options

⊙ ANOVA

SPSS OUTPUT FOR BOX 12.1 **One-way ANOVA with a random factor**

General linear model

Between-Subjects Factors

		N
LEAF	1	4
	2	4
	3	4
	4	4

Tests of Between-Subjects Effects

Dependent Variable: CACONC

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	Hypothesis	4.936	1	4.936	52.728	.005
	Error	.281	3	9.362E-02[a]		
LEAF	Hypothesis	.281	3	9.362E-02	8.702	.002
	Error	.129	12	1.076E-02[b]		

a. MS(LEAF)

b. MS(Error)

Expected Mean Squares [a,b]

	Variance Component		
Source	Var(LEAF)	Var(Error)	Quadratic Term
Intercept	4.000	1.000	Intercept
LEAF	4.000	1.000	
Error	.000	1.000	

a. For each source, the expected mean square equals the sum of the coefficients in the cells times the variance components, plus a quadratic term involving effects in the Quadratic Term cell.

b. Expected Mean Squares are based on the Type III Sums of Squares.

Variance components estimation

Factor Level Information

		N
LEAF	1	4
	2	4
	3	4
	4	4

Dependent Variable: CACONC

Variance Estimates

Component	Estimate
Var(LEAF)	2.072E-02
Var(Error)	1.076E-02

Dependent Variable: CACONC
Method: ANOVA (Type I Sum of Squares)

12.4 A two-level nested ANOVA

This second example introduces the concept of nesting. SPSS uses brackets in the DESIGN subcommand to indicate nesting. The variable `LEAF2(PLANT2)` should be read as 'Leaf2 nested within Plant2'. SPSS allows nesting only in the syntax route, so the menu route will be omitted for the next few analyses.

SPSS COMMANDS FOR BOX 12.3 **Two-way nested analysis of variance**

Syntax

```
glm CACONC2 by PLANT2 LEAF2
  /random PLANT2 LEAF2
  /design PLANT2 LEAF2(PLANT2).

varcomp CACONC2 by PLANT2 LEAF2
  /random PLANT2 LEAF2
  /method sstype(1)
  /design PLANT2 LEAF2(PLANT2).
```

Menu route

no nesting permitted in menu route

SPSS OUTPUT FOR BOX 12.3 **Two-way nested analysis of variance**

General linear model

Between-Subjects Factors

		N
PLANT2	1	6
	2	6
	3	6
	4	6
LEAF2	1	8
	2	8
	3	8

Tests of Between-Subjects Effects

Dependent Variable: CACONC2

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	Hypothesis	135.746	1	135.746	150.033	.001
	Error	2.714	3	.905[a]		
PLANT	Hypothesis	2.714	3	.905	5.736	.022
	Error	1.262	8	.158[b]		
LEAF2(PLANT2)	Hypothesis	1.262	8	.158	3.740	.020
	Error	.506	12	4.218E-02[c]		

a. MS(PLANT2)

b. MS(LEAF2(PLANT2))

c. MS(Error)

(*Contd.*)

SPSS OUTPUT FOR BOX 12.3 (*Contd.*)

Expected Mean Squares[a,b]

Source	Variance Component			
	Var(PLANT2)	Var(LEAF2 (PLANT2))	Var(Error)	Quadratic Term
Intercept	6.000	2.000	1.000	Intercept
PLANT2	6.000	2.000	1.000	
LEAF2(PLANT2)	.000	2.000	1.000	
Error	.000	.000	1.000	

a. For each source, the expected mean square equals the sum of the coefficients in the cells times the variance components, plus a quadratic term involving effects in the Quadratic Term cell.

b. Expected Mean Squares are based on the Type III Sums of Squares.

Variance components estimation

Factor Level Information

		N
PLANT2	1	6
	2	6
	3	6
	4	6
LEAF2	1	8
	2	8
	3	8

Dependent Variable: CACONC2

Variance Estimates

Component	Estimate
Var(PLANT2)	.125
Var(LEAF2(PLANT2))	5.778E-02
Var(Error)	4.218E-02

Dependent Variable: CACONC2
Method: ANOVA (Type I Sum of Squares)

12.5 Mixing random and fixed effects

It is easy to combine fixed and random factors in the same analysis. The output includes a term Q[1] which stands for:

$$\frac{\Sigma\left(\rho_i - \bar{\rho}\right)^2}{a-1}$$

where ρ_i = the true weight gain for piglets from the ith sire, $\bar{\rho}$ = true mean weight gain for all piglets, and *a*-1 are the degrees of freedom for SIRE. This follows the same format as the main text.

SPSS COMMANDS FOR BOX 12.4 **An analysis mixing fixed and random factors**

Syntax

```
glm WGAIN by SIRE DAM
  /random DAM
  /design SIRE DAM(SIRE).

varcomp WGAIN by SIRE DAM
  /random DAM
  /method sstype(1)
  /design SIRE DAM(SIRE).
```

Menu route

no nesting permitted in menu route

SPSS OUTPUT FOR BOX 12.4 **An analysis mixing fixed and random factors**

General linear model

Between-Subjects Factors

		N
SIRE	1	4
	2	4
	3	4
	4	4
	5	4
DAM	1	10
	2	10

Tests of Between-Subjects Effects

Dependent Variable: WGAIN

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	Hypothesis	27.056	1	27.056	104.952	.000
	Error	1.289	5	.258[a]		
SIRE	Hypothesis	1.034	4	.259	1.003	.484
	Error	1.289	5	.258[a]		
DAM(SIRE)	Hypothesis	1.289	5	.258	3.119	.059
	Error	.827	10	8.266E-02[b]		

a. MS(DAM(SIRE))

b. MS(Error)

Expected Mean Squares[a,b]

	Variance Component		
Source	Var(DAM (SIRE))	Var(Error)	Quadratic Term
Intercept	2.000	1.000	Intercept, SIRE
SIRE	2.000	1.000	SIRE
DAM(SIRE)	2.000	1.000	
Error	.000	1.000	

a. For each source, the expected mean square equals the sum of the coefficients in the cells times the variance components, plus a quadratic term involving effects in the Quadratic Term cell.

b. Expected Mean Squares are based on the Type III Sums of Squares.

(*Contd.*)

SPSS OUTPUT FOR BOX 12.4 (*Contd.*)

Variance components estimation

Factor Level Information

		N
SIRE	1	4
	2	4
	3	4
	4	4
	5	4
DAM	1	10
	2	10

Dependent Variable: WGAIN

Variance Estimates

Component	Estimate
Var(DAM(SIRE))	8.757E-02
Var(Error)	8.266E-02

Dependent Variable: WGAIN
Method: ANOVA (Type I Sum of Squares)

These commands will then equip you to conduct the mock analyses of section 12.6.

12.8 Exercises

Examining microbial communities on leaf surfaces

SPSS COMMANDS FOR BOX 12.10(a) **Analysis 1**

Syntax

```
glm DENSITY by TREATMNT
  /design TREATMNT.
```

Menu route

Analyze > General Linear Model > Univariate

DENSITY → Dependent Variable

TREATMNT → Fixed Factor(s)

SPSS OUTPUT FOR BOX 12.10 (a) **Analysis 1**

General linear model

Between-Subjects Factors

		N
TREATMNT	1	30
	2	30

Tests of Between-Subjects Effects

Dependent Variable: DENSITY

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	9543.248[a]	1	9543.248	12.504	.001
Intercept	576534.037	1	576534.037	755.412	.000
TREATMNT	9543.248	1	9543.248	12.504	.001
Error	44265.884	58	763.205		
Total	630343.170	60			
Corrected Total	53809.132	59			

a. R Squared = .177 (Adjusted R Squared = .163)

SPSS COMMANDS FOR BOX 12.10(b) **Analysis 2**

Syntax

```
glm DENSITY by TREATMNT PLANT
  /random PLANT
  /design TREATMNT PLANT(TREATMNT).

varcomp DENSITY by TREATMNT PLANT
  /random PLANT
  /method sstype(1)
  /design TREATMNT PLANT(TREATMNT).
```

Menu route

no nesting permitted in menu route

SPSS OUTPUT FOR BOX 12.10(b) **Analysis 2**

General linear model

Between-Subjects Factors

		N
TREATMNT	1	30
	2	30
PLANT	1	10
	2	10
	3	10
	4	10
	5	10
	6	10

Tests of Between-Subjects Effects

Dependent Variable: DENSITY

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	Hypothesis	576534.038	1	576534.038	156.054	.000
	Error	14777.813	4	3694.453[a]		
TREATMNT	Hypothesis	9543.248	1	9543.248	2.583	.183
	Error	14777.813	4	3694.453[a]		
PLANT(TREATMNT)	Hypothesis	14777.813	4	3694.453	6.765	.000
	Error	29488.071	54	546.075[b]		

a. MS(PLANT(TREATMNT))

b. MS(Error)

Expected Mean Squares[a,b]

Source	Variance Component		
	Var (PLANT (TREATMNT))	Var (Error)	Quadratic Term
Intercept	10.000	1.000	Intercept TREATMNT
TREATMNT	10.000	1.000	TREATMNT
PLANT (TREATMNT)	10.000	1.000	
Error	.000	1.000	

a. For each source, the expected mean square equals the sum of the coefficients in the cells times the variance components, plus a quadratic term involving effects in the Quadratic Term cell.

b. Expected Mean Squares are based on the Type III Sums of Squares.

(*Contd.*)

SPSS OUTPUT FOR BOX 12.10(b) (*Contd.*)

Variance components estimation

Factor Level Information

		N
TREATMNT	1	30
	2	30
PLANT	1	10
	2	10
	3	10
	4	10
	5	10
	6	10

Dependent Variable: DENSITY

Variance Estimates

Component	Estimate
Var(PLANT (TREATMNT))	314.838
Var (Error)	546.075

Dependent Variable: DENSITY
Method: ANOVA (Type I Sum of Squares)

How a nested analysis can solve problems of non-independence.

The SPSS output for this exercise may be found in the answers to exercises.