Progress exercise 10.1

1. Using rule 10.1, $\sum_{50} = \frac{50}{2}\{2 + (50-1)1\} = 1275$

2. Using rule 10.2, 11th term is $AR^{n-1} = 25(2)^{10} = 25{,}600$

3. Using rule 10.2, $\sum_{10} = \frac{A(R^n - 1)}{R-1} = \frac{1(2^{10}-1)}{2-1} = 2^{10} - 1 = 1023$

4. Using rule 10.2, $\sum_{10} = \frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^{10} - 1\right)}{\frac{1}{2} - 1} = 0.999023$ to 6 d.p.

   Using rule 10.2a, $\sum_{\infty} = \frac{A}{1-R} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$

5. (a)

month	*C*	*I*	*Y* = *C* + *I*	Δ*Y*
1	75	25	100	
2	75	35	110	10
3	82.5	35	117.5	7.5
4	88.125	35	123.125	5.625
5	92.344	35	127.344	4.219
6	95.508	35	130.508	3.164
7	97.881	35	132.881	2.373

(b) From col. 5 of the table above, we see that the increases in *Y*, ΔY, form the series

$10,\ 10 \times 0.75,\ (10 \times 0.75)0.75,\ \ldots$

This series is a GP with first term 10, common ratio 0.75. The increase after 1 year is the cumulative sum of these monthly increases. Using rule 10.2, this sum is

$$\sum_{12} = \frac{10\left(1 - (0.75)^{12}\right)}{1 - 0.75} = 38.733$$

(c) As the common ratio lies between 0 and 1, we can apply rule 10.2a. The sum to infinity is thus

$$\sum_{\infty} = \frac{A}{1-R} = \frac{10}{1-0.75} = 40$$

(d) Although rise in *Y* never quite reaches 40, (c) above gives us a close approximation to the increase in *Y* after 1 year, 2 years and so on; and is easy to calculate.

Progress exercise 10.2

1. Using rule 10.3, GDP after 25 years will be

$y = 100(1 + 0.025)^{25} = 185.39$

2. Using rule 10.3, we have $538 = 411(1 + r)^8$

$\Rightarrow \quad \left(\frac{538}{411}\right)^{\frac{1}{8}} = 1 + r = 1.03423$

So $\quad r = 1.03423 - 1 = 0.03423 = 3.423\%$

3. Using rule 10.3, we have $115 = 100(1 + r)^1$

$\Rightarrow \quad r = \frac{115}{100} - 1 = 0.15 = 15\%$

Current height is 115, so after 5 more years height will be

$y = 115(1 + 0.15)^5 = 115(2.0114) = 231.3$

4. (a) $\quad y = a(1 + r)^x = 100(1 + 0.06)^5 = 133.82$

(b) $\quad y = a\left(1 + \frac{r}{n}\right)^{nx} = 100\left(1 + \frac{0.06}{2}\right)^{2\times5} = 134.39$

(c) $\quad y = a\left(1 + \frac{r}{n}\right)^{nx} = 100\left(1 + \frac{0.06}{12}\right)^{12\times5} = 134.89$

5. Bank A: after 1 year, 1 euro becomes $y = 1(1 + 0.05)^1 = 1.05$

So effective annual rate = nominal rate $= 5\%$

Bank B: after 1 year, 1 euro becomes $y = 1\left(1 + \frac{0.049}{12}\right)^{12} = (1.00408)^{12} = 1.0501$

So effective annual rate $= 5.01\%$, while nominal annual rate $= 4.9\%$

6. $y = a(1 + r)^x$ with $a = 100{,}000$; *r* unknown ; $x = 5$; $y = 20{,}000$

$\Rightarrow \quad 20{,}000 = 100{,}000(1 + r)^5$

$$\left(\frac{20{,}000}{100{,}000}\right)^{\frac{1}{5}} = 1 + r = 0.7248$$

So $r = 0.7248 - 1 = -0.2752 = -27.52\%$

(Note: compound rather than simple interest is appropriate here because it seems reasonable to assume that the machine loses a constant percentage of its productive capacity each year, rather than a constant absolute amount of capacity.)

Progress exercise 10.3

1. (a) Using rule 10.5, $PV = y = \dfrac{20{,}000}{(1+0.05)^{10}} = \dfrac{20{,}000}{1.629} = 12{,}277.5$

   (b) I must invest $12{,}277.5$ because, from (a) above,

   $(12{,}277.5)(1+0.05)^{10} = 20{,}000$

   (c) $x = 12{,}277.5$, because from (b) above, $12{,}277.5$ now plus 10 years' interest will amount to $20{,}000$ in 10 years' time.

2. (a) Using rule 10.7, $PV = \dfrac{10{,}000}{1.05} + \dfrac{10{,}000}{(1.05)^2} + \dfrac{10{,}000}{(1.05)^3} + \dfrac{10{,}000}{(1.05)^4} + \dfrac{10{,}000}{(1.05)^5}$

   $+ \dfrac{10{,}000}{(1.05)^6} + \dfrac{10{,}000}{(1.05)^7} + \dfrac{10{,}000}{(1.05)^8} + \dfrac{10{,}000}{(1.05)^9} + \dfrac{10{,}000}{(1.05)^{10}} = 77{,}217.35$

   (b) $x = 77{,}217.35$; that is, the answer to (a) above. This is because, if I can borrow or lend freely at 5% per year, I can exchange in the market-place a lump sum of *x* now for the pension rights, and vice versa.

3. *PV* of series A: $\dfrac{50}{1.1} + \dfrac{100}{(1.1)^5} + \dfrac{150}{(1.1)^{10}} = 165.38$

   PV of series B: $\dfrac{40}{1.1} + \dfrac{100}{(1.1)^5} + \dfrac{165}{(1.1)^{10}} = 162.07$

   Although series A has lower total receipts (200 vs. 205), their *PV* is higher because they occur earlier than B's; that is, they are more "front loaded" (the term used in the business world).

4. (a) (i) Discounted at 5%, A has the higher *PV* (see table).

(ii) At 10%, B has the higher *PV*.

	Present Values		
	Series A (nuclear)	Series B (coal)	higher PV
Discounted at 5% per year	233.68	222.05	A
Discounted at 10% per year	197.47	200.43	B

(b) If series A and B are costs (not profit or revenue), then it makes sense to choose the project with the lower *PV*. The point of this question is to show that, on this criterion, which project will be chosen may depend on the discount rate. Specifically, when the discount rate is high (10%), the heavy costs of decommissioning the nuclear power plant in year 5 are more heavily discounted, so nuclear "wins" over coal. At a low discount rate (5%) the reverse is true; coal "wins".

Progress exercise 10.4

1. Using rule 10.9 to find the first repayment of principal,

$$P_1 = 5000\left[\frac{0.05}{(1.05)^5 - 1}\right] = 5000[0.18098] = 904.88$$

The first year's interest is $rk = 0.05(5000) = 250$, so the first year payment is $P_1 + rk = 1158.88$, and this also equals the total payment in every year.

2. Using rule 10.9, $P_1 = 200\left[\frac{0.015}{(1.015)^{24} - 1}\right] = 200[0.03492] = 6.98$

The first month's interest is $rk = 0.015(200) = 3$, so the monthly payments are 9.98.

3. (a) Using rule 10.9, $P_1 = 100{,}000\left[\frac{0.06}{(1.06)^{25} - 1}\right] = 100{,}000\left[\frac{0.06}{3.2919}\right] = 1822.66$

The first year's interest is $rk = 0.06(100{,}000) = 6000$, so the annual payments are $1822.66 + 6000 = 7822.66$

(b) The capital repayments are given by the series:

Year	1	2	3	4	5
Repayment of capital	P_1	$P_1(1+r)$	$P_1(1+r)^2$	$P_1(1+r)^3$	$P_1(1+r)^4$

So total repayments are the sum of these; that is,

$$P_1 + P_1(1+r) + P_1(1+r)^2 + P_1(1+r)^3 + P_1(1+r)^4$$

$$= P_1\left[1 + (1+r) + (1+r)^2 + (1+r)^3 + (1+r)^4\right] = 10{,}274.5$$

when $P_1 = 1822.66$ and $r = 0.06$

(c) From (a) above, the annual payments are 7822.66 so total payments over 25 years are $7822.66 \times 25 = 195{,}566.5$. But this includes repayment of the loan of 100,000, so the total interest is $195{,}566.5 - 100{,}000 = 95{,}566.5$.

(d) As stated in the question, at the end of each year you are charged interest on the amount owed at the beginning of that year. But if you pay monthly instalments, you are repaying some of this capital during the year. So at the end of the year you are charged interest on money that you have already repaid.