Progress exercise 11.1

(1) Without using a calculator we can say:

(a) 1, because $10^1 = 10$

(b) 2, because $10^2 = 100$

(c) If $x = \log 2$, then by definition $10^x = 2$, so *x* must be quite small, because even $10^{0.5} = \sqrt{10}$, which is more than 3 since $3^2 = 9$. So *x* must be less than 0.5, and is probably about 0.3.

(d) If $x = \log 2$, then $-x = \log\left(\frac{1}{2}\right)$; see below.

(e) If $10^x = 2$, then by definition $x = \log 2$. And if $10^x = 2$, $10^{-x} = \dfrac{1}{10^x} = \dfrac{1}{2}$. Then, by definition, $-x = \log\left(\frac{1}{2}\right)$. So $\log 2 = -\log\left(\frac{1}{2}\right)$ (and in general $\log A = -\log \frac{1}{A}$).

(2) (a) See figure 11.2(a).

(b) In figure 11.2(a) the *x*'s are the logs of the *y*'s. So $\log 500$ lies between 2 and 3 and looks to be nearer to 3 than 2. From (1)(e) above, $\log\left(\frac{1}{500}\right) = -\log 500$, so $\log\left(\frac{1}{500}\right)$ is between -2 and -3 and is nearer to -3 than to -2.

(c) See (1)(e) above.

(d) (i) Because $10^1 = 10$, $\log 10 = 1$. Similarly, because $10^0 = 1$, $\log 1 = 0$. Therefore the log of any number between 10 and 1 must lie between 1 and 0. And if $\log 1 = 0$, the log of any number which is less than 1 must be less than 0; that is, negative.

(ii) Also, since $y = 10^x$ is positive for all *x*, and the *x*'s are the logs of the *y*'s, it follows that when *y* is negative there is no corresponding *x*, hence negative numbers have no logs.

(3) (a) $\log y = 0.379$

(b) $y_0 = \dfrac{1}{y_1}$ (see (1)(e) above)

(4) (a) See figure 11.2(b)

(b) Since the *x*'s are the logs of the *y*'s, we can see that $\log 50$ lies between 1 and 2 and is probably nearer to 2 than to 1. $\log \frac{1}{50} = -\log 50$ so lies between -1 and -2.

(c) This graph is identical to that of $y = 10^x$ except that *x* is now on the vertical axis. If the labels on the two variables are interchanged, we then have the graph of $y = \log x$, which is the inverse of $x = 10^y$. (Confusing, isn't it?)

Progress exercise 11.2

(1) From calculator, $\log 2 = 0.3010$

(a) $\log 4 = \log(2 \times 2) = \log 2 + \log 2 = 0.6020$

(b) $\log(0.5) = \log(\frac{1}{2}) = -\log 2 = -0.3010$

(c) $\log \sqrt{0.5} = \log\left[(0.5)^{\frac{1}{2}}\right] = \frac{1}{2}\log(0.5) = -1505$

(d) $\log(\frac{1}{8}) = \log 1 - \log 8 = 0 - \log(2^3) = -3\log 2 = -0.9030$

(2) Assuming growth in annual jumps, $y = a(1+r)^x$, with $y = 250{,}000$, $a = 50{,}000$, $r = 0.15$, *x* unknown. Thus

$$250{,}000 = 50{,}000(1.15)^x$$
$$5 = (1.15)^x$$
$$\log 5 = \log\left[(1.15)^x\right] = x\log(1.15)$$
$$\Rightarrow \quad x = \frac{\log 5}{\log 1.15} = \frac{0.6990}{0.0607} = 11.52 \quad \text{(years)}$$

(3) Assuming growth in annual jumps, $y = a(1+r)^x$ with $y = \frac{1}{2}a$, $r = -0.05$, *x* unknown. Thus

$$\tfrac{1}{2}a = a(0.95)^x$$
$$\log(\tfrac{1}{2}) = \log\left[(0.95)^x\right] = x\log(0.95)$$
$$\Rightarrow \quad x = \frac{\log(\frac{1}{2})}{\log 0.95} = \frac{-0.3010}{-0.0223} = 13.5 \quad \text{(years)}$$

(4) (a) See figure 11.6

(b) Same as fig. 11.6 but with all numbers on *x* axis multiplied by 4.

(c) Same as fig. 11.6 but with all numbers on *x* axis divided by 2.

Comparing $y_1 = 10^{0.25x}$ with $y_0 = 10^x$, $y_1 = 10^{0.25x} = \left(10^x\right)^{0.25} = {y_0}^{0.25}$

Comparing $y_2 = 10^{2x}$ with $y_0 = 10^x$, $y_2 = \left(10^x\right)^2 = {y_0}^2$

They all have the same intercept because $10^x = 10^{0.25x} = 10^{2x} = 1$ when $x = 0$.