Progress exercise 19.1

1. 2A + 3B = $2\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}+3\begin{bmatrix}1 & 3\\4 & 2\end{bmatrix}$ = $\begin{bmatrix}2 & 4\\6 & 8\end{bmatrix}+\begin{bmatrix}3 & 9\\12 & 6\end{bmatrix}$ = $\begin{bmatrix}5 & 13\\18 & 14\end{bmatrix}$

2. AB = $\begin{bmatrix}2 & 1\\1 & 7\\5 & 3\end{bmatrix}\begin{bmatrix}1 & 3 & 6\\-1 & 2 & 4\end{bmatrix}=\begin{bmatrix}1 & 8 & 16\\-6 & 17 & 34\\2 & 21 & 42\end{bmatrix}$

BA = $\begin{bmatrix}1 & 3 & 6\\-1 & 2 & 4\end{bmatrix}\begin{bmatrix}2 & 1\\1 & 7\\5 & 3\end{bmatrix}=\begin{bmatrix}35 & 40\\20 & 25\end{bmatrix}$

(Note that AB $\neq$ BA.)

3. (a) Answer = –13 (a scalar; neither a matrix nor a vector)

(b) Answer = $ax + by$ (a scalar)

(c) Answer: the matrix, A = $\begin{bmatrix}3 & 6 & 9\\2 & 4 & 6\\1 & 2 & 3\end{bmatrix}$

(d) Answer: the column vector, A = $\begin{bmatrix}-9\\3\\-12\end{bmatrix}$

(e) Answer = 3 (scalar)

(f) Answer = $x^2 + 2y^2 + 3z^2 - 2xz + 2yz$ (scalar)

4. You should calculate each side of each equality separately. Each side should be equal to:

(a) $\begin{bmatrix}-15 & 24\\5 & 14\end{bmatrix}$

(b) $\begin{bmatrix}32 & 43\\4 & 26\end{bmatrix}$

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(c) $\begin{bmatrix} 18 & 6 \\ 18 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 59 & -75 \\ 50 & -41 \end{bmatrix}$

5. (a) Price vector (row): $\begin{bmatrix} 1.20 & 2.00 & 1.00 & 0.80 \end{bmatrix}$. Quantities (column: $\begin{bmatrix} 100 \\ 50 \\ 75 \\ 25 \end{bmatrix}$

(b) Total cost = $\begin{bmatrix} 1.20 & 2.00 & 1.00 & 0.80 \end{bmatrix}\begin{bmatrix} 100 \\ 50 \\ 75 \\ 25 \end{bmatrix}$ = 315.

Progress exercise 19.2

1. By direct multiplication, $\boldsymbol{AA}^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} . \frac{1}{(ad-bc)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$= \frac{1}{(ad-bc)}\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbf{I}$ (the identity matrix)

2. Inverses are: (a) $-\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$; (b) $\frac{1}{19}\begin{bmatrix} 3 & 7 \\ -1 & 4 \end{bmatrix}$;

(c) no inverse (because determinant = 0.

3. (a) In matrix form, the pair of simultaneous equations can be written as:

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$. Therefore we can use the inverse matrix from 2(a) above, to solve as:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} -2 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

(b) $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{19}\begin{bmatrix} 3 & 7 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 5 \end{bmatrix} = \frac{1}{19}\begin{bmatrix} 38 \\ 19 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (using same method as (a)

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(c) From 2(c) we know that matrix $\begin{bmatrix} 6 & 4 \\ 3 & 2 \end{bmatrix}$ has no inverse, therefore the given pair of sim. eqns. has no solution.

4. (a) det B = 9; det C = 10

(b) B+C = $\begin{bmatrix} 4 & 4 & 0 \\ 7 & 5 & 4 \\ 1 & -2 & 8 \end{bmatrix}$; det (B+C) = -16;

BC = $\begin{bmatrix} 2 & 7 & -1 \\ 8 & 15 & 8 \\ -4 & -8 & -7 \end{bmatrix}$; det BC = 90

5. det(A) = 1. Minors M_{ij} are:
$$\begin{matrix} M_{11}=5 & M_{12}=2 & M_{13}=11 \\ M_{21}=4 & M_{22}=2 & M_{23}=9 \\ M_{31}=6 & M_{32}=3 & M_{33}=14 \end{matrix}$$

and if A_{ij} are cofactors:
$$\begin{matrix} A_{11}=5 & A_{12}=-2 & A_{13}=11 \\ A_{21}=-4 & A_{22}=2 & A_{23}=-9 \\ A_{31}=6 & A_{32}=-3 & A_{33}=14 \end{matrix}$$

so $\mathbf{A}^{-1} = \dfrac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 5 & -4 & 6 \\ -2 & 2 & -3 \\ 11 & -9 & 14 \end{bmatrix}$

Progress exercise 19.3

1 (a) Parts requirement matrix:
$$\begin{matrix} & A & B \\ \textit{Widgets} & 2 & 6 \\ \textit{Votsits} & 3 & 1 \\ \textit{Hummets} & 1 & 2 \end{matrix}$$

(b) We need to use a row vector to represent the quantities (in order for us to use matrix multiplication):

(c) Quantities: $\begin{bmatrix} 6 & 2 & 7 \end{bmatrix}$

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Total number of parts required = $\begin{bmatrix} 6 & 2 & 7 \end{bmatrix}\begin{bmatrix} 2 & 6 \\ 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 25 & 52 \end{bmatrix}$ (that is, 25 of part A and 52 of B)

(d) Cost vector: $\begin{bmatrix} \text{unit cost of } A \\ \text{unit cost of } B \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$: Total Cost = $\begin{bmatrix} 25 & 52 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ = 206

2. $\begin{bmatrix} 18 & -1 \\ -2 & 36 \end{bmatrix}\begin{bmatrix} P_b \\ P_p \end{bmatrix} = \begin{bmatrix} 87 \\ 98 \end{bmatrix}$ $\begin{bmatrix} P_b \\ P_p \end{bmatrix} = \frac{1}{646}\begin{bmatrix} 36 & 1 \\ 2 & 18 \end{bmatrix}\begin{bmatrix} 87 \\ 98 \end{bmatrix} = \frac{1}{646}\begin{bmatrix} 3230 \\ 1938 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

3. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 & -4 & 6 \\ -2 & 2 & -3 \\ 11 & -9 & 14 \end{bmatrix}\begin{bmatrix} 12 \\ 13 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$

4. det(A) =1, det (A_1) = 2, det(A_2) = 5, det(A_3) = 1, so $x = 2, y = 5, z = 1$.

5. Verify by showing that the product of the two given matrices is an identity matrix.

6. Solution is $x_1 = 5$, $x_2 = -1$, $x_3 = 2$

7. (a) If A = $\begin{bmatrix} 1 & -1 & -1 & 0 \\ -\alpha & 1 & 0 & 0 \\ 0 & 0 & 1 & -\gamma \\ k_1 & 0 & 0 & k_2 \end{bmatrix}$; x = $\begin{bmatrix} Y \\ C \\ I \\ r \end{bmatrix}$; b = $\begin{bmatrix} G^* \\ \beta \\ \delta \\ M_S^* \end{bmatrix}$, then ***Ax = b***

We can check this by multiplying out; for example, the first row of ***A*** times the column vector ***x***, set equal to the first element of ***b***, gives:

$Y - C - I = G^*$ or $Y = C + I + G^*$

which is the first equation of the model.

(b) det A = $\begin{vmatrix} 1 & -1 & -1 & 0 \\ -\alpha & 1 & 0 & 0 \\ 0 & 0 & 1 & -\gamma \\ k_1 & 0 & 0 & k_2 \end{vmatrix} = -k_1\begin{vmatrix} -1 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & -\gamma \end{vmatrix} + k_2\begin{vmatrix} 1 & -1 & -1 \\ -\alpha & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \gamma k_1 + k_2(1-\alpha)$

(using 4th row of ***A***)

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Because r is 4th element in col. vector or unknowns, we need to find det A_4, the determinant of the sub-matrix that results from deleting the 4th column of $\boldsymbol{A}$ and replacing it with the vector of unknowns. We get:

$$\det A_4 = \begin{vmatrix} 1 & -1 & -1 & G^* \\ -\alpha & 1 & 0 & \beta \\ 0 & 0 & 1 & \delta \\ k_1 & 0 & 0 & M_S^* \end{vmatrix} = -k_1(\beta + \delta + G^*) + M_S^*(1-\alpha)$$

Therefore r = det A_4/det A = $\dfrac{-k_1(\beta + \delta + G^*) + M_S^*(1-\alpha)}{\gamma k_1 + k_2(1-\alpha)}$, the required result.

(c) From (b) we have $r = \dfrac{-k_1(\beta + \delta + G^*) + M_S^*(1-\alpha)}{\gamma k_1 + k_2(1-\alpha)}$

The partial derivative with respect to G^* is

$$\frac{\partial r}{\partial G^*} = \frac{-k_1}{\gamma k_1 + k_2(1-\alpha)}$$

Since we are told that both γ and k_2 are negative, the denominator is negative and therefore the expression is positive, so an increase in government spending causes the interest rate to rise.

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