12 Random effects

Random effects are analysed using the general linear model commands, with a few additions:

- Those explanatory variables which are random must be declared as such.
- Additional output may be requested which relates to the new concepts of expected mean squares and variance components introduced in the main text.
- It is in this context that you most commonly come across 'nested' variables.

12.3 A one-way ANOVA with a random factor

This first most simple example does not involve nesting, but introduces the extra output that may be produced with a random factor.

MINITAB COMMANDS FOR BOX 12.1 **One-way ANOVA with a random factor**

Commands

```
glm CACONC = LEAF;
random LEAF;
brief 1;
ems.
```

Menu route

Stat > ANOVA > General Linear Model
`CACONC` → Response
`LEAF` → Model
`LEAF` → Random Factors
Results…
⊙ Analysis of variance table
☑ Display expected mean squares and variance components

MINITAB OUTPUT FOR BOX 12.1 **One-way ANOVA with a random factor**

```
General Linear Model: CACONC versus LEAF
Factor          Type Levels Values
LEAF          random    4 1 2 3 4

 Analysis of Variance for CACONC, using Adjusted SS for Tests

Source          DF       Seq SS        Adj SS        Adj MS         F          P
LEAF             3      0.28086       0.28086       0.09362     8.70      0.002
Error           12      0.12910       0.12910       0.01076
Total           15      0.40996

Expected Mean Squares, using Adjusted SS

Source          Expected Mean Square for Each Term
1 LEAF             (2) + 4.0000(1)
2 Error            (2)

Error Terms for Tests, using Adjusted SS

Source           Error DF       Error MS  Synthesis of Error MS
1 LEAF              12.00        0.01076          (2)

Variance Components, using Adjusted SS

Source          Estimated Value
LEAF                    0.02072
Error                   0.01076
```

12.4 A two-level nested ANOVA

This second example introduces the concept of nesting. Minitab uses brackets to indicate nesting. The variable `LEAF2(PLANT2)`should be read as 'Leaf2 nested within Plant2'.

MINITAB COMMANDS FOR BOX 12.3 **Two-way nested analysis of variance**

Commands

```
glm CACONC2 = PLANT2 + LEAF2(PLANT2);
random PLANT2 LEAF2;
brief 1;
ems.
```

Menu route: Stat > ANOVA > General Linear Model
CACONC2 → Response
PLANT2 + LEAF2(PLANT2) → Model
PLANT2 LEAF2 → Random Factors
Results…
◉ Analysis of variance table
☑ Display expected mean squares and variance components

MINITAB OUTPUT FOR BOX 12.3 **Two-way nested analysis of variance**

General Linear Model: CACONC2 versus PLANT2, LEAF2

```
Factor                Type    Levels Values
PLANT2                random       4  1 2 3 4
LEAF2(PLANT2)         random      12  1 2 3 4 5 6 7 8 9 10 11 12

 Analysis of Variance for CACONC2, using Adjusted SS for Tests

Source                 DF       Seq SS       Adj SS       Adj MS        F        P
PLANT2                  3      2.71433      2.71433      0.90478     5.74    0.022
LEAF2(PLANT2)           8      1.26200      1.26200      0.15775     3.74    0.020
Error                  12      0.50621      0.50621      0.04218
Total                  23      4.48254

 Expected Mean Squares, using Adjusted SS

Source                     Expected Mean Square for Each Term
1 PLANT2                   (3) + 2.0000 (2) + 6.0000 (1)
2 LEAF2(PLANT2)            (3) + 2.0000 (2)
3 Error                    (3)

 Error Terms for Tests, using Adjusted SS

Source                    Error DF     Error MS    Synthesis of Error MS
1 PLANT2                      8.00      0.15775    (2)
2 LEAF2(PLANT2)              12.00      0.04218    (3)

 Variance Components, using Adjusted SS

Source                        Estimated Value
PLANT2                                0.12450
LEAF2(PLANT2)                          0.5778
Error                                  0.4218
```

12.5 Mixing random and fixed effects

To mix fixed and random factors, only the random factors are declared as such, but both are included in the model formula. The output includes a term Q[1] which stands for:

$$\frac{\Sigma(\rho_i - \bar{\rho})^2}{a-1}$$

where ρ_i = the true weight gain for piglets from the ith sire, $\bar{\rho}$ = true mean weight gain for all piglets, and $a-1$ are the degrees of freedom for SIRE. This follows the same format as the main text.

MINITAB COMMANDS FOR BOX 12.4 An analysis mixing fixed and random factors

Commands

```
glm WGAIN = SIRE + DAM(SIRE);
random DAM;
brief 1;
ems.
```

Menu route

Stat > ANOVA > General Linear Model
WGAIN → Response
SIRE + DAM(SIRE) → Model
DAM → Random Factors
Results…
⊙ Analysis of variance table
☑ Display expected mean squares and variance components

MINITAB OUTPUT FOR BOX 12.4 An analysis mixing fixed and random factors

General Linear Model: WGAIN versus SIRE, DAM

```
Factor             Type   Levels Values
SIRE               fixed       5  1 2 3 4 5
DAM(SIRE)          random     10  1 2 3 4 5 6 7 8 9 10


Analysis of Variance for WGAIN, using Adjusted SS for Tests

Source              DF       Seq SS       Adj MS       Adj SS        F         P
SIRE                 4      1.03439      1.03439      0.25860     1.00     0.484
DAM(SIRE)            5      1.28898      1.28898      0.25780     3.12     0.059
Error               10      0.82658      0.82658      0.08266
Total               19      3.14994
```

```
Expected Mean Squares, using Adjusted SS

Source                Expected Mean Square for Each Term
1 SIRE                (3) + 2.0000 (2) + Q[1]
2 DAM(SIRE)           (3) + 2.0000 (2)
3 Error               (3)

 Error Terms for Tests, using Adjusted SS

Source                 Error DF       Error MS       Synthesis of Error MS
1 SIRE                     5.00        0.25780       (2)
2 DAM(SIRE)               10.00        0.08266       (3)

 Variance Components, using Adjusted SS

Source                   Estimated Value
DAM(SIRE)                        0.08757
Error                            0.08266
```

These commands will then equip you to conduct the mock analyses of Section 12.6.

12.8 Exercises

Examining microbial communities on leaf surfaces

MINITAB COMMANDS FOR BOX 12.10(a) **Analysis 1**

Commands

```
glm DENSITY = TREATMNT;
brief 1.
```

Menu route

Stat > ANOVA > General Linear Model
DENSITY → Response
TREATMNT → Model
Results…
⊙ Analysis of variance table

MINITAB OUTPUT FOR BOX 12.10(a) **Analysis 1**

General Linear Model: DENSITY versus TREATMNT

```
Factor          Type Levels Values
TREATMNT        fixed     2 1 2

Analysis of Variance for DENSITY, using Adjusted SS for Tests

Source           DF       Seq SS       Adj SS       Adj MS          F          P
TREATMNT          1       9543.2       9543.2       9543.2      12.50      0.001
Error            58      44265.9      44265.9        763.2
Total            59      53809.1
```

MINITAB COMMANDS FOR BOX 12.10(b) **Analysis 2**

Commands

```
glm DENSITY = TREATMNT + PLANT(TREATMNT);
random PLANT;
brief 1;
ems.
```

Menu route

Stat > ANOVA > General Linear Model
DENSITY → Response
TREATMNT + PLANT(TREATMNT) -Model
PLANT → Random factors.
Results…
⊙ Analysis of variance table
☑ Display expected mean squares and variance components

MINITAB OUTPUT FOR BOX 12.10(b) **Analysis 2**

General Linear Model: DENSITY versus TREATMNT, PLANT

```
Factor                   Type   Levels Values
TREATMNT                 fixed     2 1 2
PLANT(TREATMNT)          random    6 1 2 3 4 5 6

Analysis of Variance for DENSITY, using Adjusted SS for Tests

Source                  DF      Seq SS      Adj SS      Adj MS       F         P
TREATMNT                 1      9543.2      9543.2      9543.2    2.58     0.183
PLANT(TREATMNT)          4     14777.8     14777.8      3694.5    6.77     0.000
Error                   54     29488.1     29488.1       546.1
Total                   59     53809.1
```

```
Expected Mean Squares, using Adjusted SS

Source                      Expected Mean Square for Each Term
1 TREATMNT                  (3) + 10.0000(2) + Q[1]
2 PLANT(TREATMNT)           (3) + 10.0000(2)
3 Error                     (3)

Error Terms for Tests, using Adjusted SS

Source                  Error DF       Error MS      Synthesis of Error MS
1 TREATMNT                  4.00         3694.5      (2)
2 PLANT(TREATMNT)          54.00          546.1      (3)
Variance Components, using Adjusted SS

Source                    Estimated Value
PLANT(TREATMNT)                     314.8
Error                               546.1
```

How a nested analysis can solve problems of non-independence

The Minitab output for this exercise may be found in the answers to exercises in Chapter 14.