Progress exercise 14.1

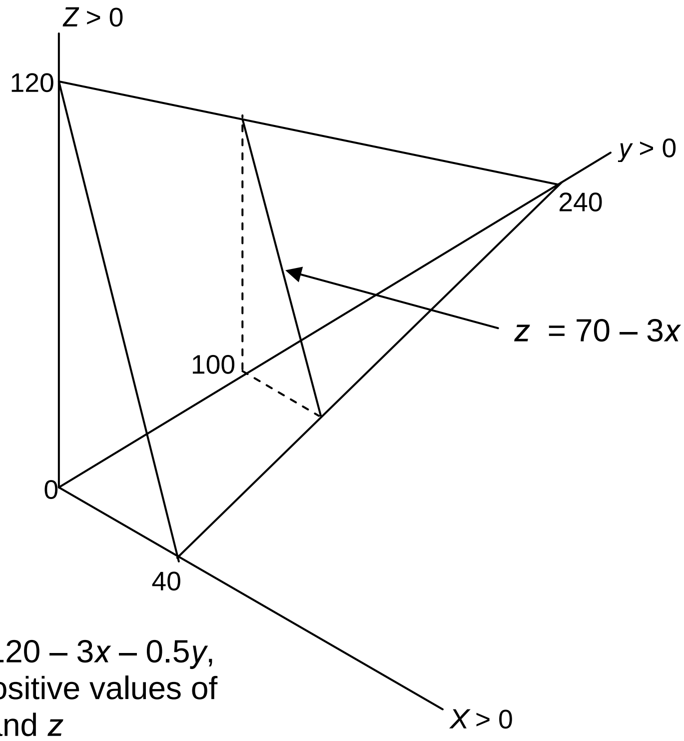
1. (a) Along an iso-*x* section, *x* is constant at, say, $x = x_0$. Then we have $z = (120 - 3x_0) - 0.5y$. Since $(120 - 3x_0)$ is constant, this is a linear relationship between *z* and *y*, with an intercept of $(120 - 3x_0)$ on the z-axis (when *y* = 0), and a slope of –0.5. Whatever the value of x_0, the slope is always –0.5, so we can infer that all iso-*x* sections are linear with slope –0.5.

Similarly an iso-*y* section has the equation $z = (120 - 0.5y_0) - 3x$, a linear relationship between *z* and *x* with an intercept on the *z* axis (when *x* = 0) of $120 - 0.5y_0$ and a slope of -3.

An iso-*z* section has the equation $z_0 = 120 - 3x - 0.5y$, which re-arrranges to $y = (240 - 2z_0) - 6x$, which again is linear with intercept $240 - 2z_0$ on the *y* axis (when *x* = 0) and slope –6.

Since we have found that the slope of the surface is linear in the *xz, yz*, and *xy* planes, the surface must be flat in all directions (that is, a plane).

(b) By definition, at any point on the *z*-axis *x* and *y* are both zero. So we can find the *z* intercept of the function by setting *x* = 0 and *y* = 0. This gives *z* = 120. Similarly at the *x* intercept, *z* = *y* = 0, giving $0 = 120 - 3x$, from which *x* = 40. Finally at the *y* intercept, *z* = *x* = 0, giving $0 = 120 - 0.5y$, from which *y* = 240. See sketch graph, next page.

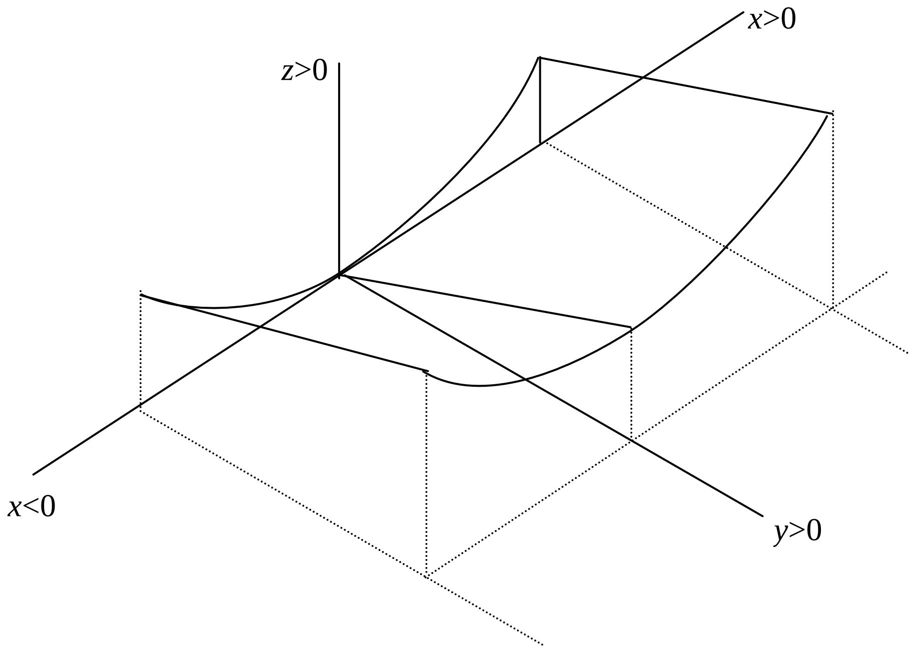


$z = 120 - 3x - 0.5y$,
for positive values of
x, y and z

2. (a) With $y = y_0$ = some constant, we have $z = x^2 + 3y_0$ which is a quadratic function relating z and x, with a z intercept of $3y_0$. With $x = x_0$ = some constant, we have $z = (x_0)^2 + 3y$ which is a linear function relating z and y, with a z intercept of $(x_0)^2$. The surface is therefore U-shaped in the xz plane, and linear in the yz plane.

(b) In general the z intercept is where $x = y = 0$. In this case, $z = 0$ when $x = y = 0$. So the z intercept is at $z = 0$. Similarly, in this example the x and y intercepts are at $x = 0$ and $y = 0$ respectively. Thus the surface passes through the origin. See figure next page.

Figure for q. 2(b)



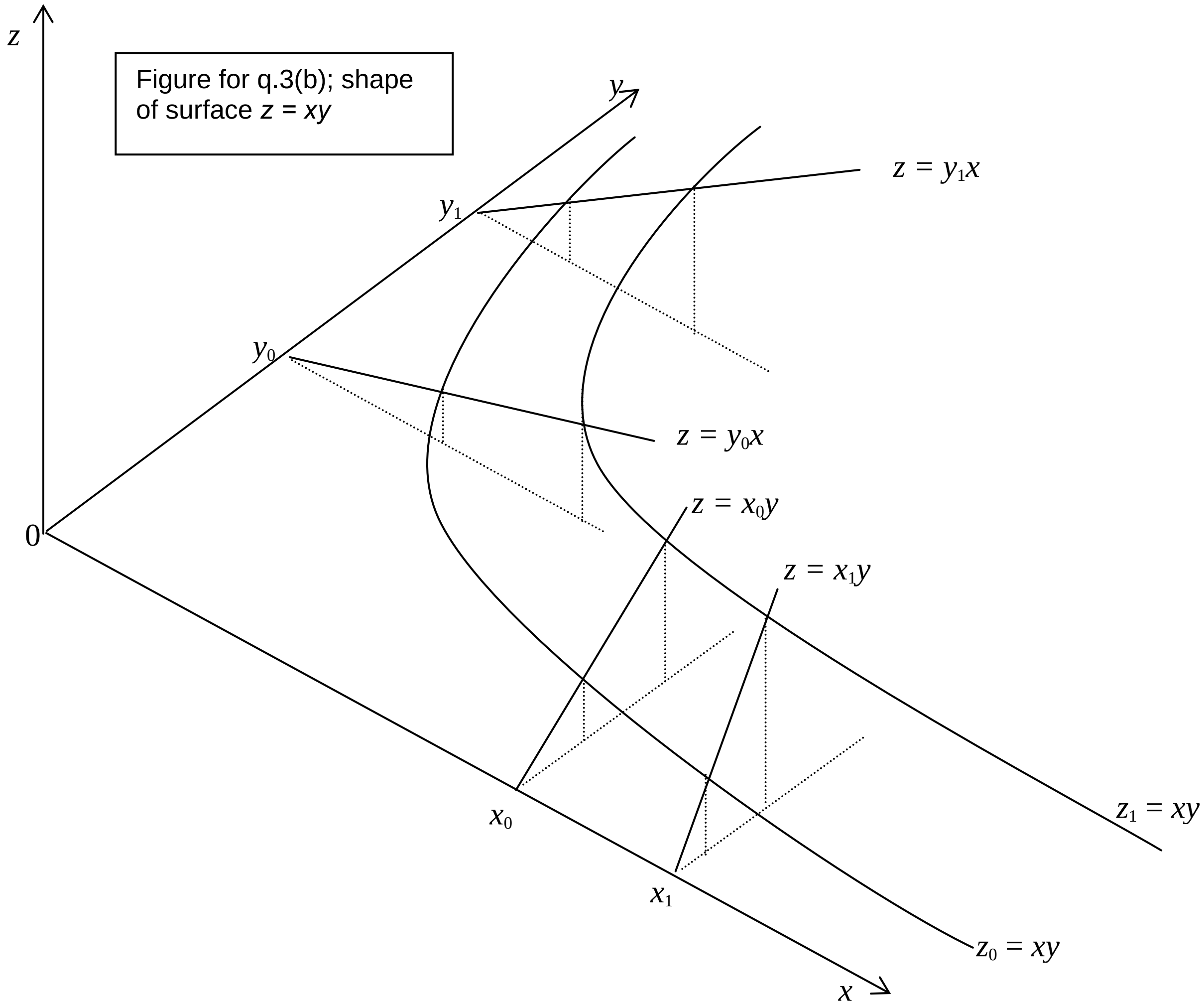
The surface $z = x^2 + 3y$. The relationship between z and x is quadratic (U-shaped); and between z and y is linear.

3. (a) Along an iso-x section we have $x = x_0$ = constant, so equation of any iso-x section is $z = x_0y$. This is a linear relationship between z and y with a slope given by x_0. Since x_0 is constant, the surface is linear in the yz plane with a slope that varies with the fixed value of x. Similarly the equation of any iso-y section is $z = y_0x$. This is a linear relationship between z and x with a slope given by y_0. The surface is thus also linear in the xz plane with a slope that varies with the fixed value of y.

Recall that, in two dimensions, at any point on the y axis, x = 0. Similarly, in three dimensions, at any point on the y axis, both $x = 0$ and $z = 0$ (see section 14.2 of book). In the case of this function, when $x = 0$, $z = 0$. Therefore the surface passes through the y axis since when $x = 0$, $z = 0$ whatever the value of y. Similarly the surface passes through the x axis since when $y = 0$, $z = 0$ whatever the value of x.

(b) From (a) we know that iso-x and iso-y sections are linear. A typical iso-z section has equation $z_0 = xy$ or $y = \frac{z_0}{x}$. This is a rectangular hyperbola.

Therefore we can deduce that surface has shape shown in figure below, which shows two iso-*x* sections, two iso-*y* sections, and two iso-*z* sections through the surface.



Progress exercise 14.2

	$\frac{\partial z}{\partial x}$:	$\frac{\partial z}{\partial y}$:	Remarks:
(a)	$2x+2y$	$2x+2y$	
(b)	$9x^2+4xy$	$2x^2+2y+1$	
(c)	$x^2-x^{-2}y^{-1}$	$0.5y^{-0.5}-y^{-2}x^{-1}$	
(d)	$\frac{3}{2}x^2\left(x^3+y^2\right)^{-0.5}$	$y\left(x^3+y^2\right)^{-0.5}$	Using function of a function rule
(e)	$\frac{(x-y)\left(3x^2\right)-\left(x^3+y^2\right)}{(x-y)^2}$	$\frac{(x-y)(2y)+\left(x^3+y^2\right)}{(x-y)^2}$	Using quotient rule
(f)	$\left(x^2+y\right)+\left(x-y^2\right)(2x)$	$\left(x^2+y\right)(-2y)+\left(x-y^2\right)$	Using product rule
(g)	$200e^{2x+3y}$	$300e^{2x+3y}$	
(h)	$\frac{3x^2}{x^3+y^2}-\frac{1}{x}$	$\frac{2y}{x^3+y^2}-\frac{1}{y}$	

Progress exercise 14.3

1. (a) and (b)

Given $z = x^3 + 5xy + 4y^2$:

$\frac{\partial z}{\partial x} = 3x^2 + 5y$ Measures slope in x-direction.

$\frac{\partial^2 z}{\partial x^2} = 6x$ Measures how slope in x-direction changes as *x* increases with *y* constant.

$\frac{\partial^2 z}{\partial y \partial x} = 5$ Measures how slope in x-direction changes as *y* increases with *x* constant.

$\frac{\partial z}{\partial y} = 5x + 8y$ Measures slope in y-direction.

$\frac{\partial^2 z}{\partial y^2} = 8$ Measures how slope in y-direction changes as *y* increases with *x* constant.

$\frac{\partial^2 z}{\partial x \partial y} = 5$ Measures how slope in y-direction changes as *x* increases with *y* constant.

2.

	$\frac{\partial z}{\partial x}$	$\frac{\partial^2 z}{\partial x^2}$	$\frac{\partial z}{\partial y}$	$\frac{\partial^2 z}{\partial x \partial y}$
(a)	$2x + 2$	2	$-3 + 2y$	0
(b)	$2x + y$	2	$x + 2y$	1
(c)	$3x^2 + 6x - 2y - y^2 + 2xy$	$6x + 6 + 2y$	$-2x - 2xy + 6y + x^2$	$-2 - 2y + 2x$
(d)	$15 + 6x - 2y$	6	$-2x - 4y + 12$	-2
(e)	$4x^{-0.6}y^{1.5}$	$-2.4x^{-1.6}y^{1.5}$	$15x^{0.4}y^{0.5}$	$6x^{-0.6}y^{0.5}$
(f)	$\alpha x^{\alpha-1}y^{\beta}$	$(\alpha - 1)\alpha x^{\alpha-2}y^{\beta}$	$\beta x^{\alpha}y^{\beta-1}$	$\alpha\beta x^{\alpha-1}y^{\beta-1}$
(g)	$\frac{0.25}{x}$	$\frac{-0.25}{x^2}$	$\frac{0.5}{y}$	0
(h)	$\frac{\alpha}{x}$	$\frac{-\alpha}{x^2}$	$\frac{\beta}{y}$	0

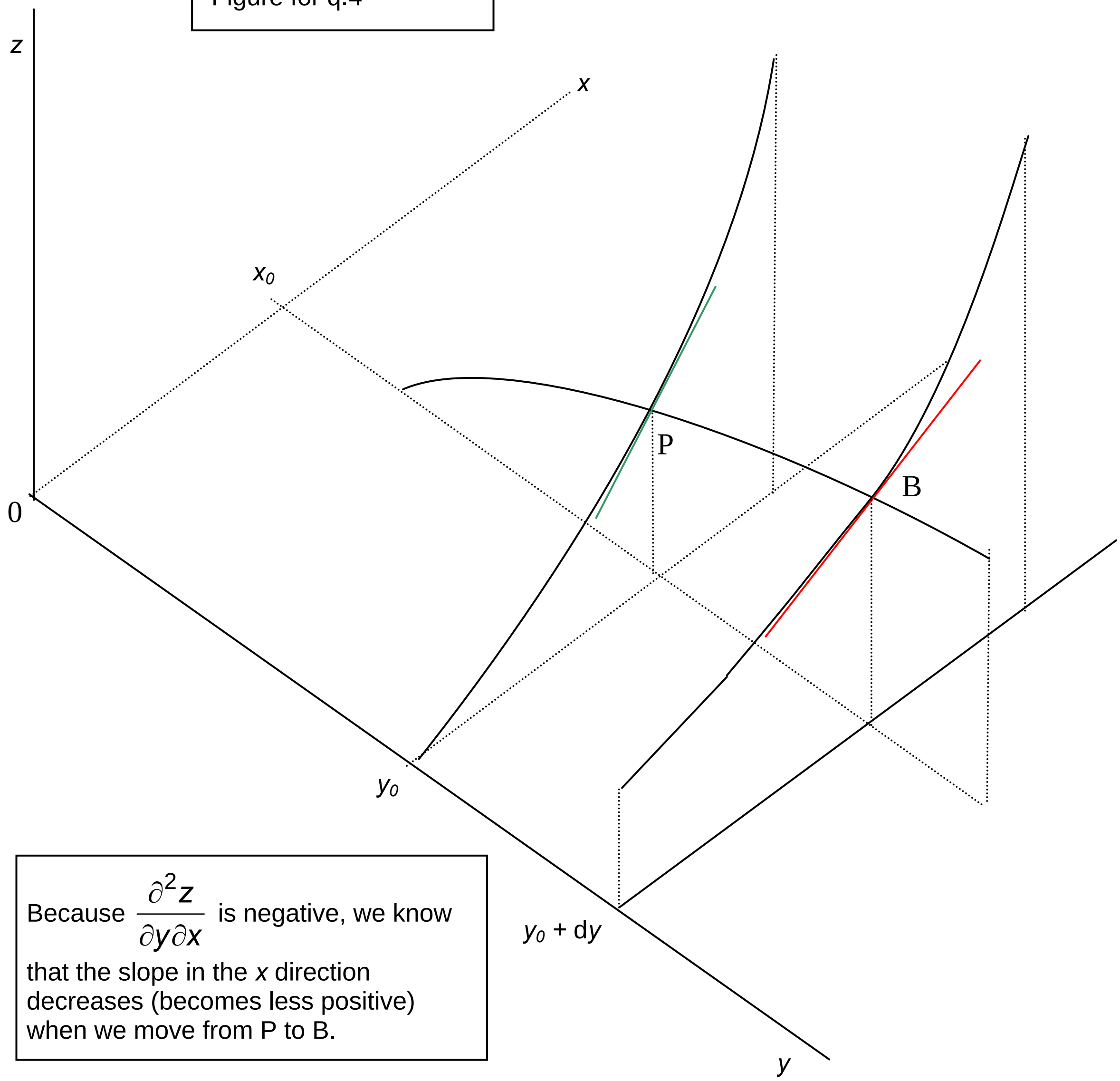
(i)	$\dfrac{2x}{x^2+3y}$	$\dfrac{(x^2+3y)(2)-2x(2x)}{(x^2+3y)^2}$	$\dfrac{3}{x^2+3y}$	$\dfrac{-6x}{(x^2+3y)^2}$
(j)	$2e^{2x+3y}$	$4e^{2x+3y}$	$3e^{2x+3y}$	$6e^{2x+3y}$

3.

	$\dfrac{\partial z}{\partial x}$	$\dfrac{\partial^2 z}{\partial x^2}$	$\dfrac{\partial z}{\partial y}$
(a)	$(2x-3y)^{-0.5}$	$-(2x-3y)^{-1.5}$	$-\frac{3}{2}(2x-3y)^{-0.5}$
(b)	$\dfrac{6x}{4y^2}$	$\dfrac{6}{4y^2}$	$\dfrac{-3x^2}{2y^3}$
(c)	$\dfrac{-2xy(1+y)}{(x^2-y)^2}$	$\dfrac{(8x^2y-2y(x^2-y))(1+y)}{(x^2-y)^3}$	$\dfrac{2y(x^2-y)+x^2+y^2}{(x^2-y)^2}$
(d)	$\dfrac{1}{(1-y^3)^{0.5}}$	0	$\dfrac{3xy^2}{2(1-y^3)^{1.5}}$
(e)	$2xe^{3y}$	$2e^{3y}$	$3x^2e^{3y}$
(f)	$\dfrac{2x}{x^2+1}$	$\dfrac{2x(x-1)^2}{(x^2+1)^2}$	$\dfrac{-2y}{y^2-1}$
(g)	$\dfrac{0.5}{x}$	$\dfrac{-0.5}{x^2}$	$\dfrac{0.25}{y}$

4. Since $\frac{\partial z}{\partial x}$ and $\frac{\partial^2 z}{\partial x^2}$ are both positive, we know that the surface is positively sloped and convex in the x-direction. Since $\frac{\partial z}{\partial y} > 0$ but $\frac{\partial^2 z}{\partial y^2} < 0$, we know that the function is positively sloped and concave in the y-direction. Since $\frac{\partial^2 z}{\partial y \partial x} < 0$, the slope in the x-direction decreases as y increases. (see figure below)

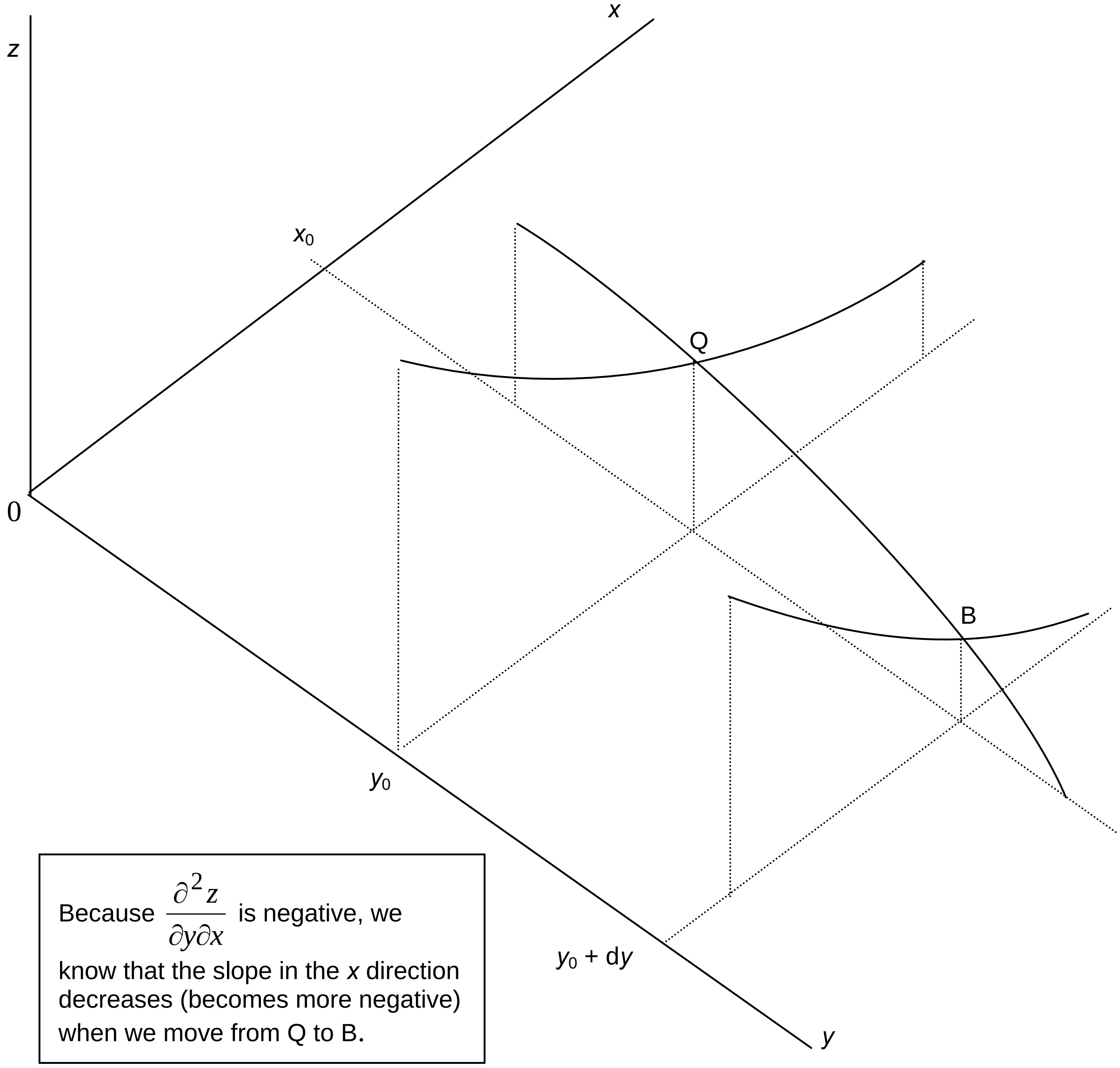
Figure for q.4



Because $\frac{\partial^2 z}{\partial y \partial x}$ is negative, we know that the slope in the *x* direction decreases (becomes less positive) when we move from P to B.

5. From the partial derivatives we can infer that the slope in the *x*-direction at *P* is negative but increasing (that is, decreasingly negative); and the slope in the *y*-direction is negative and decreasing (that is, increasingly negative). The negative sign of the cross partial derivative tells us that the slope in the *x*-direction decreases as *y* increases. Thus, in figure below, slope in *x*-direction at B is is less (more negative) than at Q

Figure for q.5



Because $\frac{\partial^2 z}{\partial y \partial x}$ is negative, we know that the slope in the *x* direction decreases (becomes more negative) when we move from Q to B.

Progress exercise 14.4

1. (a) Given $Q = 100K^{0.25}L^{0.75}$

(i) If $Q = 100$, $1 = K^{0.25}L^{0.75}$. Therefore $1^4 = \left(K^{0.25}\right)^4\left(L^{0.75}\right)^4 = KL^3 \quad \Rightarrow \quad K = \dfrac{1}{L^3}$.

(ii) On isoquant $K = \dfrac{1}{L^3}$, $\dfrac{\mathrm{d}K}{\mathrm{d}L} = -3L^{-4} = -\dfrac{3}{L^4}$ which is negative since $L^4 > 0$. So isoquant is negatively sloped. Also $\dfrac{\mathrm{d}^2K}{\mathrm{d}L^2} = 12L^{-5} = \dfrac{12}{L^5}$ which is positive since *L* is positive and so therefore is L^5.

(iii) Graph is very similar to a rectangular hyperbola, asymptotic to *K* and *L* axes.

(iv) Negative slope reflects assumption embodied in this production fn., that marginal products of *L* and *K* are always positive. Therefore if one input is increased, the other must be reduced in order to hold output constant. The positive second derivative (which tells us that the isoquant is convex) indicates diminishing absolute (that is, ignoring sign) MRS between *K* and *L*.

(v) Yes, because for any output Q_0, the isoquant is $K = \dfrac{\left(\dfrac{Q_0}{100}\right)^4}{L^3}$ and signs of the 1st and 2nd derivatives are the same as above.

(b) (i)&(ii) $MPL \equiv \dfrac{\partial Q}{\partial L} = 75K^{0.25}L^{-0.25}$ which is positive since *K* and *L* are positive and a positive number raised to any power is positive. The same is true of $MPK \equiv \dfrac{\partial Q}{\partial K} = 25K^{-0.75}L^{0.75}$.

(iii) No, it's quite possible that *MPL* could become negative when *L* is sufficiently high, for a given *K*. In effect, there are so many workers with so little capital that they get in one another's way, hence total output would increase if some of them stayed at home. And similarly for *K*, with a given *L* – too few workers with too little machinery.

(c) From (b), $MPL \equiv \dfrac{\partial Q}{\partial L} = 75K^{0.25}L^{-0.25}$. So $\dfrac{\partial}{\partial L}\left(MPL\right) \equiv \dfrac{\partial^2 Q}{\partial L^2} = \left(-0.25\right)75K^{0.25}L^{-1.25}$ which is negative since *K* and *L* are positive and a positive number raised to any power is positive (see section 2.12 of book). For the same reason,

$\frac{\partial}{\partial K}(MPK) \equiv \frac{\partial^2 Q}{\partial K^2} = (-0.75)(25)K^{-1.75}L^{0.75}$ is also negative.

(d) See figures 14.17 and 14.18 in book. The slopes are positive because *MPL* and *MPK* are positive, and concave because $\frac{\mathrm{d}MPL}{\mathrm{d}L}m$ and $\frac{\mathrm{d}MPK}{\mathrm{d}K}$ are negative.

(e) (i) By reasoning similar to (b)(iii) above, we can say that as L increases each additional new worker has less and less of the fixed quantity of *K* to assist her, hence successive new workers add less and less to production. At the same time each successive new worker means that existing workers have less and less capital to work with, and resulting fall in their output offsets the increase in output from additonal workers. And similarly with increases in *K* with *L* constant. In the case of the Cobb-Douglas production function this is true at all levels of output but this is not true for all production functions.

(ii) It is tempting to suppose that if DMP is present so that *MPL* is falling, then *APL* must be falling too. However in figure 14.21 we see that, between P and N, MPL is falling while APL is rising. The correct condition for APL to be falling is that MPL < APL.

(iii) $APL \equiv \frac{Q}{L} = 100K^{0.25}L^{-0.25}$, so $\frac{\partial \text{APL}}{\partial \text{L}} = (-0.25)100K^{0.25}L^{-1.25}$ which is negative since *K* and *L* are positive and a positive number raised to any power is positive (see section 2.12 of book). Similarly $APK = 100K^{-0.75}L^{0.75}$, so $\frac{\partial APK}{\partial K} = (-0.75)100K^{-1.75}L^{0.75}$ which is negative. Their graphs are similar to *APL* in figure 14.19b.

(f) (i) From 1(b) above, $MPL \equiv \frac{\partial Q}{\partial L} = 75K^{0.25}L^{-0.25}$, so

$\frac{\partial}{\partial K}(MPL) = \frac{\partial^2 Q}{\partial K\,\partial L} = (0.25)75K^{-0.75}L^{-0.25}$ which is positive since *K* and *L* are positive. Thus a (small) increase in the capital input increases the *MPL*. Similarly

$$\frac{\partial}{\partial L}(MPK) = \frac{\partial^2 Q}{\partial L\,\partial K} = (0.75)25K^{-0.75}L^{-0.25} > 0$$

(ii) It seems plausible that an increase in *K* should not only increase the output of existing workers (that is, increase *Q*) but also increase the amount of extra output that 1 more worker produces (that is, increase *MPL*). But it is not a logical necessity.

(g) (i) Since $(160{,}000)^{0.25} = 20$, $Q = 2000L^{0.75}$ The graph has the same shape as those in figure 14.19a.

(ii) The slope is positive indicating *MPL* positive; and slope decreases as *L* increases, indicating "law" of diminishing *MPL*.

(iii) Since $(810{,}000)^{0.25} = 30$, $Q = 3000L^{0.75}$. Since $3000L^{0.75} > 2000L^{0.75}$, this graph lies entirely above the graph of $Q = 2000L^{0.75}$, indicating that for any given *L*, *Q* is higher because *K* input is higher.

(iv) For any given *L*, the slope of $Q = 3000L^{0.75}$ is steeper than the slope of $Q = 2000L^{0.75}$, reflecting the fact that $\dfrac{\partial Q}{\partial K\,\partial L} > 0$.

(h) (i) $Q = 800{,}000K^{0.25}$. This quasi-short-run production function has the same shape as the short run production function in (g) above, except for scale. This is because the relationships between *Q* and *L*, and between *Q* and *K*, in the production function $Q = 100K^{0.25}L^{0.75}$ are identical except for the difference in their exponents. The shape of $Q = 800{,}000K^{0.2}$ reflects our earlier findings that $\dfrac{\partial Q}{\partial K}$ was always positive and $\dfrac{\partial^2 Q}{\partial K^2}$ always negative.

Exercise 14.4 (cont'd)

2. $Q = AK^{\alpha}L^{\beta}$

(a) $MPK \equiv \dfrac{\partial Q}{\partial K} = \alpha AK^{\alpha-1}L^{\beta}$. This is positive since α, *A*, *K*, and *L* are all positive.

Also $\dfrac{\partial}{\partial K}(MPK) \equiv \dfrac{\partial^2 Q}{\partial K^2} = (\alpha - 1)\alpha AK^{\alpha-2}L^{\beta}$. This is positive if $\alpha > 1$. Similarly,

$MPL \equiv \dfrac{\partial Q}{\partial L} = \beta AK^{\alpha}L^{\beta-1}$ is positive, and $\dfrac{\partial}{\partial L}(MPL) \equiv \dfrac{\partial^2 Q}{\partial L^2} = (\beta - 1)\beta AK^{\alpha}L^{\beta-2}$ is positive if $\beta > 1$.

(b) If $\alpha > 1$ this means that *MPK* increases as *K* increases with *L* constant; and if $\beta > 1$, *MPL* increases as *L* increases with *K* constant. Although each could be true over a certain range of values for *L* and *K*, it is hard to imagine how this could be true for all L and K. Thus we normally assume $\alpha < 1$ and $\beta < 1$ in the Cobb-Douglas production function.

3. Given $U = X^{0.75}Y^{1.5}$

(a) (i) Given $27 = X^{0.75}Y^{1.5}$ (the equation of the indifference curve for $U = 27$, as an implicit function). Raise both sides to power $\frac{2}{3}$ gives:

$9 = X^{0.5}Y \quad \Rightarrow \quad Y = \dfrac{9}{X^{0.5}}$ (The indifference curve as an explicit function).

(ii) From (i), $Y = 9X^{-0.5}$, so $\dfrac{dY}{dX} = -4.5X^{-1.5} = -\dfrac{4.5}{X^{1.5}}$ This is negative because $X^{1.5}$ is positive, so the indifference curve is downward sloping. We also have $\dfrac{d^2Y}{dX^2} = (-1.5)(-4.5)X^{-2.5}$ which is positive, so the slope increases as X increases (that is, becomes flatter) so the indifference curve is convex from below.

(iii) Negative slope of indifference curve reflects assumption that marginal utilities are always positive (non-satiation). Therefore if consumption of one good is increased, the other must be reduced in order to hold utility constant. The positive second derivative (which tells us that the indifference curve is convex) indicates diminishing absolute MRS between X and Y.

(iv) Shape of the indifference curve $Y = \dfrac{9}{X^{0.5}}$ is similar to that in figure 14.23 and 14.24, and is similar to a rectangular hyperbola. It is asymptotic to both axes, because $Y \to 0$ as $X \to \infty$, and there is a discontinuity at $X = 0$.

(b).(i) The indifference curve is $Y = \dfrac{4}{X^{0.5}}$. Slope is $\dfrac{dY}{dX} = -2X^{-1.5} = -\dfrac{2}{X^{1.5}}$ which is negative. We also have $\dfrac{d^2Y}{dX^2}3X^{-2.5}$ which is positive, so indifference curve is convex. Economic interpretation, and sketch, same answers as 3(a)(iii) and (iv).

(ii) The level of utility is 27 on the indifference curve in (a) and 8 on the indifference curve in (b), but these numerical values have no significance. We can only *rank* utility levels; that is, utility level 27 is higher than, and preferred to, level 8, because 27 > 8.

(iii) Given the utility function $U = X^{0.75}Y^{1.5}$, the indifference curve for $U = U_0$ has the equation $U_0 = X^{0.75}Y^{1.5}$ (implicit function) or $Y = \left(\frac{U_0}{X^{0.75}}\right)^{2/3}$ (explicit function). Thus all indifference curves have the same general shape (asymptotic to both axes)

(c) (i), (ii) $MU_X \equiv \dfrac{\partial U}{\partial X} = 0.75X^{-0.25}Y^{1.5}$, which is positive since X and Y are positive.

$MU_Y \equiv \dfrac{\partial U}{\partial Y} = 1.5X^{0.75}Y^{0.5}$ is positive for the same reason.

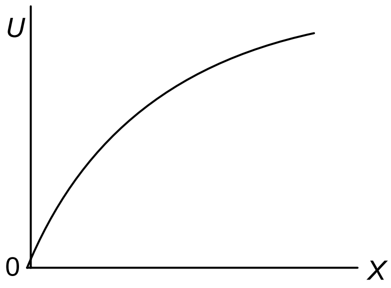
(iii) No, because an individual can become satiated with a good and increasing consumption beyond this level will, by definition, add nothing to her welfare.

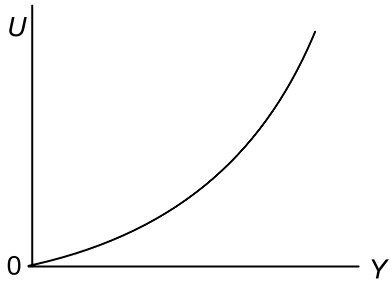
(d) (i) $\dfrac{\partial^2 U}{\partial X^2} = (-0.25)0.75X^{-1.25}Y^{1.5}$ which is negative, so consumption of good X is subject to diminishing marginal utility at all levels of consumption. But

$\dfrac{\partial^2 U}{\partial Y^2} = (0.5)(1.5)X^{0.75}Y^{0.5}$ which is positive, so consumption of good Y is subject to increasing marginal utility at all levels of consumption ("appetite grows with eating").

(ii) As the units in which U is measured are arbitrary, no significance can be attached to the absolute values of the second derivatives. For example the consumer could choose a new arbitrary unit, V, to measure her utility. Then we might have $V = U^2$, so then $V = X^{1.5}Y^3$, and $\dfrac{\partial^2 V}{\partial X^2}$ and $\dfrac{\partial^2 V}{\partial Y^2}$ are then both positive (check for yourself). Or, she could choose another unit, W, for measuring her utility, such that $W = U^{1/3}$. Then $W = X^{0.25}Y^{0.5}$ and $\dfrac{\partial^2 W}{\partial X^2}$ and $\dfrac{\partial^2 W}{\partial Y^2}$ are both negative (check for yourself).

(e) From (c) and (d) above, we have $\dfrac{\partial U}{\partial X} > 0$ and $\dfrac{\partial^2 U}{\partial X^2} < 0$ (diminishing MU). So curve giving U as function of X with Y constant is concave (see sketch below). But for Y, from (c) and (d) above, we have $\dfrac{\partial U}{\partial Y} > 0$ and $\dfrac{\partial^2 U}{\partial Y^2} > 0$ (increasing MU). So curve giving U as function of X with Y constant is convex (see sketch below).





(f) (i) Given $U = X^{0.75}Y^{1.5}$ with $\frac{\partial U}{\partial X} = 0.75X^{-0.25}Y^{1.5}$ (see (c) above)

Then $\frac{\partial^2 U}{\partial Y\,\partial X} = (1.5)(0.75)X^{-0.25}Y^{0.5}$ which is positive. Similarly

$\frac{\partial U}{\partial Y} = 1.5X^{0.75}Y^{0.5}$ (from (c) above), so $\frac{\partial^2 U}{\partial X\,\partial Y} = (0.75)1.5X^{-0.25}Y^{0.5}$ which is positive. So for this utility function a small increase in consumption of one good increases the *MU* of the other.

(ii) No significance, for reasons given in (d)(ii) above. See also Exercise 15.4, question 5, and its answer

4. (a) An "extreme" bundle (lots of *X*, not much *Y*) is $X = 256$, $Y = 9$. Then

$U = (256)^{0.75}(9)^{1.5} = 64 \times 27 = 1728$

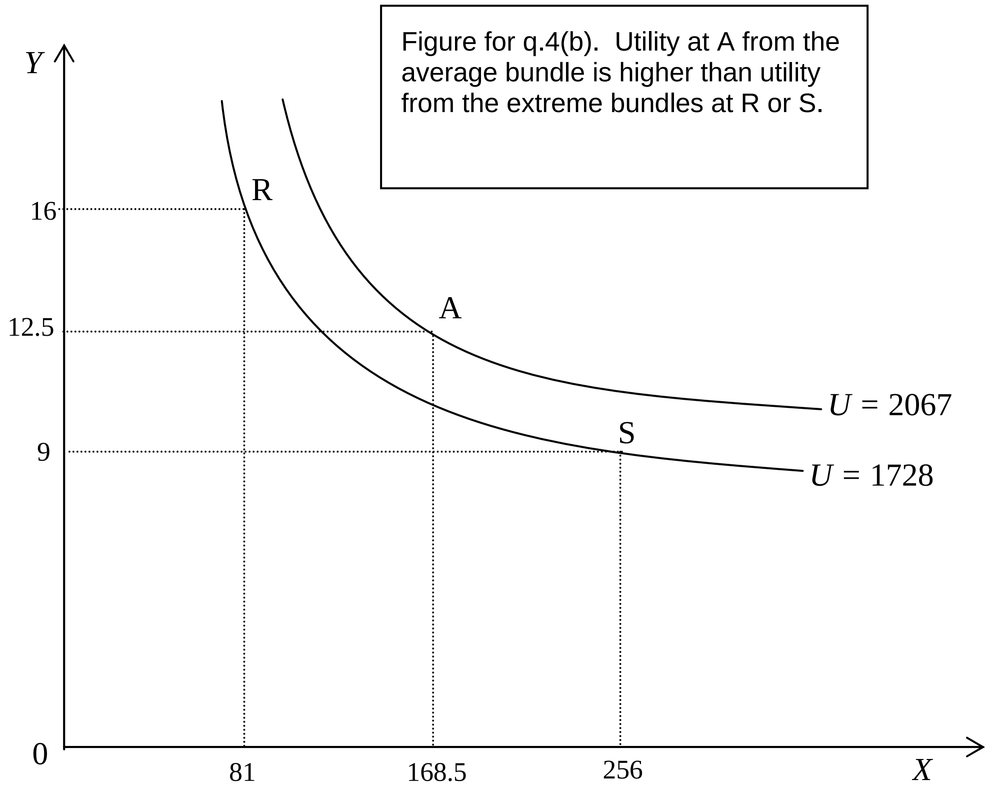
Another "extreme" bundle (lots of *Y*, not much *X*) is $X = 81$, $Y = 16$. Then

$U = (81)^{0.75}(16)^{1.5} = 27 \times 64 = 1728$

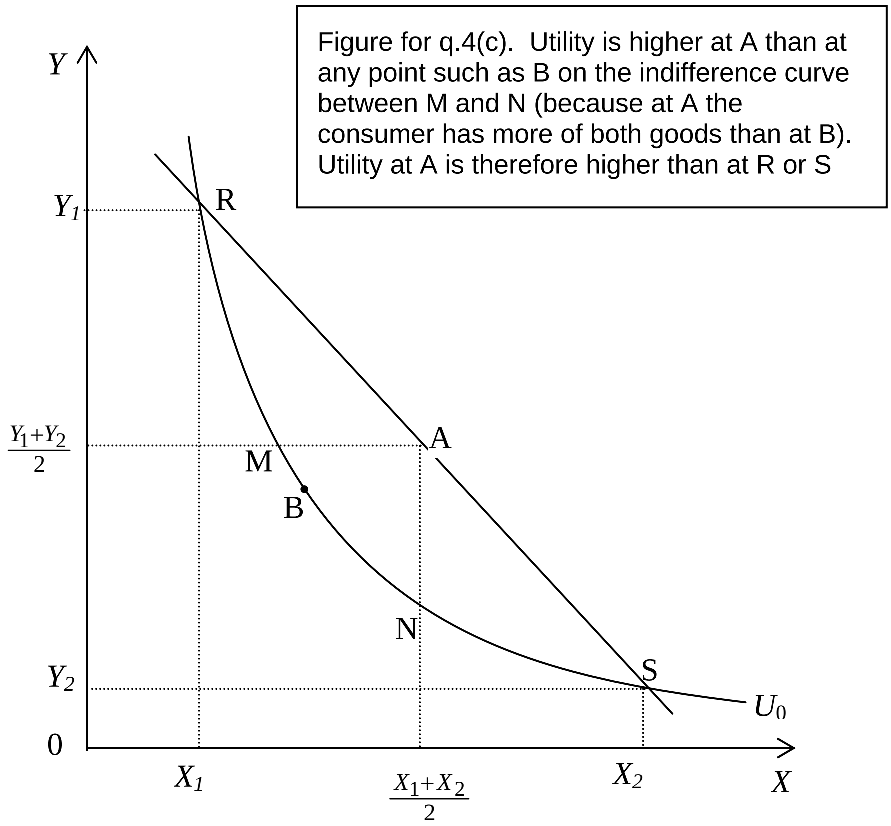
Thus these two bundles lie on the same indifference curve, $U = 1728$. The average or "middling" bundle is $X = \frac{256+81}{2} = 168.5$, $Y = \frac{9+16}{2} = 12.5$. Then

$U = (168.5)^{0.75}(12.5)^{1.5} = 2067$. So the "middling" bundle gives higher welfare than the extreme bundles.

(b) See figure below.



(c) Suppose that in the figure below the extreme bundles are point R with coordinates $(X_1\ ,\ Y_1)$ and point S with coordinates $(X_2\ ,\ Y_2)$. We can connect these two points by means of a straight line with equation $Y = a - bX$ where a and b are parameters. Thus we have $Y_1 = a - bX_1$ and $Y_2 = a - bX_2$.



Now consider the average bundle, consisting of $\frac{X_1 + X_2}{2}$ units of X and $\frac{Y_1 + Y_2}{2}$ units of Y (point A in figure above). This point also lies on the straight line $Y = a - bX$.

Proof: If the point $\left(\frac{X_1 + X_2}{2}, \frac{Y_1 + Y_2}{2}\right)$ lies on $Y = a - bX$, then

$\frac{Y_1 + Y_2}{2} = a - b\left[\frac{X_1 + X_2}{2}\right]$. After substituting $Y_1 = a - bX_1$ and $Y_2 = a - bX_2$ the left hand side becomes $\frac{(a - bX_1) + (a - bX_2)}{2} = \frac{2a - b(X_1 - X_2)}{2} = a - \frac{b(X_1 + X_2)}{2}$ = the right hand side (an identity).

Now since by assumption the indifference curve is convex and passes through (X_1, Y_1) and (X_1, Y_1), utility at A in figure above must be higher than at any point in the section of the indifference curve between M and N, such as B, because more of both goods is consumed at A than at B and both goods have positive marginal utility due to the assumption of non-satiation. However, utility *anywhere* on the indifference curve is the same as it is at B. Combining these two facts, utility at A is higher than utility anywhere on the indifference curve, including of course points R and S, as we wished to prove.

Progress exercise 14.4 (cont'd)

5(a) Repeat of question 3 for utility function *U* = *XY*:

Repeat of 3(a):

(i) Given $U = XY$, equation of indifference curve for some fixed level of utility, U^*, is

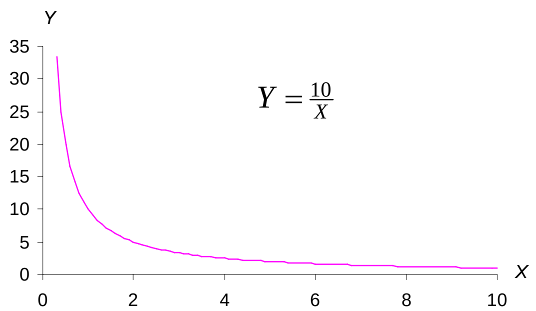
$U^* = XY$, which rearranges as $Y = \frac{U^*}{X}$ or $Y = U^* X^{-1}$

(ii) $\frac{dY}{dX} = -U^* X^{-2} = -\frac{U^*}{X^2} < 0$, so indifference curve is negatively sloped.

$\frac{d^2Y}{dX^2} = 2U^* X^{-3} = \frac{2U^*}{X^3} > 0$, so indifference curve is convex from below.

(iii) Answer is the same as answer to question 3(a)(iii) above.

(iv) See graph below of indifference curve for $U = 10$



Repeat of 3(b):

(i) Already answered in (a)(i) above.

(ii) and (iii) Already discussed in answer to 3(b) above.

Repeat of 3(c)

(i)(ii) Given $U = XY$, $MU_X \equiv \frac{\partial U}{\partial X} = Y$ and $MU_Y \equiv \frac{\partial U}{\partial Y} = X$. These are both positive assuming quantities X and Y of both goods are positive.

(iii) See answer to 3(c)(iii) above.

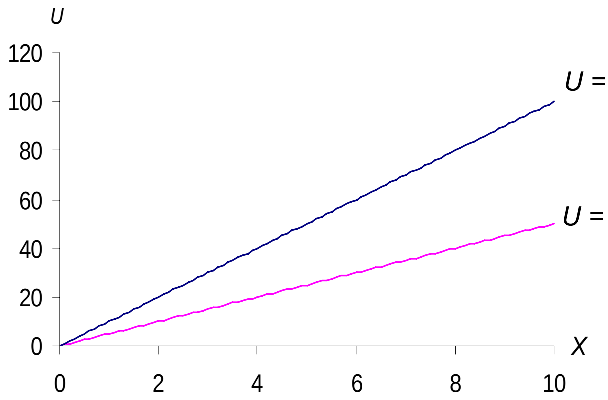
Repeat of 3(d)

(i) From (c) above, we have $\frac{\partial U}{\partial X} = Y$ so $\frac{\partial^2 U}{\partial X^2} = 0$. Similarly $\frac{\partial^2 U}{\partial Y^2} = 0$. So marginal utilities of both goods are constant.

(ii) No economic significance; see answer to 3(d) above.

Repeat of 3(e):

Graph below shows how U varies with X, when Y is fixed at 5 and 10



Repeat of 3(f):

From (d) above, $\frac{\partial U}{\partial X} = Y$, so $\frac{\partial^2 U}{\partial Y \partial X} = 1$. Similarly $\frac{\partial^2 U}{\partial X \partial Y} = 1$. No economic significance.

5(b) Repeat of question (4) for the utility function *U = XY*:

Repeat of 4(a): Suppose our "extreme" bundles are $X_0 = 1$, $Y_0 = 99$ (giving $U_0 = 1 \times 99 = 99$) and $X_1 = 99$, $Y_1 = 1$ (giving $U_1 = 99 \times 1 = 99$). Then the average of these two bundles is $X_2 = \frac{1+99}{2} = 50$ and $Y_2 = \frac{99+1}{2} = 50$, so the utility of this "middling" bundle is $U_2 = 50 \times 50 = 2500$. Thus the "middling" bundle gives higher utility that the extreme bundles, as we wished to show.

Repeat of 4(b): Sketch would be in essence the same as fig. for q. 4(b) above.

Repeat of 4(c): See answer to 4(c) and associated figure.

5(c) This utility function, although very simple, has one of the key characteristics of a plausible utility function: downward sloping and convex indifference curves. However, the fact that marginal utilities are constant is somewhat restrictive. (Note that this utitlity fn. illustrates the point made in the book that diminishing marginal utility is not a necessary condition for convexity of indifference curves.)