

Progress exercise 20.1

1.

- (a) $y_t = A2^t$ non-convergent,
- (b) $y_t = A(0.5)^t$, convergent.
- (c) $y_t = A(-0.3)^t$ oscillating and convergent,
- (d) $y_t = A(-2)^t$ oscillating and non-convergent

2.

- (a) $y_t = A2^t + 1$ non-convergent;
- (b) $y_t = A(0.5)^t + 4$ convergent to limit of 4;
- (c) $y_t = A(-0.3)^t - 3/1.3$ oscillating and convergent to $-3/1.3$;
- (d) $y_t = A(-2)^t + 100/3$ oscillating and non-convergent.

3.

- (a) $y_t = (3/2)3^t + -3/2$ non-convergent and increasing;
- (b) $y_t = (1 + 1/0.7)(0.3)^t - 1/7$ convergent and decreasing to $-1/7$;
- (c) $y_t = -(4 + c/1.4)(-0.4)^t + c/1.4$ oscillating and convergent to $c/1.4$;
- (d) $y_t = 20$ and constant since begins at equilibrium (but would be non-convergent if disturbed from this equilibrium).

4.

- (a) $y_t = (1/6)3^t + -3/2$ non-convergent and increasing;
- (b) $y_t = (1/0.3)(1 + 1/7)(0.3)^t - 1/7$ convergent and decreasing to $-1/7$;
- (c) $y_t = -(1/0.16)(4 + c/1.4)(-0.4)^t + c/1.4$ oscillating and convergent to $c/1.4$;
- (d) $y_t = 20$ and constant since is at equilibrium in period 1 (but would be non-convergent if disturbed from this equilibrium).

Progress exercise 20.2

1.

- (a) At equilibrium, $Q_t^D = Q_t^S$, and $P_{t-1} = P_t$, so that

$$120 - 0.5P = -30 + 0.3P, \text{ from which } P = 187.5.$$

- (b) By substitution, difference equation is $P_t = (1 - 0.8\alpha)P_{t-1} + 150\alpha$

$$\text{General solution: } P_t = A(1 - 0.8\alpha)^t + 187.5$$

$$\text{With } P_0 = 200, \text{ solution is } P_t = 12.5(1 - 0.8\alpha)^t + 187.5.$$

- (c) The behaviour of the solution depends on the value of $a = (1 - 0.8\alpha)$.

(i) There are no oscillations when $a \geq 0$, i.e. when $0 < \alpha \leq 1.25$.

(ii) The solution has damped oscillations when $-1 < a < 0$, i.e. when $1.25 < \alpha < 2.5$.

- (iii) The oscillations are explosive when $a < -1$, i.e. when $\alpha > 2.5$. When $\alpha = 2.5$, ($a = -1$), the solution has constant oscillations.

2.

- (a) $C_t = Y_t - I_0$, so $Y_t = \left(\frac{2}{3}\right)Y_{t-1} + (70 + I_0)$. So soln. is $Y_t = A\left(\frac{2}{3}\right)^t + 210 + 3I_0$. As t increases indefinitely, $\left(\frac{2}{3}\right)^t \Rightarrow 0$ so long run equilibrium is $Y_t = 210 + 3I_0$.

Similarly soln, for C_t is $C_t = A\left(\frac{2}{3}\right)^t + 210 + 2I_0 \Rightarrow 210 + 2I_0$ as t increases indefinitely. When $I_0 = 20$, $Y_t \Rightarrow 270$, $C_t \Rightarrow 250$.

- (b) If I_t has been 20 for many years, Y_t must be close to its long run value $Y_t = 210 + 3I_0 = 270$ (from (a) above). Then we can re-start the clock with $Y_0 = 270$ and the new level of I , 30. So the new solution for Y_t is

$Y_t = A\left(\frac{2}{3}\right)^t + 210 + 3I_0$ (from (a)) with $I_0 = 30$; that is, $Y_t = A\left(\frac{2}{3}\right)^t + 300$. So the new long run equilibrium soln is $Y_t = 300$.

To find how many years it will take for Y to rise from 270 to 290, we must solve for A . We can do this using initial condition $Y_0 = 270$. Then in the soln we have

$$270 = A\left(\frac{2}{3}\right)^0 + 300, \text{ from which } A = -30. \text{ So soln is now } Y_t = (-30)\left(\frac{2}{3}\right)^t + 300.$$

Setting $Y_t = 290$ gives $290 = (-30)\left(\frac{2}{3}\right)^t + 300$. Solving this for t gives $\left(\frac{2}{3}\right)^t = \frac{1}{3}$,

Using logs, soln is $t = 2.7$ years (approx).

3. (a) The difference equation is $Y_t = \frac{v}{v-s}Y_{t-1}$, which has solution

$$Y_t = Y_0 \left(\frac{v}{v-s} \right)^t.$$

- (b) If $v > 1$ and $0 < s < 1$, then the ratio $\frac{v}{v-s} > 1$, and therefore Y_t is explosive but non-oscillating. Income expands indefinitely.

Progress exercise 20.3

1.

(a) $y(t) = 3 - e^{-4t}$

(b) $y(t) = 1.5(1 - e^{-10t})$

(c) $y(t) = 4(1 - e^{-t})$

(d) $y(t) = 2e^{5t}$

(e) $y(t) = -1 + 8e^{7t}$

2. (a) $\frac{dp}{dt} = 2(q^D - q^S) = 2(0.25p - 5)$, so equilibrium price (where demand and supply are equal and price does not change) is 20.

(b) Solution is: $p(t) = Ae^{0.5t} + 20$, where $A = p(0) - 20$. Since $e^{0.5t}$ increases without limit as t increases, price is divergent rather than convergent. If $A > 0$, $p(t)$ increases without limit, while if $A < 0$, $p(t)$ decreases without limit. (We need to know $p(0)$ in order to determine the value of A .)

(c) When $p(0) = 21$, $A = 1$ and $p(12) = 423.4$.

(d) If $p_0 > 20$, then $A = p(0) - 20 > 0$, and the path of $p(t)$ is like figure 20.9 case 1(a).) If $p_0 < 20$, then $A = p(0) - 20 < 0$, and the path of $p(t)$ is like figure 20.9 case 1(b). Thus the market is unstable, because if the price is initially above its equilibrium value of 20, then price rises; while if the price is initially below its equilibrium value of 20, then price falls.

(e) The market is unstable (the price is divergent) because the demand function is positively sloped. This could be because buyers interpret a high current price as indicating that the future price will be even higher.

3. Demand would need to be more negatively sloped than supply for the path to be stable.