7 Interactions—getting more complex

7.2 Analysis of factorial experiments

The convention used by Minitab to represent interactions is the * sign. When using menus, this will need to be typed into the model box, just as the + signs are, while the variable names can be moved using the mouse as usual.

MINITAB COMMANDS FOR BOX 7.2 A factorial ANOVA of the yield data

Commands

```
glm YIELD=BLOCK + VARIETY + SOWRATE + VARIETY*SOWRATE;
brief 1.
```

Menu route

Stat > ANOVA > General Linear Model
YIELD → Response
BLOCK + VARIETY + SOWRATE + VARIETY*SOWRATE → Model
Results…
⊙ Analysis of variance table

An alternative notation enables you to summarise the main effects and the interaction with the | sign as below.

ALTERNATIVE MINITAB COMMANDS FOR BOX 7.2 A factorial ANOVA of the yield data

Commands

```
glm YIELD = BLOCK + VARIETY|SOWRATE;
brief 1.
```

Menu route

Stat > ANOVA > General Linear Model
YIELD → Response
BLOCK + VARIETY|SOWRATE → Model
Results…
⊙ Analysis of variance table.

Both of these sets of commands will provide you with identical output as shown below:

MINITAB OUTPUT FOR BOX 7.2 A factorial ANOVA of the yield data

General Linear Model: YIELD versus BLOCK, VARIETY, SOWRATE

```
Factor            Type Levels Values
BLOCK             fixed     3 1 2 3
VARIETY           fixed     2 1 2
SOWRATE           fixed     4 1 2 3 4

Analysis of Variance for YIELD, using Adjusted SS for Tests

Source               DF     Seq SS     Adj SS     Adj MS        F          P
BLOCK                 2     0.3937     0.3937     0.1968     0.52      0.606
VARIETY               1     2.1474     2.1474     2.1474     5.67      0.032
SOWRATE               3     5.8736     5.8736     1.9579     5.16      0.013
VARIETY*SOWRATE       3     0.0566     0.0566     0.0189     0.05      0.985
Error                14     5.3069     5.3069     0.3791
Total                23    13.7782
```

7.4 Presenting the results

Factorial experiments with insignificant interactions

If the interaction is not significant, as is the case with the *wheat* dataset, you may wish to display the means for the main effects only. These can be obtained by requesting the coefficient table with the following commands.

MINITAB COMMANDS FOR BOX 7.3 The coefficients table from the factorial analysis of Box 7.2

Commands

```
glm YIELD = BLOCK + VARIETY|SOWRATE;
brief 3.
```

Menu route

Stat > ANOVA > General Linear Model
YIELD → Response
BLOCK + VARIETY|SOWRATE → Model
Results…
⊙ In addition, coefficients for all terms.

In addition to the ANOVA table, this will give you the coefficient table. Minitab will give you the coefficients for the full model, but you ignore the deviations for the interactions. The means for the main effects may be obtained from this table as explained in the main text.

MINITAB OUTPUT FOR BOX 7.3 **The coefficients table from the factorial analysis of Box 7.2**

Term		Coef	SE Coef	T	P
Constant		8.5364	0.1257	67.92	0.000
BLOCK					
1		−0.0847	0.1777	−0.48	0.641
2		0.1810	0.1777	1.02	0.326
VARIETY					
1		0.2991	0.1257	2.38	0.032
SOWRATE					
1		−0.2804	0.2177	−1.29	0.219
2		−0.4469	0.2177	−2.05	0.059
3		−0.1034	0.2177	−0.47	0.642
VARIETY*SOWRATE					
1	1	0.0442	0.2177	0.20	0.842
1	2	0.0500	0.2177	0.23	0.822
1	3	−0.0635	0.2177	−0.29	0.775

This dataset is both orthogonal and balanced. Can we ignore the brackets of unnecessary or unwanted terms in the same way when we lose orthogonality? The answer is yes, because Minitab will take account of any imbalance, and weight its averages accordingly (always weighting the levels within a variable equally, even when the number of datapoints in the levels are different). The grand mean then becomes the average for an individual which has an equal chance of being in each level of a variable. If there were twice as many plants of variety 2 than variety 1, then the grand mean would be the average of the variety means rather than the average of all plants.

Alternatively, the means for the main effects can be obtained directly, using the following commands. We have chosen to ignore BLOCK as we are interested only in the means for the two treatment main effects in this instance.

MINITAB COMMANDS FOR BOX 7.4 **Obtaining the means for the main effects only**

Commands

```
glm YIELD = BLOCK + VARIETY|SOWRATE;
brief 1;
means VARIETY + SOWRATE.
```

Menu route

Stat > ANOVA > General Linear Model
YIELD → Response
BLOCK + VARIETY|SOWRATE → Model
Results…
⊙ Analysis of variance table.
VARIETY + SOWRATE → Display least squares means corresponding to the terms.

In addition to the ANOVA table, this will give you the following output:

MINITAB OUTPUT FOR BOX 7.4 **Summarising the results when the interaction is not significant**

```
Least Squares Means for YIELD

SOWRATE                       Mean              SE Mean
1                            8.256               0.2514
2                            8.089               0.2514
3                            8.433               0.2514
4                            9.367               0.2514
VARIETY
1                            8.836               0.1777
2                            8.237               0.1777
```

Factorial experiments with significant interactions

The *tulips* dataset provides an example of an experiment of factorial design with a significant interaction. The coefficient table may be obtained in the usual way:

MINITAB COMMANDS FOR BOX 7.5 **Analysing a factorial experiment with a significant interaction**

Commands

```
glm BLOOMS = BED + WATER|SHADE;
brief 3.
```

Menu route

Stat > ANOVA > General Linear Model
`BLOOMS` → Response
`BED + WATER|SHADE` → Model
Results…
⊙ In addition, coefficients for all terms.

MINITAB OUTPUT FOR BOX 7.5 **Analysing a factorial experiment with a significant interaction**

General Linear Model: `BLOOMS` versus `BED`, `WATER`, `SHADE`

```
Factor             Type Levels Values
BED               fixed      3 1 2 3
WATER             fixed      3 1 2 3
SHADE             fixed      3 1 2 3
```

```
Analysis of Variance for BLOOMS, using Adjusted SS for Tests

Source                DF      Seq SS      Adj SS      Adj MS         F         P
BED                    2       13811       13811        6906      3.88     0.042
WATER                  2      103626      103626       51813     29.11     0.000
SHADE                  2       36376       36376       18188     10.22     0.001
WATER*SHADE            4       41058       41058       10265      5.77     0.005
Error                 16       28477       28477        1780
Total                 26      223348

Term                            Coef        SE Coef            T           P
Constant                     128.994          8.119        15.89       0.000
BED
1                            -31.87          11.48        -2.78       0.014
2                             13.59          11.48         1.18       0.254
WATER
1                            -77.72          11.48        -6.77       0.000
2                              3.85          11.48         0.33       0.742
SHADE
1                             51.44          11.48         4.48       0.000
2                            -19.67          11.48        -1.71       0.106
WATER*SHADE
1     1                      -72.67          16.24        -4.47       0.000
1     2                       12.94          16.24         0.80       0.437
2     1                       29.92          16.24         1.84       0.084
2     2                       -6.48          16.24        -0.40       0.695

Unusual Observations for BLOOMS

Obs          BLOOMS         Fit        SE Fit       Residual       St Resid
 12          19.870      92.809        26.928        -72.939         -2.25R

R denotes an observation with a large standardized residual.
```

The means for all combinations of WATER and SHADE can be calculated from the coefficient table as described in the main text. To obtain these means directly, use the following commands:

MINITAB COMMANDS FOR BOX 7.6 Summarising the results with a significant interaction

Commands

```
glm BLOOMS = BED + WATER|SHADE;
brief 1;
means WATER*SHADE.
```

Menu route

Stat > ANOVA > General Linear Model
BLOOMS → Response
BED + WATER|SHADE → Model
Results…
⊙ Analysis of variance table
WATER*SHADE → Display least squares means corresponding to the terms.

MINITAB OUTPUT FOR BOX 7.6 **Summarising the results with a significant interaction**

```
Least Squares Means for BLOOMS

WATER*SHADE                          Mean             SE Mean
1     1                             30.04               24.36
1     2                             44.55               24.36
1     3                             79.22               24.36
2     1                            214.20               24.36
2     2                            106.69               24.36
2     3                             77.63               24.36
3     1                            297.05               24.36
3     2                            176.74               24.36
3     3                            134.82               24.36
```

7.5 Extending the concept of interactions to continuous variables

Mixing continuous and categorical variables

To include interactions between continuous and categorical variables, the same symbols and commands are used. This is illustrated with the *leprosy* dataset.

MINITAB COMMANDS FOR BOX 7.7
Analysing interactions with categorical and continuous variables: the leprosy dataset

Commands

```
glm BACAFTER = BACBEF|TREATMT;
covariate BACBEF;
brief 3.
```

Menu route

Stat > ANOVA > General Linear Model
`BACAFTER` → Response
`BACBEF|TREATMT` → Model
Covariates…
`BACBEF` → Covariates
Results…
⊙ In addition, coefficients for all terms

MINITAB OUTPUT FOR BOX 7.7
Analysing interactions with categorical and continuous variables: the leprosy dataset

General Linear Model: BACAFTER versus TREATMT

```
Factor          Type Levels Values
TREATMT         fixed     3 1 2 3

Analysis of Variance for BACAFTER, using Adjusted SS for Tests

Source                  DF      Seq SS     Adj SS     Adj MS        F        P
BACBEF                   1      587.48     482.63     482.63    30.57    0.000
TREATMT                  2       83.35       5.83       2.91     0.18    0.833
TREATMT*BACBEF           2        1.25       1.25       0.62     0.04    0.961
Error                   24      378.90     378.90      15.79
Total                   29     1050.98

Term                           Coef        SE Coef           T            P
Constant                     -0.126          1.955       -0.06        0.949
BACBEF                       0.8894         0.1609        5.53        0.000
TREATMT
1                            -0.946          2.520       -0.38        0.711
2                            -0.896          2.701       -0.33        0.743
BACBEF*TREATMT
      1                     -0.0611         0.2174       -0.28        0.781
      2                      0.0167         0.2128        0.08        0.938

Unusual Observations for BACAFTER

Obs          BACAFTER         Fit       SE Fit        Residual      St Resid
 13           16.0530      6.5897       1.5745          9.4633         2.59R
 28            3.3710      4.4134       3.1467         -1.0424        -0.43 X

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large influence.
```

To compare the three treatments, there is the extra complication of the covariate BACBEF in the model. However, if you request the three treatment means with the same commands, Minitab will adjust them for the mean value of BACBEF (for details see main text).

MINITAB COMMANDS FOR BOX 7.8
Obtaining adjusted means to compare treatments, when the model contains a covariate

Commands

```
glm BACAFTER = BACBEF|TREATMT;
covariate BACBEF;
brief 1;
means TREATMT.
```

Menu route

Stat > ANOVA > General Linear Model
BACAFTER → Response
BACBEF|TREATMT → Model
Covariates…
BACBEF → Covariates
Results…
⊙ Analysis of variance table.
TREATMT → Display least squares means corresponding to the terms

MINITAB OUTPUT FOR BOX 7.8
Obtaining adjusted means to compare treatments, when the model contains a covariate

```
Means for Covariates

Covariate                          Mean                    StDev
BACBEF                            11.19                    4.904

Least Squares Means for BACAFTER

TREATMT                            Mean                 SE Mean
1                                 8.201                   1.313
2                                 9.121                   1.286
3                                12.171                   1.262
```

The means produced by Minitab are automatically adjusted. By requesting the means for treatment, it is noted by Minitab that a covariate is involved, so the treatment means are automatically adjusted, and the mean value of the covariate, which was used in the adjustment, is also given.

The analysis of the G*reat tits* dataset uses the same commands, so we move on to the next section.

Interactions between continuous variables

To test for an interaction between two continuous variables, the same commands and symbols are used, as illustrated with the *trees* dataset.

MINITAB COMMANDS FOR BOX 7.10 **Testing for interactions between two continuous variables**

Commands

```
glm VOLUME = DIAMETER|HEIGHT;
covariate DIAMETER HEIGHT;
brief 3.
```

Menu route

Stat > ANOVA > General Linear Model
VOLUME → Response
DIAMETER|HEIGHT → Model
Covariates…
DIAMETER HEIGHT → Covariates
Results…
⊙ In addition, coefficients for all terms

MINITAB OUTPUT FOR BOX 7.10 **Testing for interactions between two continuous variables**

General Linear Model: VOLUME versus

```
Factor              Type Levels Values

Analysis of Variance for VOLUME, using Adjusted SS for Tests

Source                    DF     Seq SS      Adj SS      Adj MS        F         P
DIAMETER                   1     7581.8        68.1        68.1     9.29     0.005
HEIGHT                     1      102.4       128.6       128.6    17.52     0.000
DIAMETER*HEIGHT            1      223.8       223.8       223.8    30.51     0.000
Error                     27      198.1       198.1         7.3
Total                     30     8106.1

Term                             Coef        SE Coef            T           P
Constant                        69.40          23.84         2.91       0.007
DIAMETER                       -5.856          1.921        -3.05       0.005
HEIGHT                         -1.2971         0.3098       -4.19       0.000
DIAMETER*HEIGHT                0.13465        0.02438        5.52       0.000

Unusual Observations for VOLUME

Obs            VOLUME           Fit        SE Fit       Residual      St Resid
 18           27.4000       33.9821        1.0264        -6.5821        -2.63R
 31           77.0000       77.2472        2.0407        -0.2472        -0.14 X

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence
```

7.8 Exercises

Antidotes

This exercise involves a factorial analysis with two categorical variables. Data are provided in the *Antidotes* dataset

MINITAB COMMANDS FOR BOX 7.11 Interaction in the *antidotes* data

Commands

```
glm BLOOD = ANTIDOTE|DOSE;
brief 3.
```

Menu route

Stat > ANOVA > General Linear Model
BLOOD → Response
ANTIDOTE|DOSE → Model
Results…
⊙ In addition, coefficients for all terms

MINITAB OUTPUT FOR BOX 7.11 Interaction in the *antidotes* data

General Linear Model: BLOOD versus ANTIDOTE, DOSE

```
Factor              Type Levels Values
ANTIDOTE            fixed     2 1 2
DOSE                fixed     4 5 10 15 20

Analysis of Variance for BLOOD, using Adjusted SS for Tests

Source                 DF      Seq SS      Adj SS      Adj MS         F         P
ANTIDOTE                1     1396.90     1396.90     1396.90     23.68     0.000
DOSE                    3     1070.09     1070.09      356.70      6.05     0.006
ANTIDOTE*DOSE           3      835.88      835.88      278.63      4.72     0.015
Error                  16      943.68      943.68       58.98
Total                  23     4246.55

Term                          Coef          SE Coef            T             P
Constant                     8.698            1.568         5.55         0.000
ANTIDOTE
1                            7.629            1.568         4.87         0.000
DOSE
 5                          -8.186            2.715        -3.01         0.008
10                          -4.119            2.715        -1.52         0.149
15                           3.098            2.715         1.14         0.271
ANTIDOTE*DOSE
1         5                 -7.244            2.715        -2.67         0.017
1        10                 -3.551            2.715        -1.31         0.209
1        15                  2.573            2.715         0.95         0.358

Unusual Observations for BLOOD

Obs             BLOOD          Fit       SE Fit        Residual        St Resid
  4           19.9800       33.7567       4.4340        -13.7767          -2.20R
  8           54.3200       33.7567       4.4340         20.5633           3.28R

R denotes an observation with a large standardized residual
```

The results could be summarised using the coefficients table to calculate the means for each ANTIDOTE by DOSE combination, or the means could be requested directly from Minitab using the following commands:

ALTERNATIVE MINITAB COMMANDS FOR MEANS OF BOX 7.11

Commands

```
glm BLOOD = ANTIDOTE|DOSE;
brief 1;
means ANTIDOTE*DOSE.
```

Menu route

Stat > ANOVA > General Linear Model
BLOOD → Response
ANTIDOTE|DOSE → Model
Results…
⊙ Analysis of variance table
ANTIDOTE*DOSE → Display least squares means corresponding to the terms

MINITAB OUTPUT GIVING MEANS OF BOX 7.11

```
Least Squares Means for BLOOD

ANTIDOTE*DOSE                    Mean          SE Mean
1         5                    0.8967            4.434
1        10                    8.6567            4.434
1        15                   21.9967            4.434
1        20                   33.7567            4.434
2         5                    0.1267            4.434
2        10                    0.5000            4.434
2        15                    1.5933            4.434
2        20                    2.0533            4.434
```

Weight, fat and sex

See Minitab output for this exercise in the answers to exercises in Chapter 14.