Progress exercise 9.1
Question 1

(a) (i) Arc elasticity $\equiv \frac{\frac{\Delta q}{\Delta p}}{\frac{q_0}{p_0}} = \frac{2}{\frac{5}{5}} = 2$

(ii) $\frac{2}{1.9} = 1.052632$

(b) (i) Point elasticity $\equiv \frac{\frac{\mathrm{d}q}{\mathrm{d}p}}{\frac{q}{p}} = \frac{2}{\frac{5}{5}} = 2$

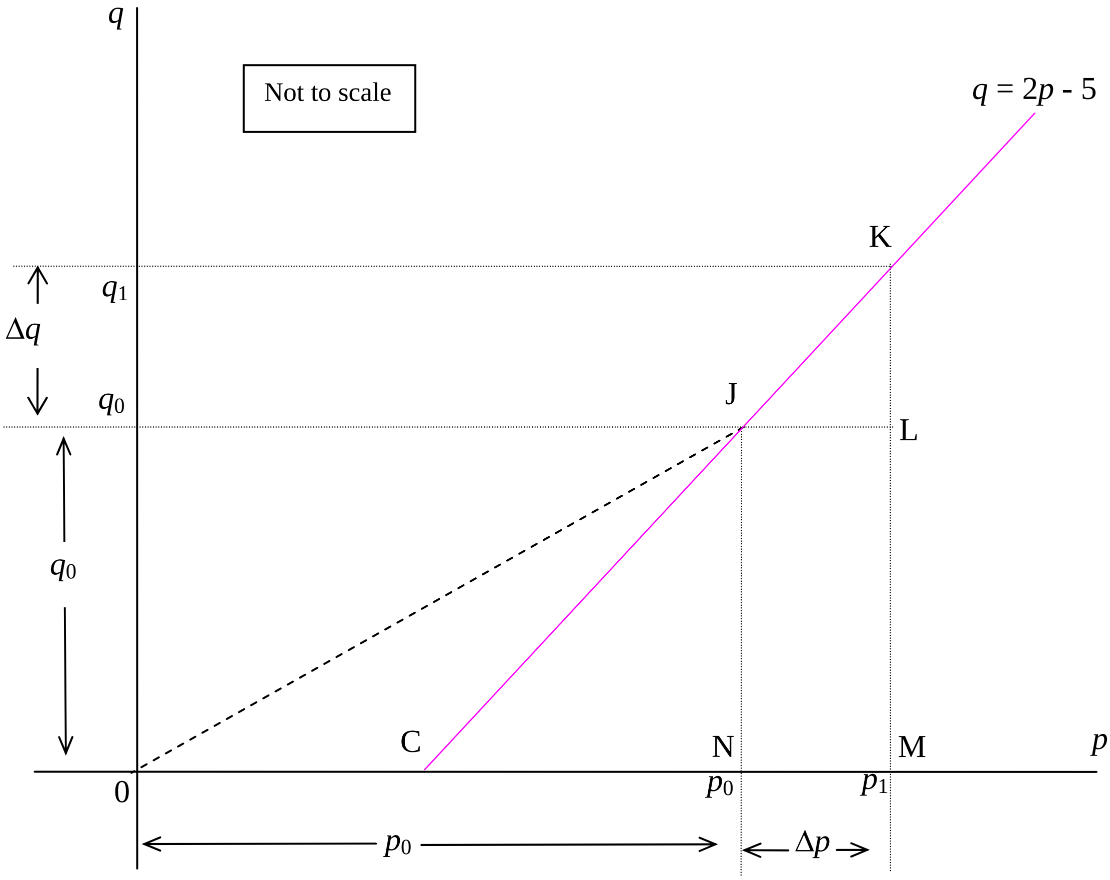
(ii) $\frac{2}{1.9} = 1.052632$

(c) Point elasticity $\equiv \frac{\frac{\mathrm{d}q}{\mathrm{d}p}}{\frac{q}{p}} = \frac{2}{\frac{2p-5}{p}} = \frac{2p}{2p-5}$

As *p* increases without limit, the difference between $2p$ and $2p-5$ becomes insignificant, and their ratio approaches 1.

(d) See figure next page.

Progress exercise 9.1 question 1(d)



Given the supply curve CK with equation $q = 2p - 5$, the arc elasticity of supply at J is:

$$E_A^S \equiv \frac{\frac{\Delta q}{\Delta p}}{\frac{q_0}{p_0}} = \frac{\frac{KL}{JL}}{\frac{JN}{0N}} = \frac{\text{slope of JK}}{\text{slope of OJ}}$$

For example, given $p_0 = 5$ and $p_1 = 6$, then $q_0 = 5$, $q_1 = 7$. So slope of JK = 2 and slope of OJ = 1, so the arc elasticity of supply is $\frac{2}{1} = 2$.

The point elasticity of supply is $$E^S \equiv \frac{\frac{dq}{dp}}{\frac{q_0}{p_0}} = \frac{\text{slope of tangent at J}}{\text{slope of OJ}} = \frac{\text{slope of JK}}{\text{slope of OJ}}$$

Thus in this case, because the supply curve is linear, the point elasticity is identical to the arc elasticity.

Progress exercise 9.1
Question 2

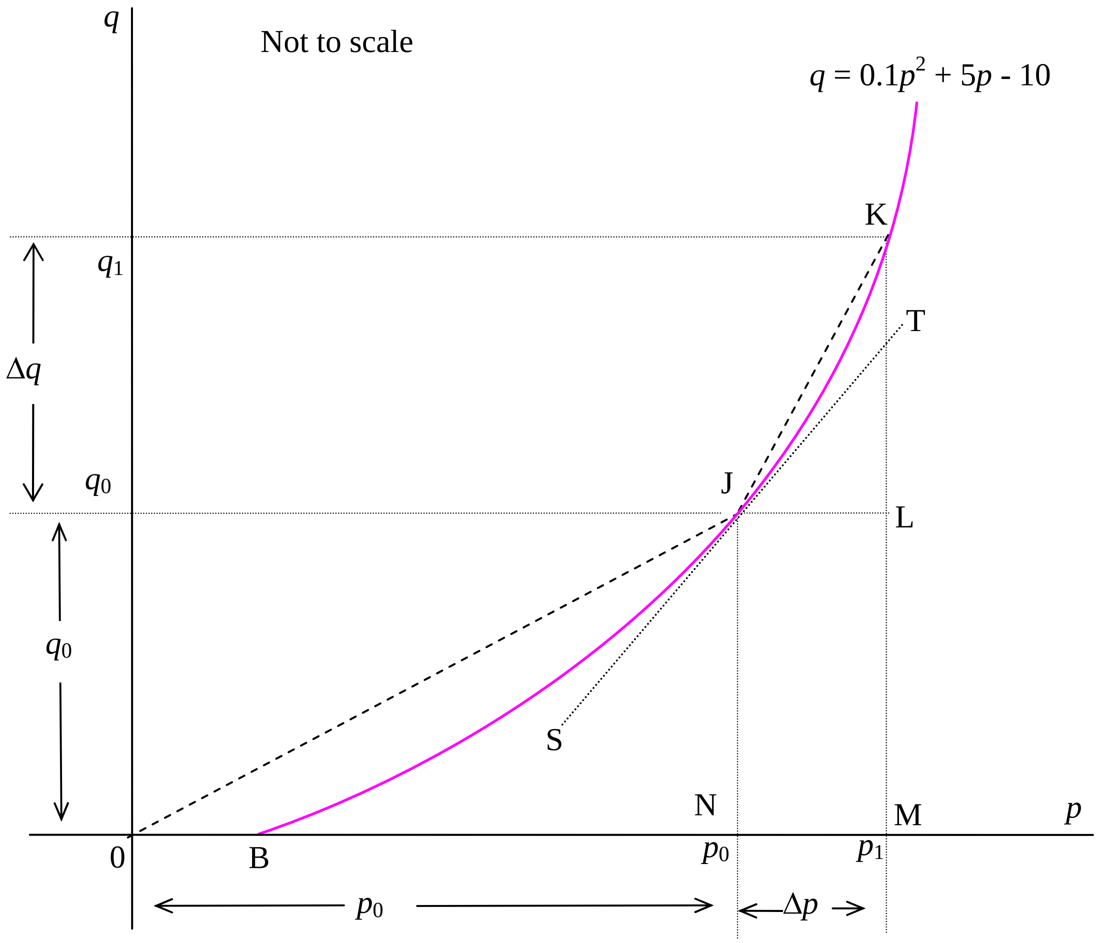
(a) Arc elasticity $= \dfrac{\frac{\Delta q}{\Delta p}}{\frac{q_0}{p_0}} = \dfrac{7.1}{5} = 1.42$

Point elasticity $= \dfrac{\frac{\mathrm{d}q}{\mathrm{d}p}}{\frac{q}{p}} = \dfrac{7}{5} = 1.4$

The two elasticity measures are very similar because, in the demand function, the coefficient of p^2 (0.1) is small relative to the coefficient of p (5), so the function is almost linear when p is small.

(b) See figure next page.

Progress exercise 9.1 question 2(b)



Given the supply curve BK with equation $q = 0.1p^2 + 5p - 10$, the arc elasticity of supply at J is:

$$E_A^S \equiv \frac{\frac{\Delta q}{\Delta p}}{\frac{q_0}{p_0}} = \frac{\frac{KL}{JL}}{\frac{JN}{0N}} = \frac{\text{slope of arc JK}}{\text{slope of OJ}}$$

For example, given $p_0 = 5$ and $p_1 = 6$, then $q_0 = 12.5$, $q_1 = 16.4$. So $q = 3.9$, $p = 1$, slope of JK = 3.9 and slope of OJ = 12.5 ÷ 5 = 2.5. So the arc elasticity of supply is $\frac{3.9}{2.5} = 1.56$.

The point elasticity of supply is $E^S \equiv \frac{\frac{dq}{dp}}{\frac{q_0}{p_0}} = \frac{\text{slope of tangent at J}}{\text{slope of OJ}} = \frac{\text{slope of ST}}{\text{slope of OJ}}$

In this example, because the slope of ST is less steep than the slope of the arc JK, the point elasticity is smaller than the arc elasticity. Check this for yourself by calculating the point elasticity at $p = 5$.

Progress exercise 9.1
Question 3

(a) For $p_0 = 10$, $p_1 = 11$, arc elasticity is $\frac{4.37}{3.7} = 1.181$ to 3 decimal places.

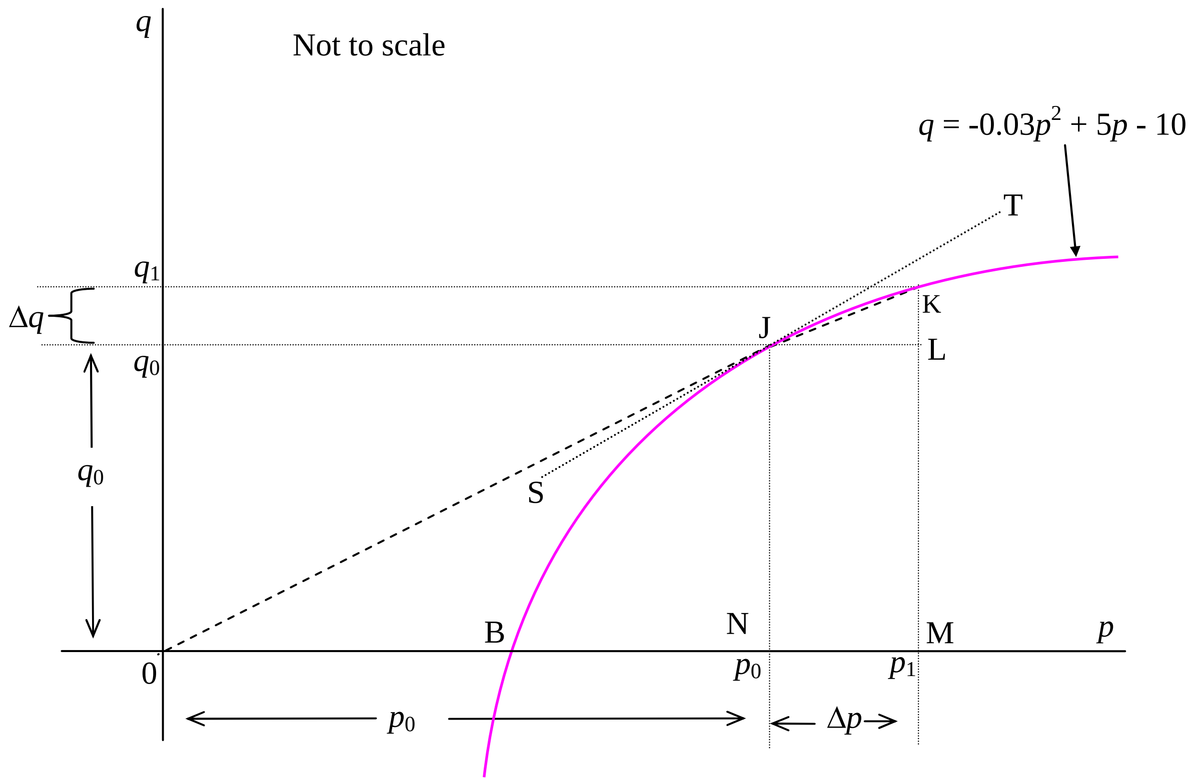
Point elasticity at $p = 10$ is $\frac{\frac{dq}{dp}}{\frac{q}{p}} = \frac{-0.06p + 5}{3.7}$

$= \frac{4.4}{3.7} = 1.189$ to 3 d.p.

As in (2), the elasticity measures are close because, for small *p*, the demand function is almost linear.

(b) See figure next page.

Progress exercise 9.1 question 3(b)



As in the previous example, given the supply curve BK with equation $q = -0.03p^2 + 5p - 10$, the arc elasticity of supply at J is:

$$E_A^S \equiv \frac{\frac{\Delta q}{\Delta p}}{\frac{q_0}{p_0}} = \frac{\frac{KL}{JL}}{\frac{JN}{0N}} = \frac{\text{slope of arc JK}}{\text{slope of OJ}}$$

and the point elasticity of supply at J is

$$E^S \equiv \frac{\frac{dq}{dp}}{\frac{q_0}{p_0}} = \frac{\text{slope of tangent at J}}{\text{slope of OJ}} = \frac{\text{slope of ST}}{\text{slope of OJ}}$$

In this example, because the slope of the tangent ST is steeper than the slope of the arc JK, the point elasticity is greater than the arc elasticity.

Note also that, because the slope of the arc JK is slightly less than the slope of 0J, the arc elasticity is slightly less than 1. Conversely, because the slope of the tangent ST is slightly greater than the slope of 0J, the point elasticity is slightly greater than 1.

Progress exercise 9.2
Question 1

(a) $E_A^D =$ arc elasticity of demand $= \dfrac{\frac{\Delta q}{\Delta p}}{\frac{q_0}{p_0}} = \dfrac{\Delta q}{\Delta p} \cdot \dfrac{p_0}{q_0}$

(i) $p_0 = 16,\ \ p_1 = 17$

$\Rightarrow \quad q_0 = 128\ ,\ \ q_1 = 126$

$E_A^D = \dfrac{-2}{\frac{128}{16}} = -2 \cdot \dfrac{16}{128} = -0.25$

(ii) $p_0 = 48,\ \ p_1 = 49 \quad \Rightarrow \quad q_0 = 64,\ \ q_1 = 62$

$E_A^D = \dfrac{-2}{\frac{64}{48}} = -2 \cdot \dfrac{48}{64} = -1.5$

(b) $E^D =$ point elasticity of demand $= \dfrac{\frac{\mathrm{d}q}{\mathrm{d}p}}{\frac{q}{p}} = \dfrac{\mathrm{d}q}{\mathrm{d}p} \cdot \dfrac{p_0}{q_0}$

(i) $p_0 = 16,\ \ q_0 = 128,\ \ \dfrac{\mathrm{d}q}{\mathrm{d}p} = -2$

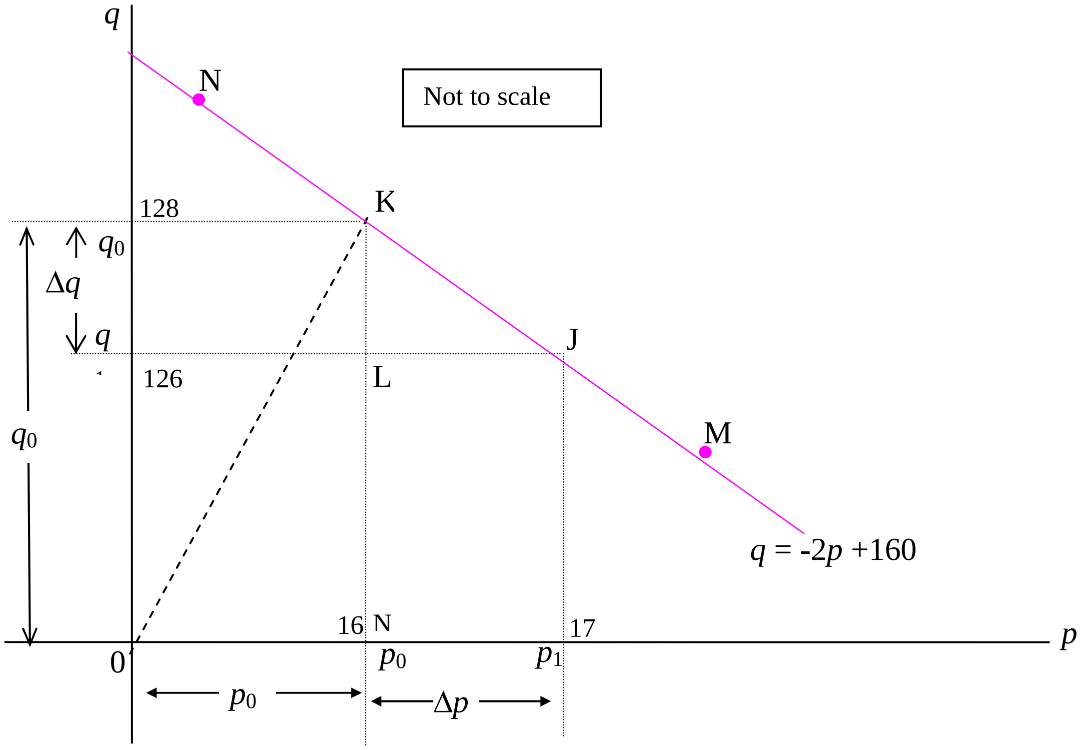
$E^D = \dfrac{-2}{\frac{128}{16}} = -0.25$

(ii) $p_0 = 48,\ \ q_0 = 64,\ \ \dfrac{\mathrm{d}q}{\mathrm{d}p} = -2$

$E^D = \dfrac{-2}{\frac{64}{48}} = -1.5$

(iii) See figure next page.

Progress exercise 9.2 question 1(c)



Given the demand curve with equation $q = 2p - 5$, the arc elasticity of demand at K is:

$$E_A^D \equiv \frac{\frac{\Delta q}{\Delta p}}{\frac{q_0}{p_0}} = \frac{\frac{KL}{LJ}}{\frac{KN}{0N}} = \frac{\text{slope of KJ}}{\text{slope of OK}}$$

(a) (i) Given $p_0 = 16$ and $p_1 = 17$, then $q_0 = 128$, $q_1 = 126$. So slope of KJ = -2 and slope of OK = 8, so the arc elasticity of supply is $\frac{-2}{8}$ = -0.25

(a) (ii) Here p increases from 48 to 49. The diagrammatic treatment is the same as the figure above, with $p_o = 48$ and $p_1 = 49$. The arc elasticity is -1.5.

(b) The point elasticity of demand at K is

$$E_D \equiv \frac{\frac{dq}{dp}}{\frac{q_0}{p_0}} = \frac{\text{slope of tangent at K}}{\text{slope of OK}} = \frac{\text{slope of KJ}}{\text{slope of OK}}$$

Thus in this case, because the demand curve is linear, the point elasticity is identical to the arc elasticity.

(d) When p is high (such as M in the diagram above) $\frac{q_0}{p_0}$ is small, and since this is the denominator of the arc and point elasticities, it follows that both are large in absolute value; that is, elastic. When p is low (such as N in the diagram above) $\frac{q_0}{p_0}$ is large, so demand is inelastic. Note that the numerator of both the arc and point elasticities is constant because the demand curve is linear.

Progress exercise 9.2 question 1(d)

(d) In both E_A^D and E^D, $\frac{q_0}{p_0}$ appears in the denominator. When p_0 is low (and consequently q_0 is high), then $\frac{q_0}{p_0}$ is large and therefore E_A^D and E^D are small (in absolute value). When p_0 is high, the reverse is true. So even though the slope of the demand function in this example is constant, the elasticity increases in absolute value (that is, demand becomes more elastic) as the price increases. In figure below, we can see that the slope of $\frac{q}{p}$ falls as *p* increases.

Progress exercise 9.2 question 2

(a) $E_A^D = \dfrac{\frac{\Delta q}{\Delta p}}{\frac{q_0}{p_0}} = \dfrac{\Delta q}{\Delta p} \cdot \dfrac{p_0}{q_0}$

(i) $p_0 = 5,\ p_1 = 6\ ;\ q_0 = 684.5,\ q_1 = 639$

$E_A^D = \dfrac{\frac{-45.5}{1}}{\frac{684.5}{5}} = -45.5 \cdot \dfrac{5}{684.5} = -0.3324$ (to 4 decimal places)

(ii) $p_0 = 12,\ p_1 = 13\ ;\ q_0 = 198,\ q_1 = 96.5$

$E_A^D = \dfrac{\frac{-101.5}{1}}{\frac{198}{12}} = -6.1515$

(b) $E^D = \dfrac{\mathrm{d}q}{\mathrm{d}p} \cdot \dfrac{p_0}{q_0}$

(i) $p_0 = 5,\ q_0 = 684.5,\ \dfrac{\mathrm{d}q}{\mathrm{d}p} = -8p - 1.5 = -41.5$ when $p = 5$

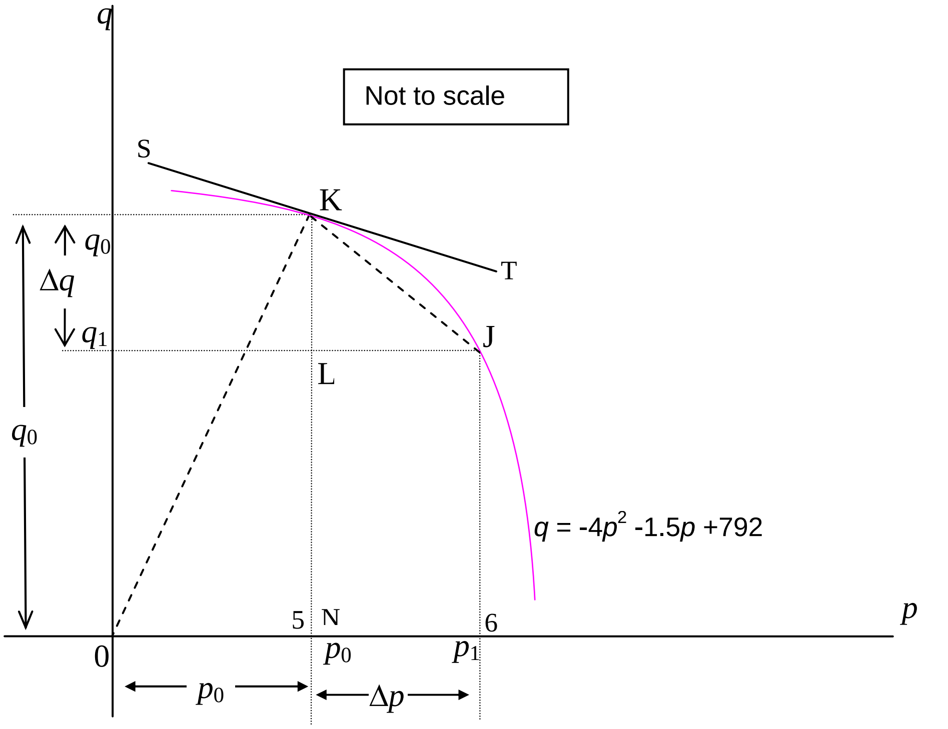
$E^D = \dfrac{5}{664.5}(-41.5) = -0.3123$ (to 4 dp)

(ii) $p_0 = 12,\ q_0 = 198,\ \dfrac{\mathrm{d}q}{\mathrm{d}p} = -97.5$ when $p = 12$

$E^D = \dfrac{12}{198}(-97.5) = -5.9091$

(c) See figure next page.

Exercise 9.2 question 2(c)



In figure above, the arc elasticity of demand at K is:

$$E_A^D \equiv \frac{\frac{\Delta q}{\Delta p}}{\frac{q_0}{p_0}} = \frac{\frac{KL}{LJ}}{\frac{KN}{0N}} = \frac{\text{slope of arc KJ}}{\text{slope of OK}}$$ (note that Δq is negative)

Considering for simplicity only question (a)(i), the arc elasticity is found by substituting $p_0 = 5$ and $p_1 = 6$, and the corresponding values of q_0 and q_1, into the expression above.

In figure above, the point elasticity of demand at K is

$$E_D \equiv \frac{\frac{dq}{dp}}{\frac{q_0}{p_0}} = \frac{\text{slope of tangent at K}}{\text{slope of OK}} = \frac{\text{slope of ST}}{\text{slope of OK}}$$

Again considering only (b)(i), given $q = -4p^2 - 1.5p + 792$, we have $\frac{dq}{dp} = -8p - 1.5 =$ -41.5 when $p = p_0 = 5$.
The point elasticity is found by substituting this expression into the expression for E_D above, with $p_0 = 5$ and with the corresponding value of q_0.

Progress exercise 9.2 question 2(d)

In this example, as p increases the demand curve becomes steeper, so both $\frac{dq}{dp}$ and $\frac{\Delta q}{\Delta p}$ increase in absolute value (though negative, of course). At the same time, the slope of 0K (= $\frac{q}{p}$) decreases. Thus both arc and point elasticities increase in absolute value because the numerator is increasing and the denominator is decreasing. Demand becomes more elastic. (Specifically, the absolute value of the point elasticity is 0.3123 when *p* = 5, but 5.9091 when *p* = 12.) Note that the demand elasticity increases, as *p* increases, more rapidly than in the case of linear demand function of question (1).

Progress exercise 9.2 question 3

(a) Demand function is $q = \frac{50}{p+2}$

(b) Using quotient rule, $\frac{\mathrm{d}q}{\mathrm{d}p} = \frac{(p+2)(0) - 50(1)}{(p+2)^2} = \frac{-50}{(p+2)^2}$

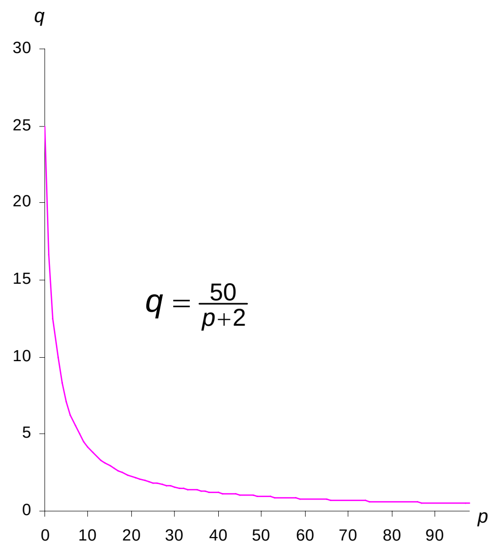
So $E^D = \frac{p}{q}\frac{\mathrm{d}q}{\mathrm{d}p} = \frac{p}{\frac{50}{p+2}} \cdot \frac{-50}{(p+2)^2} = \frac{-50p}{(p+2)^2} \cdot \frac{p+2}{50} = -\frac{p}{p+2}$

Because $p + 2 > p$ (assuming $p > 0$), it follows that $\frac{p}{p+2}$ is positive but less than 1, that is:

$0 < \frac{p}{p+2} < 1$ Multiplying by -1 we get

$0 > -\frac{p}{p+2} > -1 \quad \Rightarrow \quad 0 > E^D > -1$; that is, inelastic demand.

Progress exercise 9.2 question 3(c)



Progress exercise 9.2 question 4

(a) Demand function is $q = \dfrac{50}{p-2} = 50(p-2)^{-1}$

(b) Using function of a function rule, $\dfrac{\mathrm{d}\,q}{\mathrm{d}\,p} = -50(p-2)^{-2}$

So $$E^D \equiv \frac{p}{q}\frac{\mathrm{d}\,q}{\mathrm{d}\,p} = \frac{p}{50(p-2)^{-1}} \cdot \frac{-50}{(p-2)^{-2}} = -\frac{p}{p-2}$$

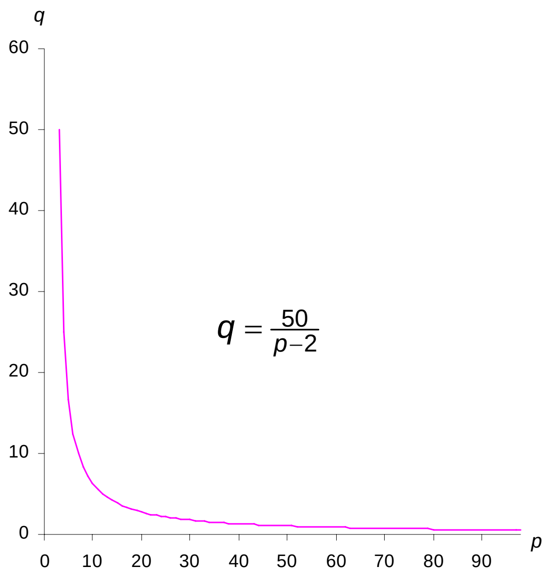
Assuming $p > 0$, $\dfrac{p}{p-2} > 1.$ Multiplying by -1,

$$-\frac{p}{p-2} < -1 \quad \Rightarrow \quad E^D < -1 \quad ;$$

that is, elastic demand.

(c) See figure below.

Progress exercise 9.2 question 4(c)



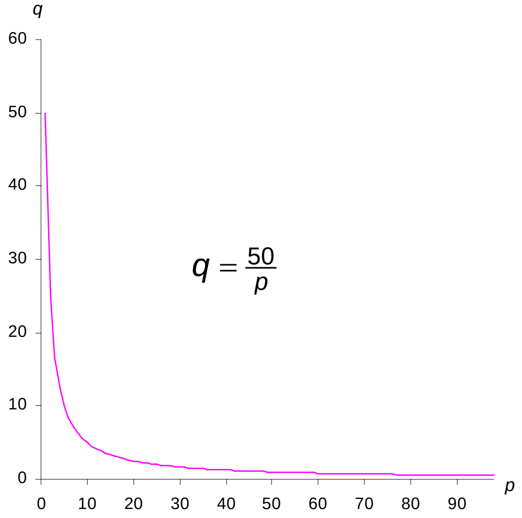
Progress exercise 9.2 question 5

(a) Demand function $q = \dfrac{50}{p} = 50p^{-1}$ so $\dfrac{\mathrm{d}\,q}{\mathrm{d}\,p} = -50p^{-2}$

(b) $E^D = \dfrac{p}{q}\dfrac{\mathrm{d}\,q}{\mathrm{d}\,p} = \dfrac{p}{50p^{-1}} - 50p^{-2} = \dfrac{-50p^{-1}}{50p^{-1}} = -1$

(c) See figure next page.

Progress exercise 9.2 question 5(c)



Progress exercise 9.3 question 1

Given $q = -2p + 160$ so inverse fn. is $p = -\frac{1}{2}q + 80$

(a) $TR \equiv pq = -\frac{1}{2}q^2 + 80q$

$$MR \equiv \frac{\mathrm{d}\,TR}{\mathrm{d}\,q} = -q + 80$$

(b) *TR* max. when $MR = -q + 80 = 0 \;\Rightarrow\; q = 80$. Also $\frac{\mathrm{d}^2\,TR}{\mathrm{d}\,q^2} = -1 < 0$ when $q = 80$ so 2nd order condition for a max. is satisfied.

Thus $q^* = 80$ and $p^* = -\frac{1}{2}q^* + 80 = 40$

(c) $E^D \equiv \frac{p}{q}\frac{\mathrm{d}\,q}{\mathrm{d}\,p} = \frac{p}{-2p+160}(-2) = \frac{-2p}{-2p+160} = \frac{p}{p-80}$

So when $p = p^* = 40$, $E^D = \frac{40}{40-80} = -1$

(d) If $p > 40$, then $p - 40 > 0$. We can suppose that $p - 40 = h$, where *h* is some unknown but positive number. (Here *h* is called a "slack variable").

Since $p - 40 = h,\; p = 40 + h$, so

$$E^D = \frac{40+h}{40+h-80} = \frac{40+h}{h-40} = -\frac{40+h}{40-h}$$

Since $40+h>40-h$ (because *h* is positive), we have $\frac{40+h}{40-h}>1$. Multiplying by -1 gives

$E^D=-\frac{40+h}{40-h}<-1$ so demand is elastic when $p>40$.

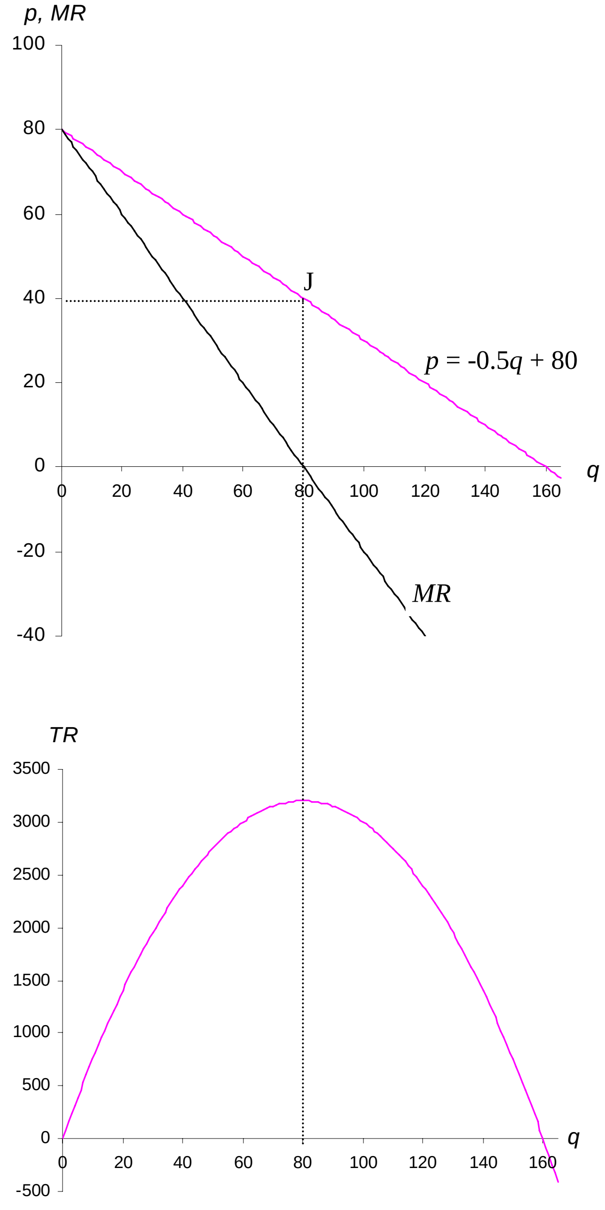
(e) If $p<40$ we can repeat all the steps above, but this time *h* is negative, so $40+h<40-h$ and therefore $\frac{40+h}{40-h}<1$. Multiplying by -1 gives

$E^D=-\frac{40+h}{40-h}>-1$ so demand is inelastic when $p<40$.

(f) See figure below.

Verbal explanation of relationship between marginal revenue and demand elasticity: when demand is elastic, this means that following a price reduction the gain in total revenue from the increase in *q* is greater than the loss of revenue from the fall in *p*, hence total revenue rises. But if total revenue rises, then by definition marginal revenue is positive. So when demand is elastic, marginal revenue is positive. Similarly when demand is inelastic, marginal revenue is negative.

Progress exercise 9.3 question 1(f)



From the algebra of the answer to this question, we know that along the demand curve, demand is elastic above J (where $p > 40$), and inelastic below J (where $p < 40$). At J, demand has unit elasticity.

Progress exercise 9.3 question 2

(a) Given $q = -4p^2 - 1.5p + 792$

$TR \equiv pq = -4p^3 - 1.5p^2 + 792p$ (as a fn. of p)

(b) $MR = -12p^2 - 3p + 792 = 0$ for max. TR

Using formula to solve this quadratic gives p = –8.25 or 8.

$\dfrac{d^2TR}{dp^2} = -24p - 3 < 0$ when $p = 8$ so 2nd order condition for max. satisfied.

When $p = 8$, $q = 524$, and $TR = pq = 4192$

(c) From (a), $\dfrac{dq}{dp} = -8p - 1.5$, so

$$E^D = \frac{p}{q}\frac{dq}{dp} = \frac{p}{-4p^2 - 1.5p + 792}\cdot(-8p - 1.5) = \frac{-8p^2 - 1.5p}{-4p^2 - 1.5p + 792}$$

(d) From (c), $E^D = -1 \Rightarrow 8p^2 + 1.5p = -4p^2 - 1.5p + 792$

$\Rightarrow 12p^2 + 1.5p - 792 = 0$

But from (b) above, we know that that when $12p^2 + 1.5p - 792 = 0$, p = 8 and TR at its maximum. So $E^D = -1$ when TR at its max.

(e) Elastic demand $\Rightarrow$ $E^D > -1$. From (d), $E^D > -1 \Rightarrow 8p^2 + 1.5p > -4p^2 - 1.5p + 792$

$\Rightarrow 12p^2 + 1.5p > 792$

But from (b) we know that that when p = 8, $12p^2 + 1.5p = 792$ and $E^D = -1$. So clearly when p > 8, $12p^2 + 1.5p > 792$ and $E^D > -1$.

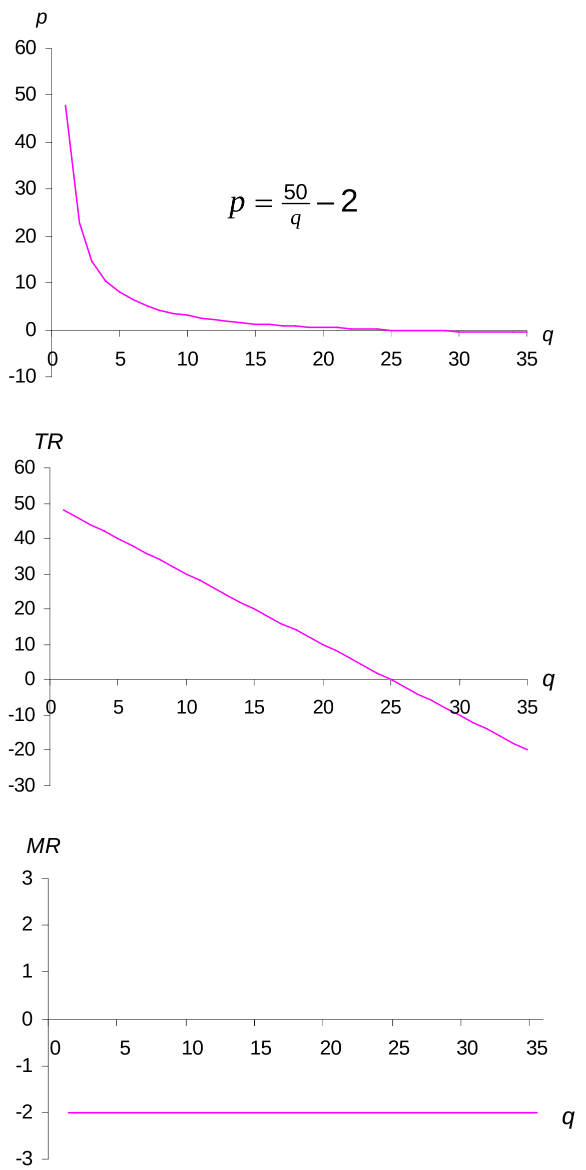
So $E^D > -1$ when p > 8. In the same way, we can show that $E^D < -1$ (inelastic demand) when p < 8.

Progress exercise 9.3 question 3

(a) Given $p = \frac{50}{q} - 2$, $TR = pq = 50 - 2q$. See below for graphs.

(b) From (a), MR = –2. Since MR always negative, this implies that TR function always negatively sloped. This also follows from the fact that demand is inelastic at any price, as we found in Ex. 9.2, q3. Inelastic demand means that a price reduction, and associated quantity increase, reduces *TR*.

Figures for exercise 9.3 question 3(a)



In this example, because demand is inelastic at any price, a price reduction always reduces total revenue. Thus marginal revenue is always negative. Note that we have not plotted *TR* or *MR* for $q < 1$, as in the inverse demand function *p* increases without limit at *q* approaches zero.

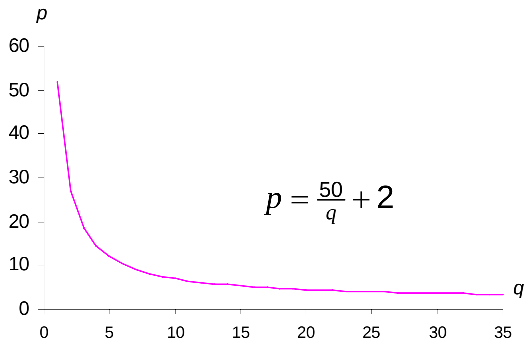
Progress exercise 9.3 question 4

(a) In this case we have $TR = pq = 50 + 2q$. See below for graphs.

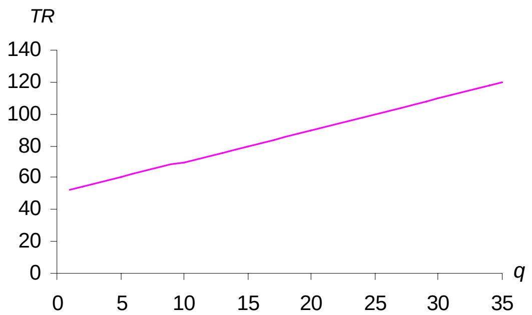
Therefore *MR* = +2. Since *MR* is always positive, this implies that *TR* function always positively slopes. This also follows from the fact that demand is elastic at any price, as we found in Ex. 9.2, q4. Elastic demand means that a price reduction, and associated quantity increase, increases *TR*.

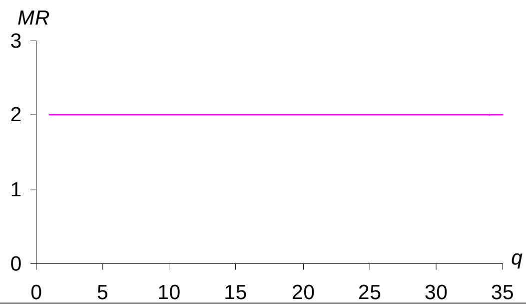
(b) It seems very unlikely that any demand curve would have constant elasticity at <u>every</u> price, as in these two examples. But it is not unreasonable to suppose that a demand curve might have constant elasticity over a range of values of *p*, and this assumption is frequently made to simplify problems in economic analysis, both theoretical and applied.

Figures for exercise 9.3 question 4



In this example, because demand is elastic at any price, a price reduction always increases total revenue. Thus marginal revenue is always positive. Note that we have not plotted *TR* or *MR* for *q* < 1, as in the inverse demand function *p* increases without limit at *q* approaches zero.





Progress exercise 9.4 question 1

Given $TC = 9q^2 + 2q + 8100$

(a) $MC \equiv \dfrac{\mathrm{d}\,TC}{\mathrm{d}\,q} = 18q + 2$

$$AC \equiv \frac{TC}{q} = 9q + 2 + \frac{8100}{q}$$

(b) *AC* minimum where $\dfrac{\mathrm{d}\,AC}{\mathrm{d}\,q} = 0$ and $\dfrac{\mathrm{d}^2\,AC}{\mathrm{d}\,q^2} > 0$

$$\frac{\mathrm{d}\,AC}{\mathrm{d}\,q} = 9 - \frac{8100}{q^2} = 0 \quad \Rightarrow \quad 9q^2 = 8100 \quad \Rightarrow \quad q = 30$$

$\dfrac{\mathrm{d}^2\,AC}{\mathrm{d}\,q^2} = \dfrac{16200}{q^3} > 0$ when $q = 30$ so this is a minimum.

$$MC = AC \quad \Rightarrow \quad 18q + 2 = 9q + 2 + \frac{8100}{q} \quad \Rightarrow \quad 9q^2 = 8100 \quad \Rightarrow \quad q = 30$$

(c) $MC < AC \quad \Rightarrow \quad 18q + 2 < 9q + 2 + \dfrac{8100}{q} \quad \Rightarrow \quad 9q^2 < 8100 \quad \Rightarrow \quad q < 30$

Also, *AC* falling when $\dfrac{\mathrm{d}\,AC}{\mathrm{d}\,q} = 9 - \dfrac{8100}{q^2} < 0 \quad \Rightarrow \quad 9q^2 < 8100 \quad \Rightarrow \quad q < 30$. So when $q < 30,\; MC < AC$ and *AC* falling. Similarly we can show that when $q > 30,\; MC > AC$ and *AC* rising.

(d) $E_{TC} \equiv \dfrac{\frac{\mathrm{d}\,TC}{\mathrm{d}\,q}}{\frac{TC}{q}} \equiv \dfrac{MC}{AC}$

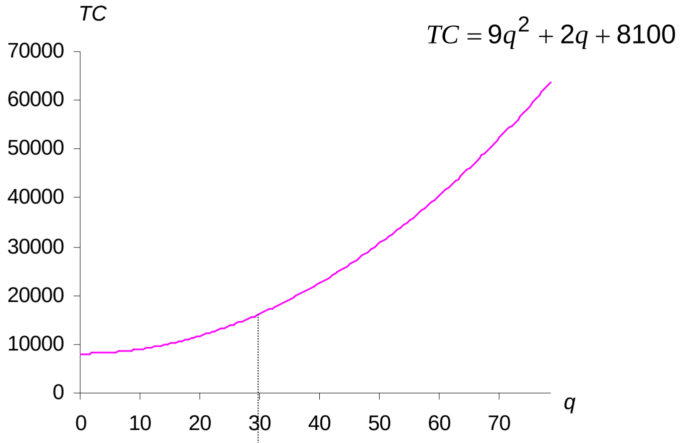
(e) $E_{TC} < 1 \quad \Rightarrow \quad \dfrac{MC}{AC} < 1 \quad \Rightarrow \quad MC < AC$

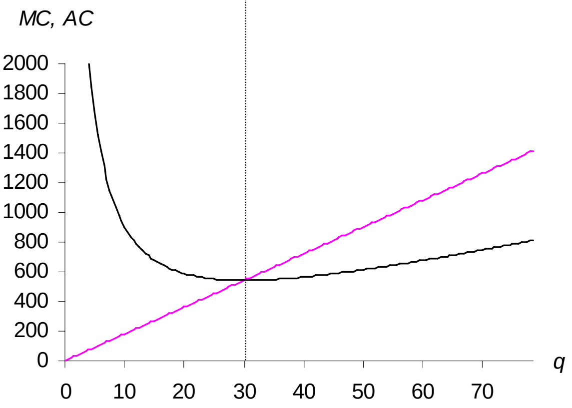
But, from (c) $MC < AC \Rightarrow AC$ falling. So $E_{TC} < 1 \Rightarrow AC$ falling.

Similarly, $E_{TC} > 1 \Rightarrow AC$ rising.

(f) See figure next page.

Progress exercise 9.4 question 1(f)





When q is less than 30, the elasticity of TC is less than 1, hence TC is rising proportionately more slowly than output. When q is greater than 30, the elasticity of TC is greater than 1, hence TC is rising proportionately faster than output.

Progress exercise 9.4 question 2

Given $TC = aq^2 + bq + c$

(a) $MC \equiv \dfrac{\mathrm{d}TC}{\mathrm{d}q} = 2aq + b$

$AC \equiv \dfrac{TC}{q} = aq + b + \dfrac{c}{q}$

(b) AC minimum where $\dfrac{\mathrm{d}\,AC}{\mathrm{d}\,q} = 0$ and $\dfrac{\mathrm{d}^2\,AC}{\mathrm{d}\,q^2} > 0$

$\dfrac{\mathrm{d}AC}{\mathrm{d}q} = a - \dfrac{c}{q^2} = 0 \quad \Rightarrow \quad q = \sqrt{\tfrac{c}{a}}$

$\dfrac{\mathrm{d}^2\,AC}{\mathrm{d}q^2} = \dfrac{2c}{q^3}$. We assume q > 0. If there are fixed costs, c is also positive, so then $\dfrac{\mathrm{d}^2\,AC}{\mathrm{d}q^2} > 0$ so the AC curve has a minimum (rather than a max.).

$MC = AC \quad \Rightarrow \quad 2aq + b = aq + b + \dfrac{c}{q} \quad \Rightarrow \quad aq = \dfrac{c}{q} \quad \Rightarrow \quad q = \sqrt{\tfrac{c}{a}}$. But from above, we know that min. *AC* is where $q = \sqrt{\tfrac{c}{a}}$. So *MC* = *AC* at min. *AC*.

(c) $MC = AC \quad \Rightarrow \quad 2aq + b < aq + b + \dfrac{c}{q} \quad \Rightarrow \quad aq < \dfrac{c}{q} \quad \Rightarrow \quad aq < \tfrac{c}{q}$

If we multiply both sides of this inequality by q^2 (which does not reverse the direction of inequality because q^2 is positive); and also multiply both sides by *a* (which also does not reverse the direction of inequality if we assume *a* > 0, which is a reasonable assumption), then the inequality becomes $q^2 < \dfrac{c}{a}$, or $q = \sqrt{\tfrac{c}{a}}$

Also, *AC* falling when $\dfrac{\mathrm{d}AC}{\mathrm{d}q} = a - \dfrac{c}{q^2} < 0 \quad \Rightarrow \quad a < \dfrac{c}{q^2}$. Again we multiply both sides by q^2 and obtain $q^2 < \dfrac{c}{a}$, or $q = \sqrt{\tfrac{c}{a}}$. So we have shown that when *AC* is falling, *MC* < *AC*. In the same way, we can show that when *AC* is rising, *MC* > *AC*.

(d) $E_{TC} \equiv \dfrac{\frac{\mathrm{d}TC}{\mathrm{d}q}}{\frac{TC}{q}} \equiv \dfrac{MC}{AC}$

(e) $E_{TC} < 1 \quad \Rightarrow \quad \dfrac{MC}{AC} < 1 \quad \Rightarrow \quad MC < AC$

But, from (c) $MC < AC \Rightarrow AC$ falling. So $E_{TC} < 1 \Rightarrow AC$ falling.

Similarly, $E_{TC} > 1 \Rightarrow AC$ rising.

(f) The figure is identical to that in part (f) of the previous question, except that we cannot calibrate the axes. Recall the assumptions we made in proving results above; that in the general quadratic *TC* function, $TC = aq^2 + bq + c$, both *a* and *c* were positive.

Progress exercise 9.4 question 3

Given $C = 0.9Y + 100$

(a) $MPC \equiv \dfrac{\mathrm{d}C}{\mathrm{d}Y} = 0.9 \quad : \quad APC \equiv \dfrac{C}{Y} = 0.9 + \dfrac{100}{Y}$

(b) $MPC < APC \quad \Rightarrow \quad 0.9 < 0.9 + \dfrac{100}{Y} \quad \Rightarrow \quad 0 < \dfrac{100}{Y}$

which is true if $Y > 0$ (as we assume is the case).

Since $\dfrac{100}{Y}$ decreases as Y increases, *APC* approaches *MPC* as Y increases without limit.

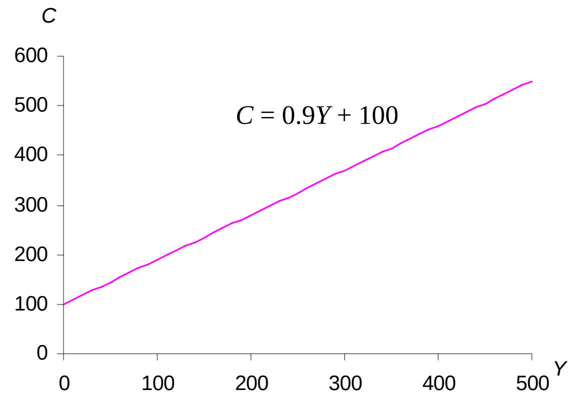
(c) Elasticity by definition equals $\dfrac{Y}{C}\dfrac{\mathrm{d}C}{\mathrm{d}Y} \equiv \dfrac{\frac{\mathrm{d}C}{\mathrm{d}Y}}{\frac{C}{Y}} \equiv \dfrac{MPC}{APC}$

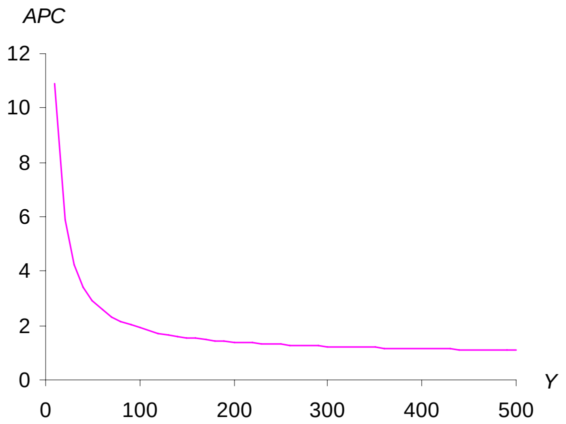
Given $C = 0.9Y + 100$, elasticity $= \dfrac{0.9}{\frac{0.9 \cdot 100}{Y}} = \dfrac{0.9Y}{0.9Y + 100}$

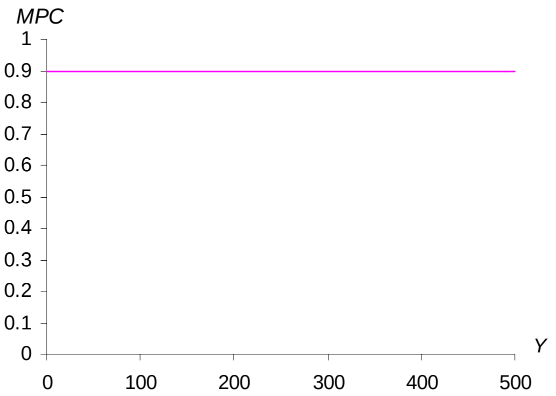
With $Y > 0,\ 0.9Y < 0.9Y + 100$, so elasticity is less than 1.

(d) See figure next page.

Progress exercise 9.4 question 3(d)







In this example the *APC* is always greater than the *MPC* due to the additive constant in the consumption function. However the *APC* falls continuously as *Y* increases, and approaches the *MPC* as *Y* approaches infinity.
Note that $C > Y$ when *Y* is small, implying the consumer is able to borrow or draw on past savings.

Progress exercise 9.4 question 4

(a) Given $C = aY + b$

$$MPC \equiv \frac{dC}{dY} = a \quad : \quad APC \equiv \frac{C}{Y} = a + \frac{b}{Y}$$

So $MPC > APC \quad \Rightarrow \quad a > a + \frac{b}{Y} \quad \Rightarrow \quad 0 > \frac{b}{Y}$

which is true if $b < 0$ (since we assume $Y > 0$). The consumption function would then have a negative intercept on the Y axis. This implies there is some positive level of income at which consumption falls to zero, which seems very unlikely.

(b) $MPC = APC \quad \Rightarrow \quad a = a + \frac{b}{Y} \quad \Rightarrow \quad b = 0$. This consumption function passes through the origin.