

### Progress exercise 18.1

- (a)  $\frac{1}{4}x^4 + c$
- (b)  $-0.5x^{-2} + c$
- (c)  $2x^{-0.5} + c$
- (d)  $0.8x^{1.25} + c$
- (e)  $4x^{0.25} + c$
- (f)  $5\ln x + c$
- (g)  $2e^x + c$
- (h)  $x^5 + e^x - 8x^{0.5} + c$
- (i)  $-0.5x^{-4} + 3x^3 + 0.625x^{1.6} + c$
- (j)  $6x^{5/3} - 0.5\ln x + c$
- (k)  $1000e^{0.1x} + c$
- (l)  $-200e^{-0.05x} + c$
- (m)  $(2x^2 + \ln x)^4 + c$
- (n)  $\frac{2}{3}(x^{0.5} + 6x + 5)^{3/2} + c$
- (o)  $\ln(x^4 + 5x^2 - 2e^x) + c$
- (p)  $xe^x - e^x + c$
- (q)  $0.5xe^{2x} - 0.25e^{2x} + c$

### Progress exercise 18.2

- (1) 2500      (2) 2.9957      (3) 108      (4) 126.4241      (5) 6811.5333  
(6) 187.112      (7) e      (8) 1

### Progress exercise 18.3

1. To answer this, we find the definite integral of  $4 + 4q$  from 25 to 50. Answer is 3850.
2. To answer this, we find the definite integral of  $MR$  from 144 to 400, which equals 4000. So answer is  $-4000$ ; that is, a loss.
3. To answer this, we first find the consumer's valuation, which is the area under the inverse demand curve between  $q = 0$  and  $q = 8$ . This is

$$\int_0^8 (100 - q^2) dq = 629\frac{1}{3}$$

Then we subtract what the consumer has to pay for 8 units. When  $q = 8$ , then from the inverse demand function  $p = 100 - q^2$  we have  $p = 36$ . So consumer pays  $8 \times 36 = 288$  for 8 units. Her consumer surplus is therefore

$$629\frac{1}{3} - 288 = 341\frac{1}{3}$$

4. The  $PV$  is the integral:  $PV = \int_0^x ae^{-rx} dx$ . This can be written as

$$PV = \frac{a}{r}(1 - e^{-rx}) \quad (\text{see rule 18.6 and 18.6A})$$

5. Expected profit is \$10 million per year for 20 years. The present value ( $PV$ ) of the expected profit (see rule 18.6 and 18.6A), discounted continuously, is

$$PV = \int_0^x ae^{-rx} dx = \frac{a}{r}(1 - e^{-rx}) \text{ million dollars}$$

Where  $a$  = expected profit per year,  $x$  = number of years,  $r$  = discount rate.

- (a) Given  $r = 0.05$ , with  $x = 20$  and  $a = 20$ ,  $PV = \$252.85$  million  
(b) when  $r = 0.1$ , with  $x = 20$  and  $a = 20$ ,  $PV = \$172.93\text{m}$  (31.3% less than (i))  
(c) when  $r = 0.1$ ,  $x = 5$  and  $a = 20$ ,  $PV = \$78.69\text{m}$