INTERNATIONAL TRADE AND THE WORLD ECONOMY

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Answers to * exercises in chapter 4 of the Study Guide

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The * exercises in chapter 4 are: 4.2 and 4.8.

Question 4.2

4.2A.

The exponents of the factors of production (capital and labour) in the production function sum to one. This characteristic makes the function a constant returns to scale production function. If the exponents would be larger (smaller) than one, we have an increasing (decreasing) returns to scale function.

4.2B.

First, compute, for example, the output for one unit of capital and one unit of labour, which is simply equal to 1:

$$M = 1^{0.2} 1^{1-0.2} = 1$$

Then do the same for two units of capital and two units of labour, a doubling of inputs. Not surprisingly, the output of Manufactures also doubles and becomes equal to two:

$$M = 2^{0.2} 2^{1-0.2} = 2$$

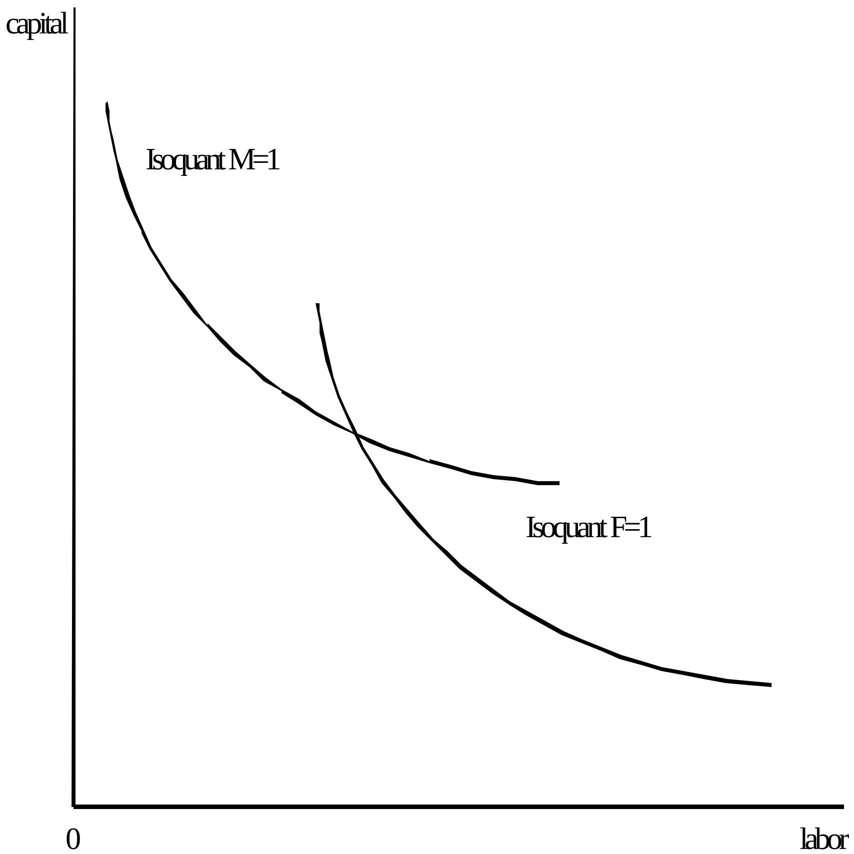
4.2C.

An increase in the capital intensity parameter a_m makes capital more productive in production and labour less so. The production function becomes more responsive to capital changes and less to labour changes. Because capital is more important in the latter case we need less capital to compensate for the drop in labour input if $a_m = 0.5$ than if $a_m = 0.2$.

4.2D.

Because the production of Manufactures is more capital intensive than the production of Food, the capital intensity parameter of this sector should always be higher in the Manufactures sector than in the Food sector. So we know that $a_m > a_f$.

When drawing the isoquants of the two sectors in capital - labour space we should make sure that the Manufactures sector is indeed more capital intensive than the Food sector. An example is given below.



Question 4.8

4.8A.

A production function is characterised by constant returns to scale when an increase in the use of all factors of production by multiplicative factor κ, leads to an increase in output by the same factor κ. It can be shown that the given CES production function is also characterised by constant returns to scale.

First, take an arbitrary choice of inputs (K_f, L_f) and calculate the output level F_0:

$$F_0 = \left[a_f (K_f)^{-r_f} + (1-a_f)(L_f)^{-r_f}\right]^{\frac{-1}{r_f}}$$

Second, increase the inputs by the same factor k and call the resulting output F_1:

$$F_1 = \left[a_f (kK_f)^{-r_f} + (1-a_f)(kL_f)^{-r_f}\right]^{\frac{-1}{r_f}}$$

Third, use the properties of the production function to conclude that $F_1 = kF_0$:

$$F_1 = \left[a_f (kK_f)^{-r_f} + (1-a_f)(kL_f)^{-r_f}\right]^{\frac{-1}{r_f}} = \left[k^{-r_f}\left(a_f K_f^{-r_f} + (1-a_f)L_f^{-r_f}\right)\right]^{\frac{-1}{r_f}}$$

$$= k\left(a_f K_f^{-r_f} + (1-a_f)L_f^{-r_f}\right)^{\frac{-1}{r_f}} = kF_0$$

4.8B.

We want to minimise the cost of production under the constraint that the production of food should at least equal to 1. To solve this problem we define the Lagrangean Γ:

$$\Gamma = rK_f + wL_f + l\left[1 - \left(a_f K_f^{-r_f} + (1-a_f)L_f^{-r_f}\right)^{\frac{-1}{r_f}}\right]$$

The first order derivatives with respect to K and L are:

$$\frac{\partial \Gamma}{\partial K_f} = r - l\left(\frac{-1}{r_f}\right)\left(a_f K_f^{-r_f} + (1-a_f)L_f^{-r_f}\right)^{\frac{-1}{r_f}-1}\left(-r_f\right)a_f K_f^{-r_f-1} = 0$$

$$\frac{\partial \Gamma}{\partial L_f} = w - l\left(\frac{-1}{r_f}\right)\left(a_f K_f^{-r_f} + (1-a_f)L_f^{-r_f}\right)^{\frac{-1}{r_f}-1}\left(-r_f\right)(1-a_f)L_f^{-r_f-1} = 0$$

or:

$$r = l\left(a_f K_f^{-r_f} + (1-a_f)L_f^{-r_f}\right)^{\frac{-1}{r_f}-1} a_f K_f^{-r_f-1}$$

$$w = l\left(a_f K_f^{-r_f} + (1-a_f)L_f^{-r_f}\right)^{\frac{-1}{r_f}-1}(1-a_f)L_f^{-r_f-1}$$

4.8C.

Taking the ratio of the two equations derived in 4.8B gives:

$$\frac{r}{w} = \frac{a_f}{1-a_f}\left(\frac{K_f}{L_f}\right)^{-r_f-1}$$

Which allows us to determine the capital-labour ratio:

$$\frac{K_f}{L_f} = \left(\frac{1-a_f}{a_f}\frac{r}{w}\right)^{\frac{-1}{r_f+1}}$$

4.8D.

In 4.8C we have derived the optimal capital-labour ratio. Rewriting gives:

$$L_f = K_f \left(\frac{1-a_f}{a_f} \frac{r}{w} \right)^{\frac{1}{r_f+1}}$$

Substituting this equation in the CES production function and using F = 1 gives:

$$1 = \left(a_f K_f^{-r_f} + (1-a_f) \left[K_f \left(\frac{1-a_f}{a_f} \frac{r}{w} \right)^{\frac{1}{r_f+1}} \right]^{-r_f} \right)^{\frac{-1}{r_f}}$$

$$1 = K_f^{-r_f} \left(\frac{r}{a_f} \right)^{\frac{-r_f}{r_f+1}} \left[a_f^{\left(1-\frac{r_f}{r_f+1}\right)} r^{\frac{r_f}{r_f+1}} + (1-a_f)^{\left(1-\frac{r_f}{r_f+1}\right)} w^{\frac{r_f}{r_f+1}} \right]$$

$$K_f = \left(\frac{a_f}{r} \right)^{\frac{1}{r_f+1}} \left[a_f^{\frac{1}{r_f+1}} r^{\frac{r_f}{r_f+1}} + (1-a_f)^{\frac{1}{r_f+1}} w^{\frac{r_f}{r_f+1}} \right]^{\frac{1}{r_f}}$$

This equation gives the optimal choice of capital for the production of one unit of food. The optimal choice of labourers can be found by substituting this function in the function of the capital-labour ratio.

$$L_f = \left(\frac{1-a_f}{w} \right)^{\frac{1}{r_f+1}} \left[a_f^{\frac{1}{r_f+1}} r^{\frac{r_f}{r_f+1}} + (1-a_j)^{\frac{1}{r_f+1}} w^{\frac{r_f}{r_f+1}} \right]^{\frac{1}{r_f}}$$

4.8E.

In 4.8D we have calculated the optimal use of capital and labour to minimise unit production costs. Substitution in total costs gives the unit cost function:

$$c_f = rK_f + wL_f$$

$$= \left[r \left(\frac{a_f}{r} \right)^{\frac{1}{r_f+1}} + w \left(\frac{1-a_f}{w} \right)^{\frac{1}{r_f+1}} \right] \cdot \left[a_f^{\frac{1}{r_f+1}} r^{\frac{r_f}{r_f+1}} + (1-a_f)^{\frac{1}{r_f+1}} w^{\frac{r_f}{r_f+1}} \right]^{\frac{1}{r_f}}$$

$$= \left[a_f^{\frac{1}{r_f+1}} r^{\frac{r_f}{r_f+1}} + (1-a_f)^{\frac{1}{r_f+1}} w^{\frac{r_f}{r_f+1}} \right]^{\frac{1+r_f}{r_f}}$$