WebAppendix 9. Money Demand and Money Supply

A9.1. An Inventory Model of the Demand for Money

Here, we present a simple inventory model of money demand by individual households. It is due to Baumol (1952) and Nobel Prize laureate James Tobin (1956). (A more elaborate and difficult model for the firm's optimal holding of money can be found in Miller and Orr (1966).)

A9.1.1. Formulation of the Household's Problem

Consider a household which receives a nominal income of PY in each period and spends this income on consumption at a constant rate over the period. There are no savings carried over from one period to the next. Income is paid directly into the household's bank account at the beginning of each period, say, a month. Let's assume that the household purchases goods with cash (banknotes), so it must visit the bank at least once, in order to withdraw money from the bank account, so that it is available for spending.[1] In general, the household will probably choose to deposit some part of the money it doesn't need to spend immediately in an interest-bearing account.

Now each visit to the bank or transaction has a nominal cost c attached to it, and c should be thought of as the opportunity cost of time spent queuing, 'shoe leather' used up while running to the bank, using an online banking service or bank fees for such transactions. If n is the number of visits to the bank during the month, the monthly cost will be nc. To reduce such costs, the household might hold more cash or sight deposits, but faces an opportunity cost in the form of lost interest payments. If i is the monthly interest rate served on the savings account, holding an average nominal balance M over the month implies an opportunity cost iM. If money is spent continuously over the period, the point-in-time bank balance is given by the upper side of the triangles in Figure A9.1 and the average money balance held over a unit period is simply the area of all the triangles in that period.

Figure A9.1 shows the amount of money that the agent has in her checking account for various numbers of monthly trips to the bank. The height of each triangle represents the amount withdrawn from the savings account and deposited in the checking account or held as cash. For evenly-spaced trips to the bank, it is PY/n.

[1] We could instead assume that businesses accept debit cards which debit the bank account directly, although then the structure of transaction costs would have to be modified slightly.

These occur every (30/n)th day of the month, the area of each triangle represents her average money held between two trips. Using the formula for the area of a triangle:

$$\text{Average money held between two trips} = \frac{1}{2} \times \frac{PY}{n} \times \frac{1}{n} = \frac{PY}{2n^2}.$$

The opportunity cost is the interest forgone on this average money holding:

$$\text{Opportunity cost} = \underset{\text{interest rate}}{i} \times \underset{\text{no. of trips}}{n} \times \underset{\text{av. holding between two trips}}{(PY/2n^2)}$$

Total costs TC (trips plus opportunity costs of holding M1) are therefore

(A9.1) $$TC = i\frac{PY}{2n} + cn.$$

Table A9.1 presents an example of the total costs of holding money and their breakdown as given by equation (A9.1), when PY = ECU 2000 per month, with a cost of transferring money of c = 1 euro and a nominal interest rate of 12% (about 1% per month). Total costs are minimized for three trips to the bank over the month.

Table A9.1. Breakdown of total costs of holding money balances

Number of trips = n	Interest cost $(PY/2n)i$	Trip's cost (cn)	Total cost
1	10.00	1.00	11.00
2	5.00	2.00	7.00
3	3.33	3.00	6.33
4	2.50	4.00	6.50
5	2.00	5.00	7.00
6	1.67	6.00	7.67

A9.1.2. Mathematical version of the Baumol-Tobin Model

A more general mathematical formulation of the cost minimization problem is:

(A9.2) $$\min_n TC = i\frac{PY}{2n} + cn\,.$$

Ignoring the fact that n must be a whole number, the optimum is obtained by setting the marginal cost equal to zero:[2]

(A9.3) $$\frac{\partial TC}{\partial n} = -\frac{iPY}{2n^2} + c = 0\,.$$

The optimal n is:

(A9.4) $$n^* = \sqrt{\frac{iPY}{2c}}\ ,$$

which leads to the 'square root' formula describing the optimal holding money stock:

(A9.5) $$M^* = \frac{PY}{2n^*} = \sqrt{\frac{PYc}{2i}}\,.$$

As the demand-for-money function (8.1), this expression states that the optimal average money holding is:

- a positive function of real economic activity Y
- a positive function of the price level P
- a positive function of transactions costs c
- a negative function of the nominal interest rate i

If we further define the *real* cost of transactions c^R as $c^R \equiv c/P$, the square root formula can be expressed in terms of real money demand:

(A9.6) $$\left(\frac{M}{P}\right)^* = \sqrt{\frac{Yc^R}{2i}}\,.$$

We can further rewrite (A9.6) as

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[2] This is a local minimum because the second-order condition is satisfied.

(A9.7) $$\left(\frac{M}{P}\right)^* = Y\sqrt{\frac{c^R/Y}{2i}}\,.$$

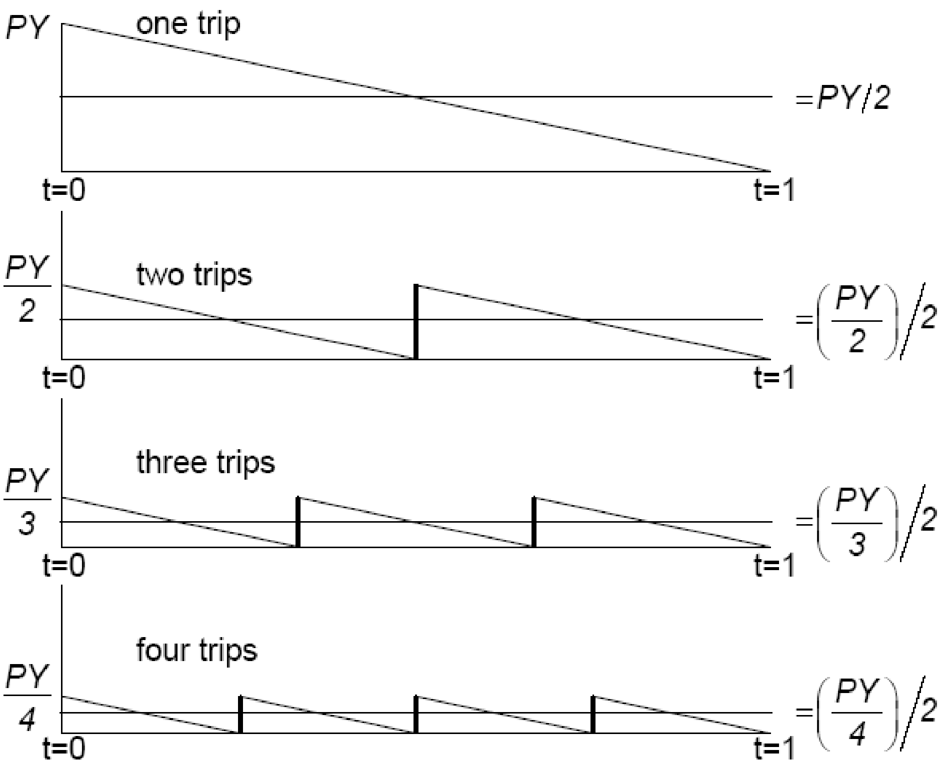
If the transaction cost is a fixed proportion of income, c^R/Y is constant and the demand for money is proportional to the real GDP. The income elasticity of money demand is unity, and the elasticities with respect to (c^R/Y) and i are both 0.5:

$$\frac{\partial(M/P)^*/\partial(c^R/Y)}{(M/P)^*/(c^R/Y)} = \frac{1}{2}$$

and

$$\frac{\partial(M/P)^*/\partial i}{(M/P)^*/i} = -\frac{1}{2}.$$

Figure A9.1 Trips to the bank and average money holdings



A9.2. The Money and Credit Multiplier Process

In this section we explore the money creation process in more detail, and highlight the role of credit creation in the money multiplier mechanism. The first step tracks down the process, bank after bank. The second step consolidates the banking sector and focuses on the end result.

A9.2.1. Step-by-step money creation

Imagine that a customer of Dresdner Bank finds €10,000 in banknotes in her garden. Once considered to be money, the cash stopped circulating a long while ago, and it has been since written off.[3] When the customer deposits the cash into her checking account, she effectively re-creates currency. Panel (*a*) in Figure A9.1 shows that Dresdner Bank increases simultaneously its liabilities (the new deposit of the customer) and its assets (the cash deposited, now part of its bank reserves) by €10,000.[4]

The Dresdner Bank will not sit on these reserves, since they do not bear interest. Suppose the required or desired reserves ratio is 5%. The bank then keeps €500 in reserves (cash in the vault, or, more likely, deposits this amount at the central bank). The remaining €9500 is lent to company X AG as shown in panel (*b*). The bank credits the loan customer's bank account by €9500. This is spent on a new car, so Dresdner 'loses' the €9500 in currency/deposits exactly as if the loan had been paid out in cash. This is shown in panel (*c*).

The car dealer, A GmbH, takes the sale proceeds to his own bank, Berliner Bank. Berliner finds itself in the same position as Dresdner did before, i.e. with a new deposit and new loanable funds. This is depicted in panel (*c*). Like Dresdner, Berliner holds 5% of the new reserves €9500 as reserves (=€475) and lends out the remaining €9025 (panel (*d*)). The same process occurs again: the €9025 of banknotes/bank reserves that are loaned are deposited with some other bank (Commerzbank) and so on (panel (*e*)).

In Figure A9.1, M1 increases by €10,000 in panels (*a*) and (*b*) (the new deposit at Dresdner), then by €9500 in panel (*c*) (a new deposit at Berliner), and by €9025 in panel (*d*) (a new deposit at Commerzbank). So far, the total increase in M1 is €28,525

[3] How does the central bank know? It does not know for sure, of course, but central banks keep careful track what they consider depreciation or loss of previously issued currency by destruction, wear and tear, etc.

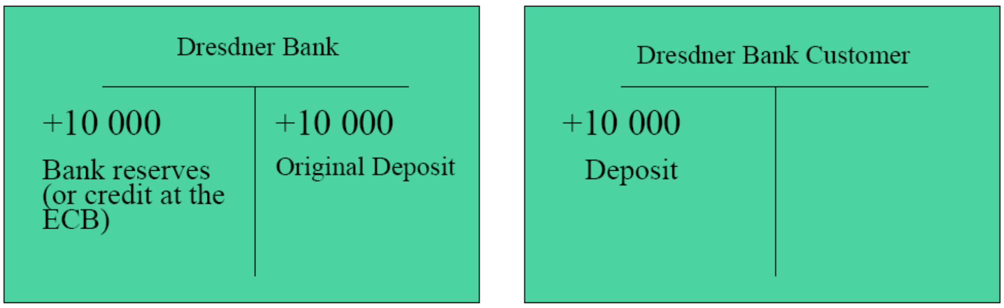
[4] The original money creation — the unearthing of €10,000— increased M1, first in the form of currency, then in the form of bank deposits. Note that as long as the cash remains in the bank's vault, it is not circulating and therefore is not counted as money.

(10,000+9,500+9,025). Letting the process continue *ad infinitum* in the same fashion, the total increase in M1 turns to be €200,000.[5] The commercial banks have ‘multiplied’ the initial deposit tenfold. As Equation (9.3) of the text established, with a 5% required reserves ratio, the reserves multiplier is 20. In the absence of currency holdings, the money multiplier is also equal to 20. These new deposits are “backed” by €10,000 of reserves (high powered money, or monetary base) plus €190.000 of loans to the nonbank sector.

Figure A9.1. The Money Supply Process

In panel (*a*) the Dresdner Bank receives a currency deposit of 10,000. Its assets (reserves) and liabilities (deposits) increase by 10,000. With a reserve requirement or preference of 5%, the bank lends 9,500 to another customer in panel (*b*). The loan customer spends the 9,500, which is next deposited at Berliner Bank (panel (*d*)). Berliner Bank in turn lends out 95% (19/20) of the new deposit, and holds 5% (1/20) in reserves at the central bank. This process repeats itself, in the next round at the Commerzbank (panel (*d*)). The total increase in money will be the sum of all such increments. While the number of potential increments is infinite, the sum of them is finite. The total increase in the money supply is equal to 200,000. Loans increase by 190,000 (200,000 less 10,000).

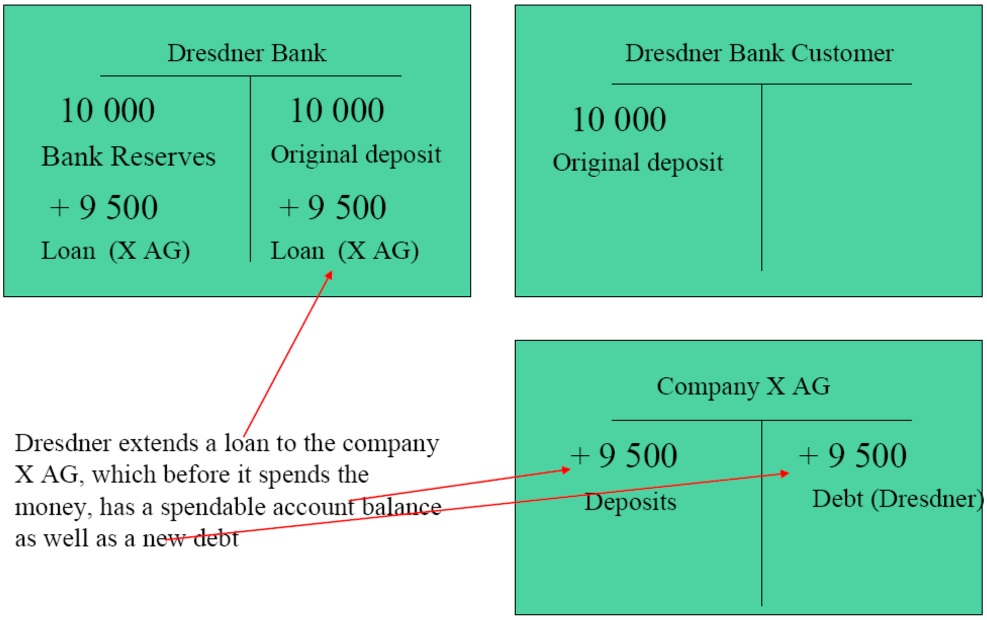
a)



[5] This follows from the formula for a geometric series: for any x such that $-1 < x < 1$, we have $1 + x + x^2 + x^3 + \ldots = \ldots \sum_{i=0}^{\infty} x^i = 1/(1-x)$. Here, with a reserve ratio r , an initial deposit D gives rise to a first loan of $D(1-r)$, and a second loan of $D(1-r)^2$; etc. The total increase in the money supply is: $D[1 + (1-r) + (1-r)^2 + (1-r)^3 + \ldots] = D/[1-(1-r)] = D/r$.

Figure A9.1. The Money Supply Process (continued)

b)



c)

...Company X AG buys a company car with the credit and pays for it by transferring the funds to the car dealer A GmbH, which has an account at the Berliner Bank…

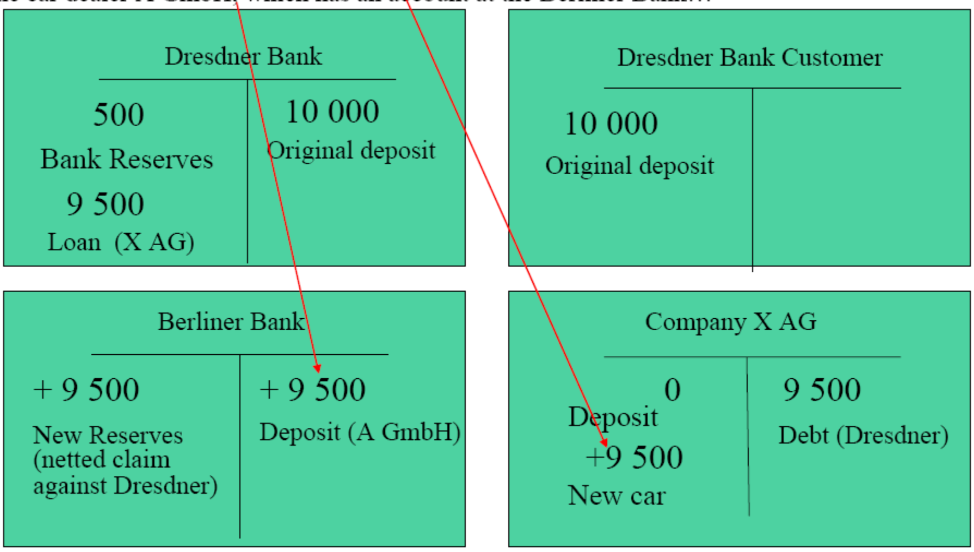
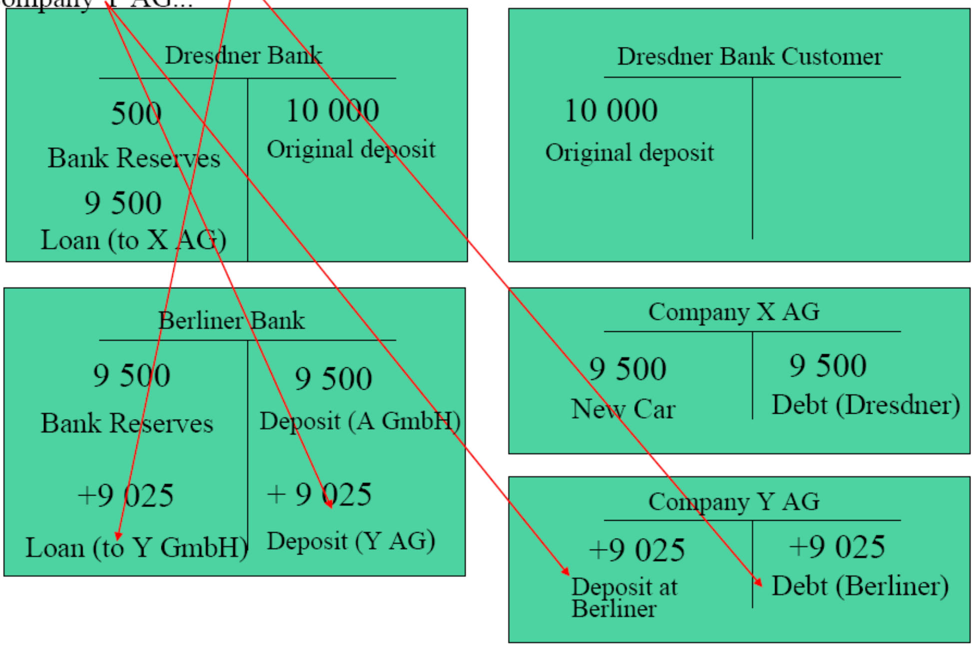


Figure A9.1. The Money Supply Process (continued)

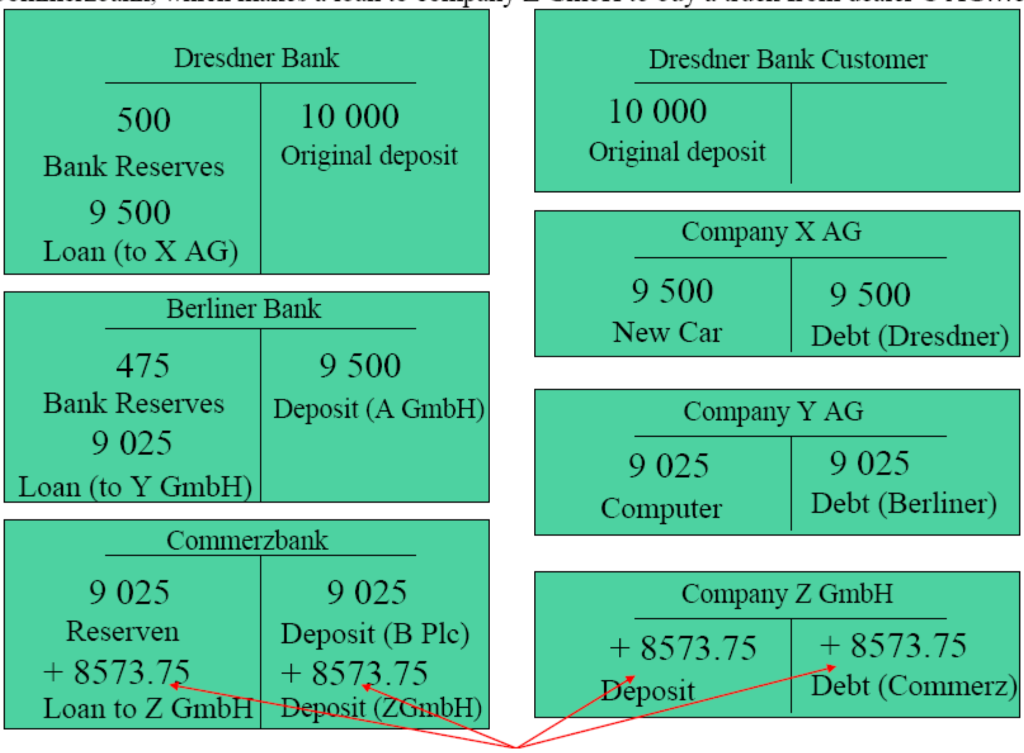
d)

...which now may lend 95% of the new reserves (.95 x 9500=9025), this time to company Y AG...



e)

... which buys a computer from computer store B Plc, which has an account with the Commerzbank, which makes a loan to company Z GmbH to buy a truck from dealer C AG…etc.



A9.2.2. Step-by-step money creation

In theory, the money-creation process could be achieved in a single step, if all German banks were consolidated into a single entity, German Bank AG. The manager of German Bank AG who receives the initial €10,000 will reason differently from the previous example. The bank manager can simply keep the cash in a vault or deposit it at the central bank. As Figure A9.2 shows, either way German Bank AG now has €10,000 additional reserves which could potentially 'support' €200,000 of additional deposits. Given the initial €10,000 deposited, it can extend more loans and can credit the corresponding amounts to customers' accounts up to €190,000. The initial €10,000 increase in reserves (the cash reintroduced) again triggers a total increase in money ten times larger. By keeping the reserves, German Bank AG has captured all the money-creating potential of the initial reintroduction of €10,000. As the only bank in town, it cannot lose reserves to other banks: the German Bank AG reproduces the successive actions of the whole banking sector in a single step.

Figure A9.2. The Money Supply Process in One Step
If there were only one commercial bank— which might indeed represent the banking system as a whole—it would not lose any reserves to other banks. The bank would create a loan and credit the account of its customer. When the customer pays for her expenditures by writing cheques, these cheques would all be redeposited in the same bank and none of the newly created money would leak out. Knowing this, the single bank keeps in the form of reserves all the 10,000 received as cash, and grants loans ten times that amount. The final outcome is the same as in Figure 9.4 of the text, the limit of the process described in Figure A9.1, but it is accomplished in a single step.

German Bank	
+10 000 Bank reserves	+10 000 Original Deposit
+190 000 Loans	+190 000 Other Deposits

Consolidated Private Nonbank Sector	
+10 000 Original Deposit	+10 000 Net Worth
+190 000 Other Deposits	+190 000 Debt to banks