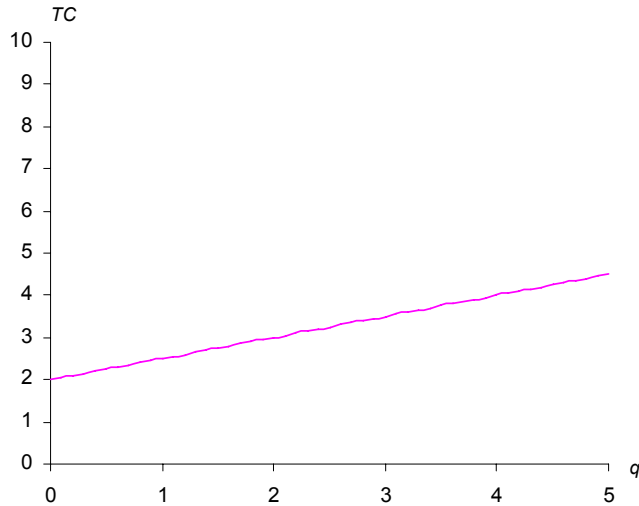


**Progress exercise 8.1 question 1**

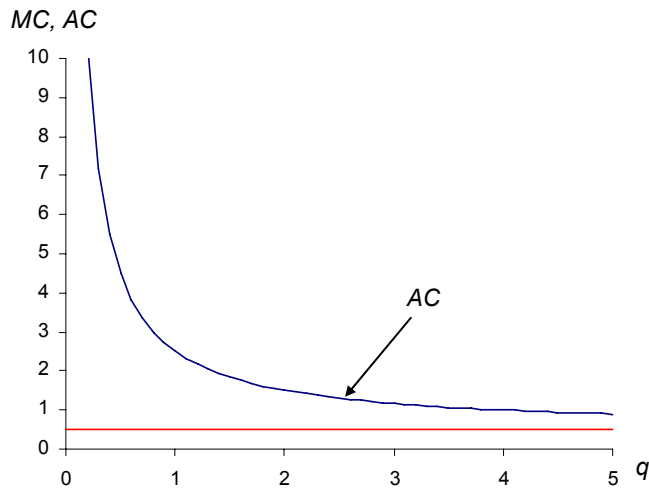
(a)

Ex 8.1 question 1



(b)  $MC \equiv \frac{dTC}{dq} = 0.5$ .  $AC \equiv \frac{TC}{q} = \frac{0.5q + 2}{q} = 0.5 + \frac{2}{q}$ .

Exercise 8.1 question 1

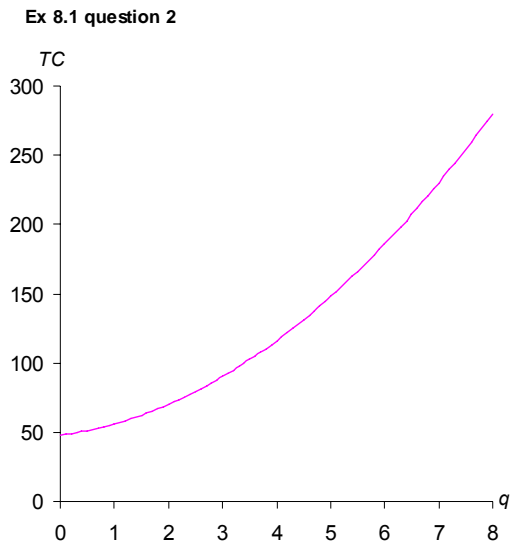


(c) AC includes fixed costs per unit of output ( $= \frac{2}{q}$  in this example); MC does not.

(d) If  $TC$  is linear, say  $TC = aq + b$ , we have  $MC = a$  and  $AC = a + \frac{b}{q}$ . So  $MC = AC$  only if  $b = 0$ ; that is, no fixed costs.

**Progress exercise 8.1 question 2**

(a)

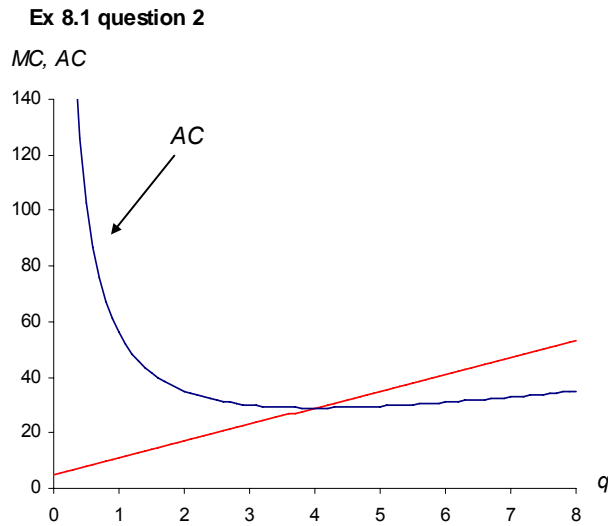


(b)  $MC = 6q + 5$ .  $AC = 3q + 5 + \frac{48}{q}$

(c) Minimum AC when  $q = 4$  (ignoring negative root). Then,  $\frac{dAC}{dq} = 0$  and

$$\frac{d^2 AC}{dq^2} > 0. \text{ When } q = 4, MC = AC = 29.$$

(d)



**Progress exercise 8.1 question 3**

(a) When  $q = 2$ ,  $\frac{dAC}{dq} = 4q - 2 - 24q^{-2} = 0$ . And  $\frac{d^2AC}{dq^2} = 4 + \frac{48}{q^3} > 0$ . So a minimum of

AC.

(b) When  $q = \frac{1}{3}$ ,  $MC \equiv \frac{dTC}{dq} = 6q^2 - 4q + 5 = 0$  and  $\frac{d^2MC}{dq^2} = 12$  which is  $> 0$ . So

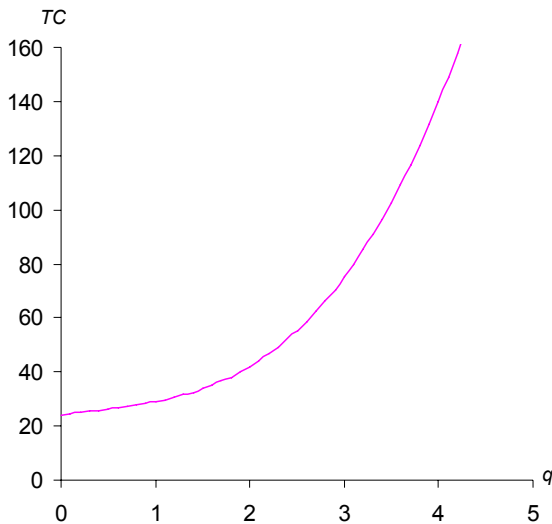
$q = \frac{1}{3}$  is a minimum of MC.

(c) AC minimum when  $q = 2$  (from (a) above). When  $q = 2$ ,

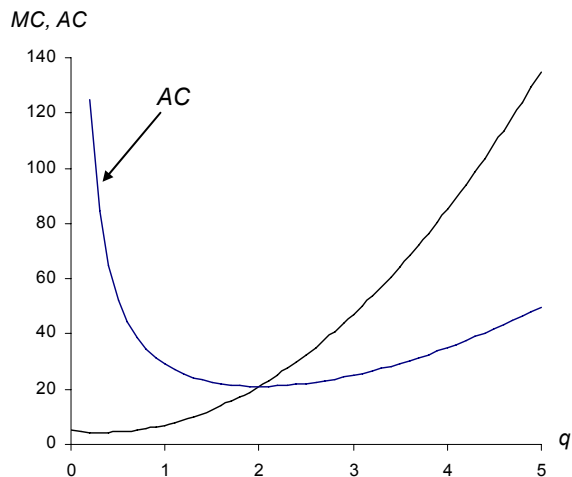
$$AC = 6(2)^2 - 20(2) + 25 + \frac{24}{2} = 21, \text{ and } MC = 6(2)^2 - 4(2) + 5 = 21$$

(d), (e)

Ex 8.1 question 3



Ex 8.1 question 3



**Progress exercise 8.1 question 4**

4.

(a)  $\frac{dAC}{dq} = 12q - 20 - 144q^{-2}$ . When  $q = 3$ ,  $\frac{dAC}{dq} = 0$  and  $\frac{d^2AC}{dq^2} = 12 + \frac{288}{q^3}$  is

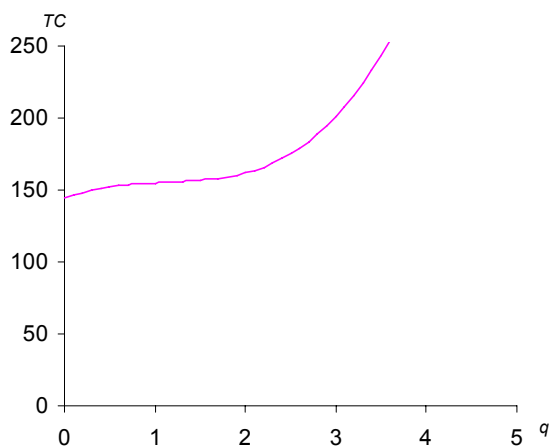
positive. So  $q = 3$  is a minimum of  $AC$ .

(b) From (a), min.  $AC$  at  $q = 3$ . When  $q = 3$ ,  $AC = 67$  and  $MC = 67$ . So  $MC = AC$ .

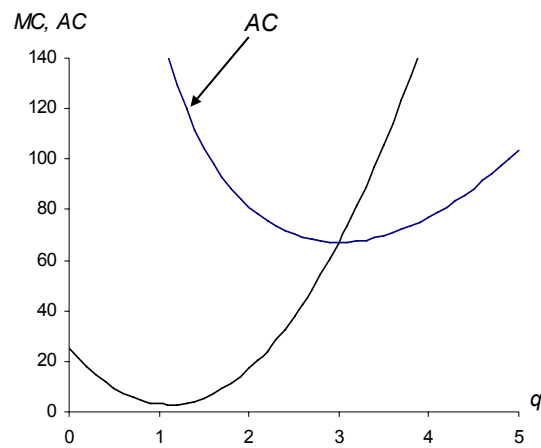
(c)  $\frac{dMC}{dq} = 36q - 40 = 0$  when  $q = \frac{40}{36} = 1\frac{1}{9}$ , and  $\frac{d^2MC}{dq^2} = 36 > 0$ . So  $q = 1\frac{1}{9}$  is a minimum of  $MC$ .

(d), (e)

Ex 8.1 question 4



Ex 8.1 question 4



**Progress exercise 8.2 question 1**

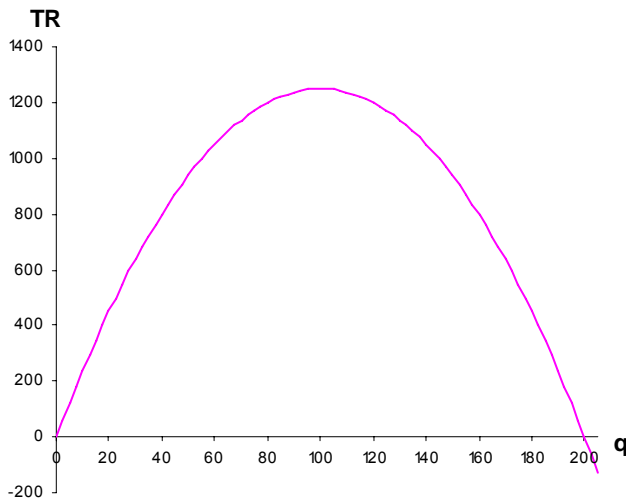
(a)  $TR \equiv pq = \left(-\frac{1}{8}q + 250\right)q = -\frac{1}{8}q^2 + 250q$ ; and  $MR \equiv \frac{dTR}{dq} = -\frac{1}{4}q + 250$ .

(b)  $MR = 0$  when  $q = 1000$ , and  $\frac{d^2TR}{dq^2} = \frac{dMR}{dq} = -\frac{1}{4} < 0$ .

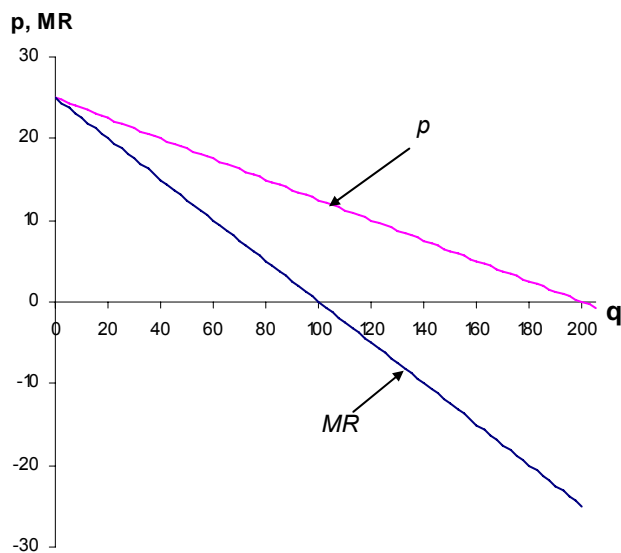
So  $q = 1000$  is a max of  $TR$ ..

(c)

**Ex 8.2 question 1**



**Ex 8.2 question 2**

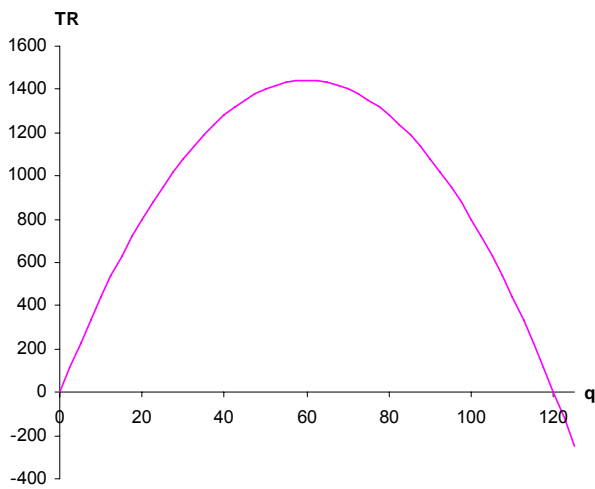


**Progress exercise 8.2 question 2**

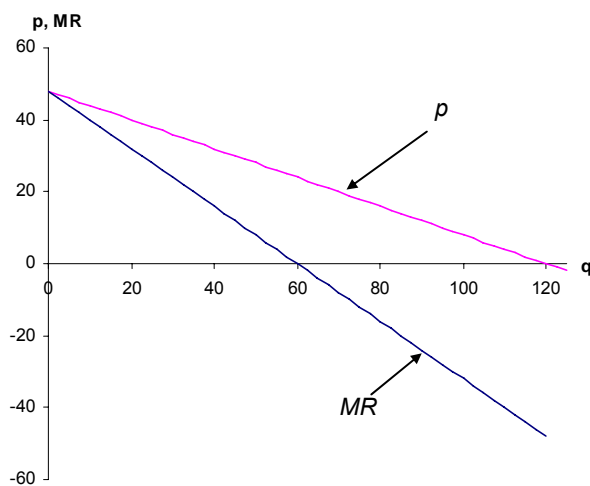
- (a) Inverse demand function is  $p = -0.4q + 48$ . So  $TR \equiv pq = -0.4q^2 + 48q$ , and  

$$MR \equiv \frac{dTR}{dq} = -0.8q + 48$$
- (b) Setting  $MR = 0$ , soln is  $q = 60$ ; and  $\frac{d^2TR}{dq^2} = \frac{dMR}{dq} = -0.8 < 0$ . So  $q = 60$  is a  
 max of  $TR$ . When  $q = 60$ ,  $p = -0.4q + 48 = 24$ .
- (c). (d)

Ex 8.2 question 2



Ex 8.2 question 2

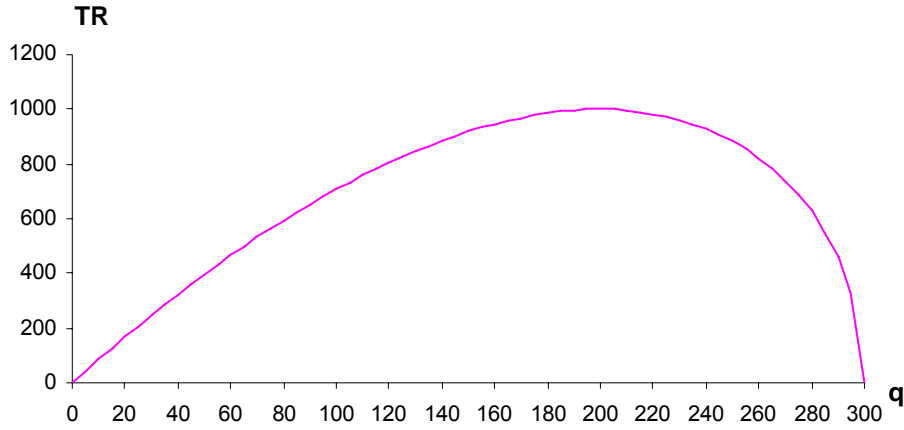


**Progress exercise 8.2 question 3**

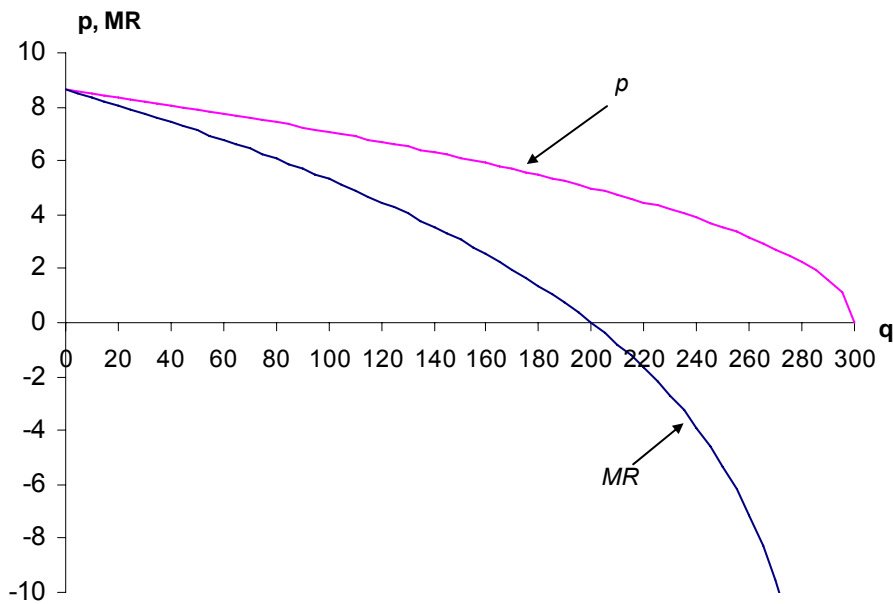
- (a)  $q = 300 - 4p^2$  (demand function). Because  $TR$  is, by convention, defined as a function of  $q$ , in order to find  $TR$  we first have to find the inverse demand function (as in questions 1 and 2 above). From  $q = 300 - 4p^2$  we can get inverse demand fn as:  $p = (75 - \frac{1}{4}q)^{\frac{1}{2}}$ . Therefore,  $TR \equiv pq = (75 - \frac{1}{4}q)^{\frac{1}{2}} \cdot q$  (a function of  $q$ , as required). The derivative of this function then gives us  $MR$  as:  $MR \equiv \frac{dTR}{dq} = (75 - \frac{1}{4}q)^{\frac{1}{2}} - \frac{1}{8}q(75 - \frac{1}{4}q)^{-\frac{1}{2}}$  (using both the function of a function and product rules).
- (b)  $TR$  maximum requires  $MR = (75 - \frac{1}{4}q)^{\frac{1}{2}} - \frac{1}{8}q(75 - \frac{1}{4}q)^{-\frac{1}{2}} = 0$  But this equation is not easy to solve so, following hint in question, we find  $TR$  in terms of  $p$  as:  $TR \equiv pq = p(300 - 4p^2) = 300p - 4p^3$  (using the demand function to eliminate  $q$ ).
- When  $p = 5$ ,  $\frac{dTR}{dp} = 300 - 12p^2 = 0$  and  $\frac{d^2TR}{dp^2} = -24p < 0$ . So  $p = 5$  gives max  $TR$ . From demand function with  $p = 5$ ,  $q = 300 - 4(5)^2 = 200$ .
- (c) See next page.

(c)

**Ex 8.2 question 3**



**Ex 8.2 question 3**



**Progress exercise 8.2 question 4**

(a) Given  $p = \frac{16}{q+2} - 2$ . The  $p$  intercept is where  $q=0 \Rightarrow p = \frac{16}{0+2} - 2 = 6$ . The  $q$

intercept is where  $p=0 \Rightarrow p = \frac{16}{q+2} - 2 = 0 \Rightarrow q = 6$

(b)  $TR \equiv pq = \frac{16q}{q+2} - 2q$ . So  $MR \equiv \frac{dTR}{dq} = \frac{(q+2)16 - 16q}{(q+2)^2} - 2$ .

(c) Max  $TR$  when  $MR = \frac{32 - 2(q+2)^2}{(q+2)^2} = 0$ . This is true when  $32 - 2(q+2)^2 = 0$ .

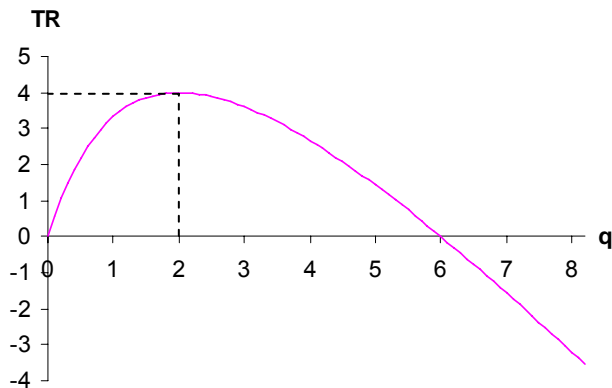
This rearranges as  $-2q^2 - 8q + 24 = 0$ . This quadratic eqn has roots  $q = 2$  or  $-6$

(but we discard the negative solution). When  $q = 2$ ,  $p = \frac{16}{2+2} - 2 = 2$ . Max

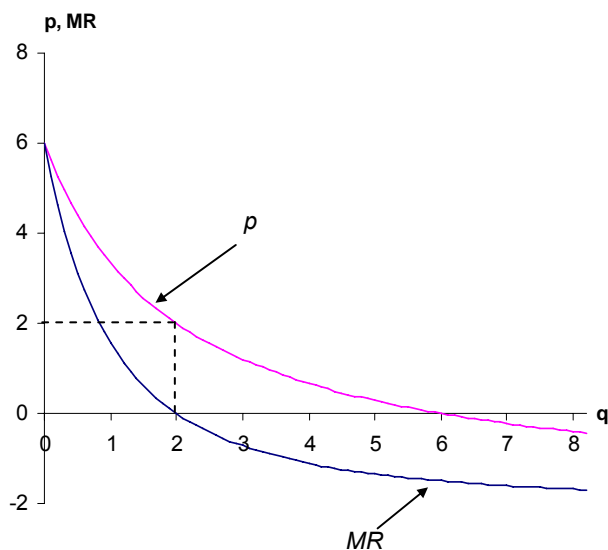
$TR \equiv pq = 4$

(d)

Ex 8.2 question 4



Ex 8.2 question 4

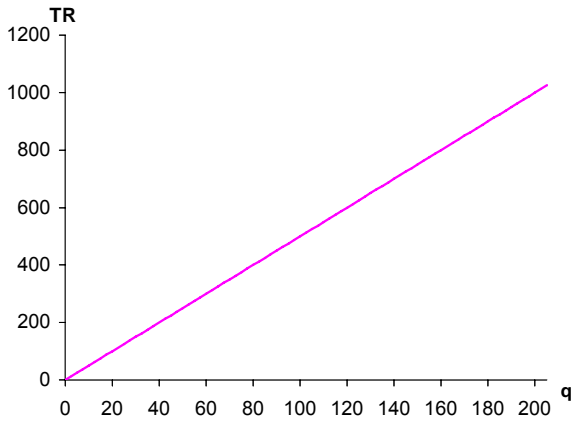


**Progress exercise 8.2 question 5**

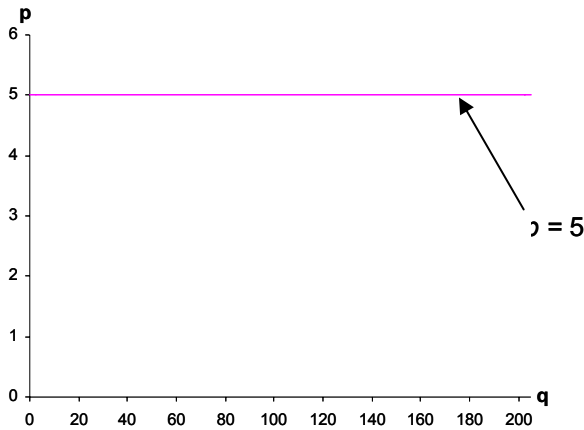
Inverse demand function is  $p = 5$ . Therefore  $TR \equiv pq = 5q$  and  $MR \equiv \frac{dTR}{dq} = 5$ .

(This is the inverse demand function for the product of a perfectly competitive firm.)

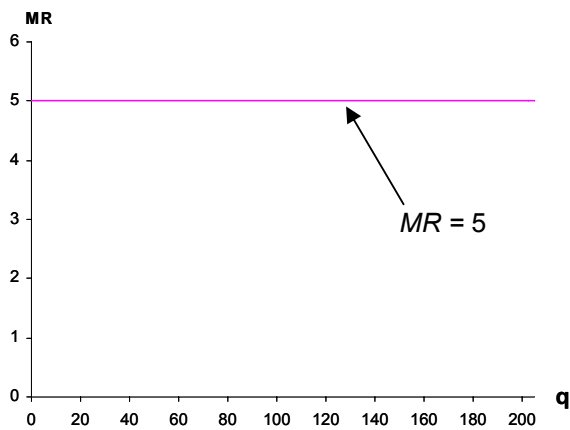
Ex 8.2 question 5



Ex 8.2 question 5



Ex 8.2 question 5



**Progress exercise 8.2 question 6**

(a) In the figure below, consider a movement down the demand function from A to B. Quantity sold rises by 1 unit, and price falls from  $p_0$  to  $p_0 + dp$ . (It is slightly confusing that we *add*  $dp$  when the price *falls*, but this is explained by the fact that  $dp$  itself is negative.)

The resulting change in total revenue can be decomposed into two elements: the change in total revenue due to selling one more unit; and the change in total revenue due to the price reduction.

The additional unit of quantity is sold for a price of  $p_0 + dp$ , which approximately equals  $p_0$  when  $dp$  is small. So the addition to total revenue is approximately  $p_0 \times 1$  which equals the area of the rectangle J.

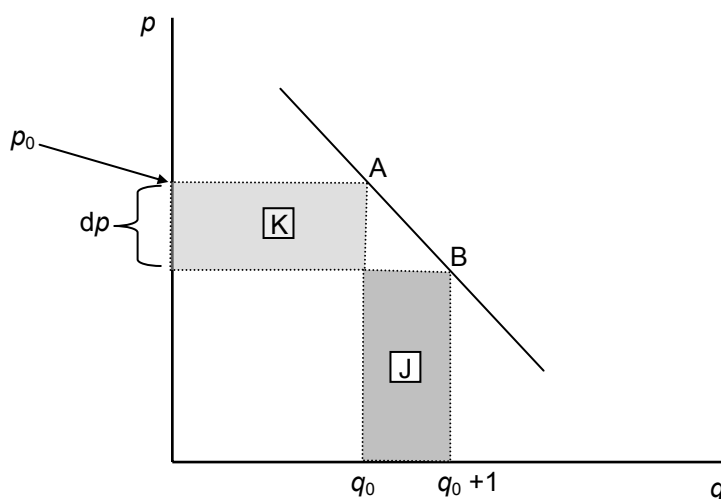
The price reduction of  $dp$  means that revenue from the initial quantity,  $q_0$ , falls by  $q_0 \times dp$ . This equals the area of the rectangle K.

Marginal revenue is therefore the sum of these two effects:

$$MR = (p_0 \times 1) + (q_0 \times dp) \text{ which simplifies to } p_0 + q_0 dp$$

This is necessarily less than  $p_0$  because  $dp$  is negative. Thus marginal revenue is necessarily less than the (initial) price,  $p_0$ . The only exception is when the demand function is horizontal so that  $dp = 0$ .

Graphically,  $MR$  is measured by area of rectangle J + area of rectangle K, with area K counting as negative because  $dp$  is negative. So  $MR$  is positive if area J is greater than area K



(b) See appendix 8.2 to chapter 8, especially rule 8.4.

**Progress exercise 8.3 question 1**

(a) Profit function is  $\Pi = 67q - (q^2 - 3q + 500) = -q^2 + 70q - 500$ . For maximum

$\Pi$  we require  $\frac{d\Pi}{dq} = 0$  and  $\frac{d^2\Pi}{dq^2} < 0$ . When  $q = 35$ ,  $\frac{d\Pi}{dq} = -2q + 70 = 0$  and

$\frac{d^2\Pi}{dq^2} = -6 < 0$  so  $q = 35$  is a maximum. Max  $\Pi = -(35)^2 + 70(35) - 500 = 725$

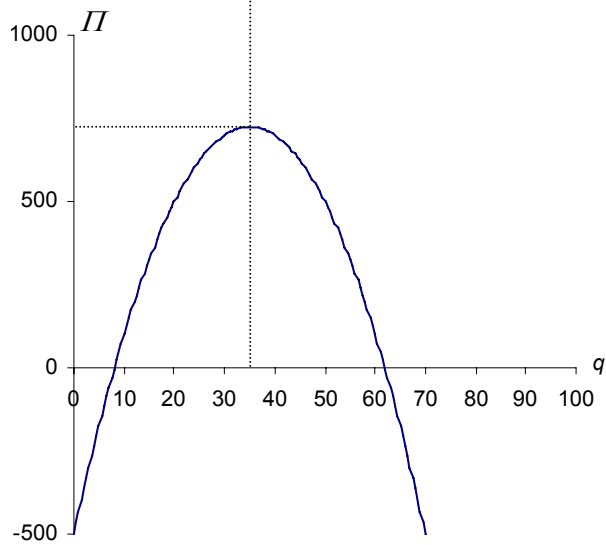
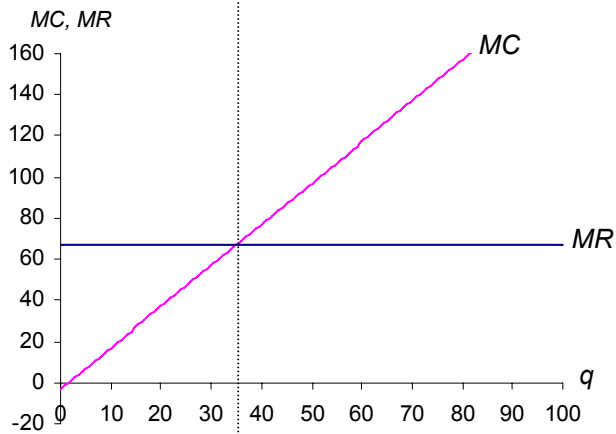
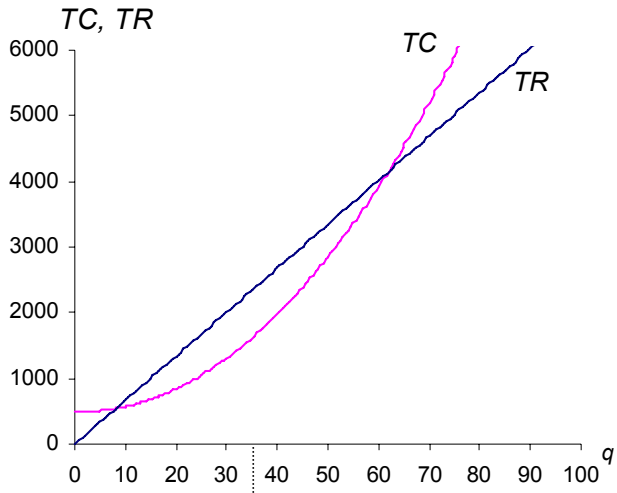
(b)  $AC = q - 3 + \frac{500}{q}$ . Min  $AC$  when  $\frac{dAC}{dq} = 1 - 500q^{-2} = 0 \Rightarrow q = 22.36$ . This is a

minimum because  $\frac{d^2AC}{dq^2} = \frac{1000}{q^3} > 0$  when  $q = 22.36$ .

The most profitable output ( $q = 35$ ) > output at which  $AC$  is minimised ( $q = 22.36$ ). In general there is no reason to suppose the two will coincide as profit depends on  $TR$  and  $TC$ .  $AC$  in itself has no relevance.

(c) & (d) See next page

Ex 8.3 question 1



**Progress exercise 8.3 question 2**

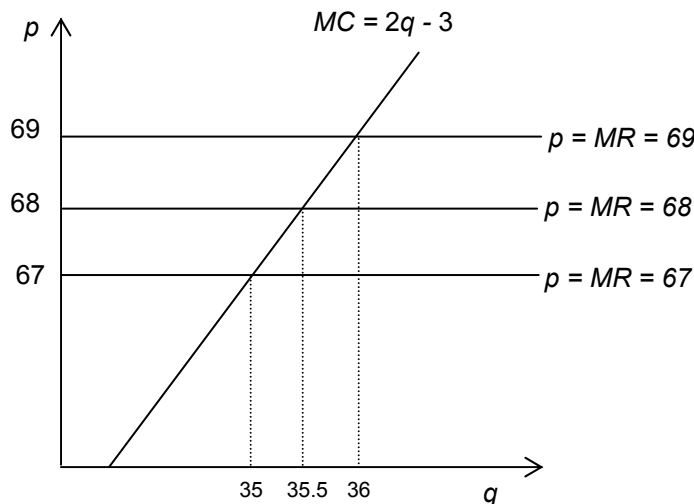
(a) In question 1,  $MC \equiv \frac{dTR}{dq} = 2q - 3$ . We also have  $TR = 67q$  so

$$MR \equiv \frac{dTR}{dq} = 67. \text{ We know } \Pi \text{ maximum requires } MC = MR \text{ (see section 8.16}$$

above). So  $\Pi$  max requires  $2q - 3 = 67 \Rightarrow q = 35$ . (We assume second order condition for max is satisfied.)

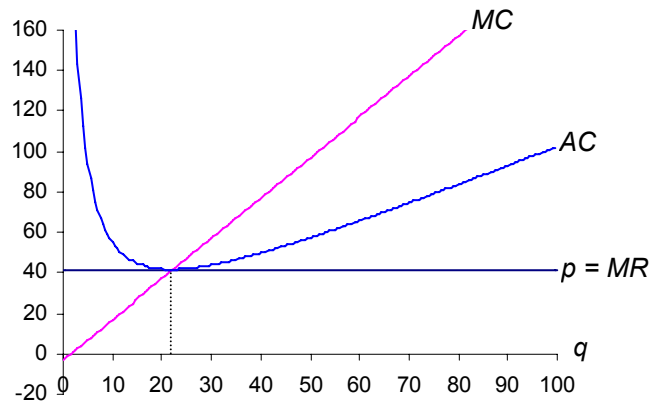
If  $p$  rises to 68, the  $MC = MR$  condition becomes  $2q - 3 = 68 \Rightarrow q = 35\frac{1}{2}$ . If  $p$  rises to 69 this becomes  $2q - 3 = 69 \Rightarrow q = 36$  and so on (see graph below).

(b) If price has some unspecified value  $p^*$ , the  $MC = MR$  condition becomes  $2q - 3 = p^*$ . This can be re-arranged as  $q = \frac{1}{2}p^* - \frac{3}{2}$ . This is the firm's supply curve, as it tells us what quantity the firm will choose to supply, at any given market price  $p^*$ , in order to maximise profit. (Note: this assumes perfect competition.)



As the ruling market price (= marginal revenue) increases from 67 to 68 and then to 69, so the firm moves up its marginal cost curve in order to maintain  $MR = MC$  as required for profit maximisation. Thus the most profitable output increases from 35 to 35.5 and then to 36. In effect, its marginal cost curve is its supply curve since it tells us how much the firm will produce and sell at any given market price.

- (c)  $AC$  minimised when  $q = 22.36$  (from 1(b) above). Firm produces output  $q = \frac{1}{2}p^* - \frac{3}{2}$  (from 2(b) above). So firm will produce output  $q = 22.36$  only if  $22.36 = \frac{1}{2}p^* - \frac{3}{2}$ , from which  $p^* = 41.72$ . When  $q = 22.36$ ,  $p^* = AC = 41.72$ .  
 For graph, see next page.



The most profitable output is always where  $MC = MR$ . If the ruling market price is 41.72, then  $MC = MR$  when  $q = 22.36$ . It happens that this is also the output at which  $AC$  is at its minimum, but this does not influence the firm's choice of output.

However, since at this point we have  $p = AC$ , it follows that  $pq = AC \times q$ . But the left hand side of this equation is total revenue, and the right hand side is total cost; so profits are zero.

### Progress exercise 8.3 question 3

- (a) From 1(a) above,  $\Pi = -q^2 + 70q - 500$  where 500 = fixed costs. To find  $\Pi$  max, we solve the equation  $\frac{d\Pi}{dq} = -2q + 70 = 0$ . An increase in fixed costs from 500 to 1000 (or any other level) has no effect on  $\frac{d\Pi}{dq}$  and therefore does not change the profit maximising value of  $q$ . However, fixed costs do appear in our expression for  $\Pi$  above, so an increase in fixed costs from 500 to 1000 will reduce profits by 500.
- (b) If fixed costs rise to 1250,  $q = 35$  remains the profit maximising output, but maximum profit is now -25 (= minimum loss).

**Progress exercise 8.3 question 4**

(a)  $MC = \frac{dTC}{dq} = 2q - 3$ . Also,  $MR = -2q + 105$ . Setting  $MC = MR$  for max  $\Pi$

gives  $q = 27$  (and  $p = -q + 105 = 78$ )

The second order condition for a maximum of  $\Pi$ ,  $\frac{dMC}{dq} > \frac{dMR}{dq}$ , is satisfied

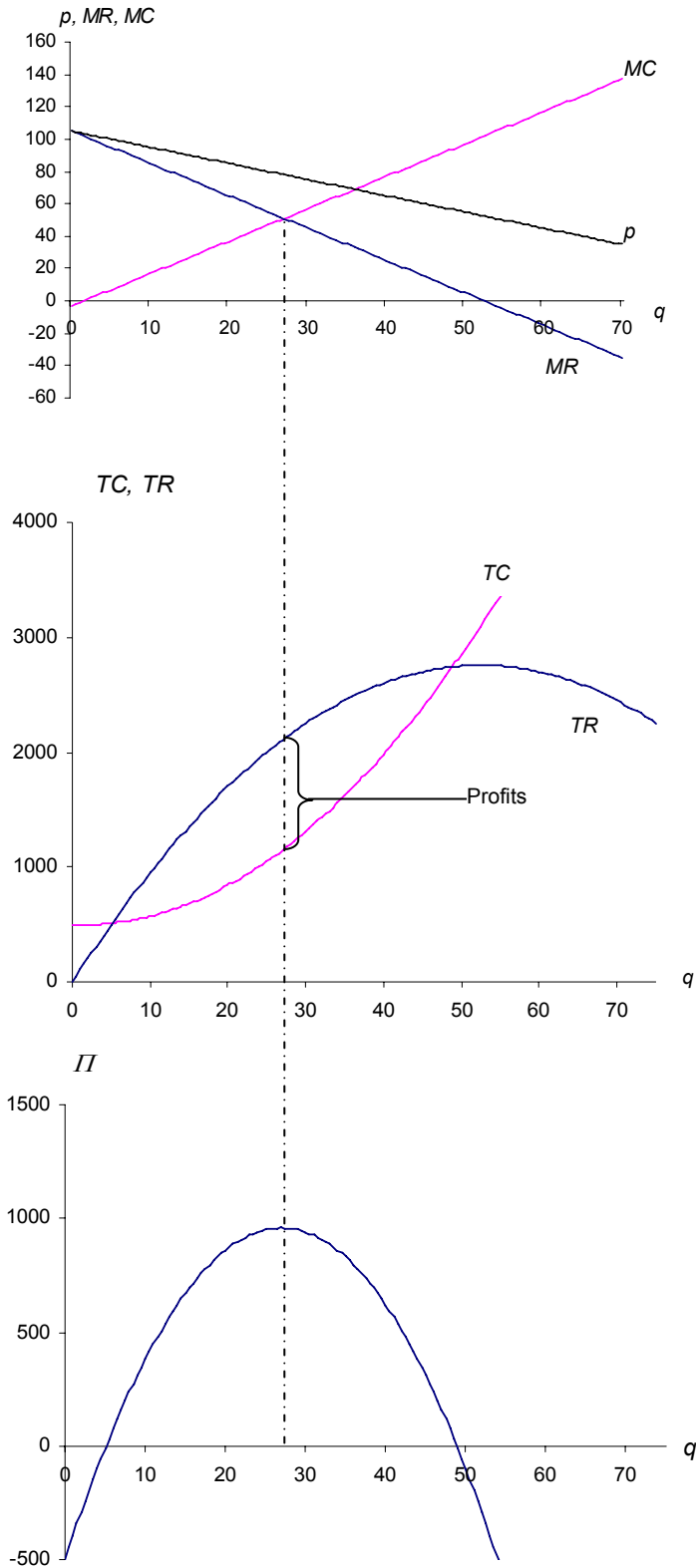
since  $\frac{dMC}{dq} = 2$  and  $\frac{dMR}{dq} = -2$  (see equation 8.8 in section 8.20 above)

When  $q = 27$ ,  $\Pi = TR - TC = -q^2 + 105q - (q^2 - 3q + 500) = 958$

(b) From 1(b), minimum  $AC$  is at  $q = 22.36$  which is less than  $q = 27$  which maximises profit.

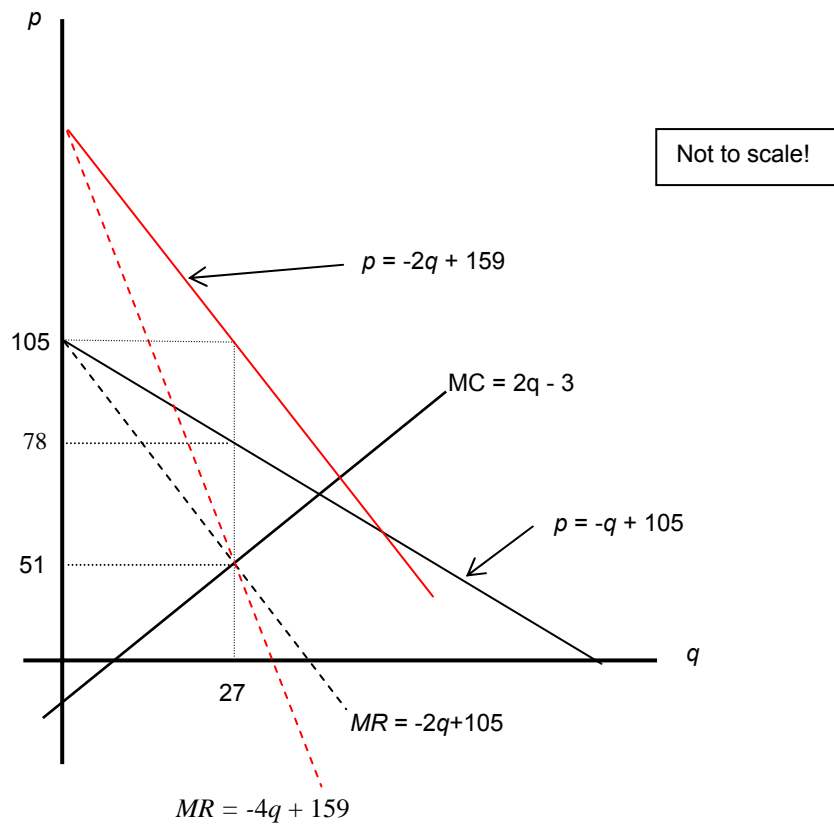
(c) & (d) See next page.

Ex 8.3 question 4(c)



**Progress exercise 8.3 question 5**

- (a) New  $TR$  is  $TR = -2q^2 + 159q$ . So new  $MR$  is  $MR = -4q + 159$ .  $TC$  is unchanged. Setting  $MR = MC$  gives  $-4q + 159 = 2q - 3$ , with solution  $q = 27$  (no change). From new inverse demand function,  $p = -2(27) + 159 = 105$  (compared with old price, 78). New  $\Pi$  is  $\Pi = -2q^2 + 159q - (q^2 - 3q + 500)$   
 $= -3q^2 + 162q - 500 = 1687$  when  $q = 27$  (compared with old profits, 958).
- (b) (See graphs, next page). New  $MR$  curve cuts unchanged  $MC$  curve at same point as old  $MR$  curve, hence output unchanged. But new inverse demand function and new  $MR$  function have higher intercept and steeper slope. Consequently, for any given  $MR$ ,  $p$  is now higher. In turn, this means that profits are higher since  $TR$  is higher but  $TC$  unchanged.
- (c) See next page



The "old" marginal revenue curve and the "new" marginal revenue curve (after the advertising campaign) both cut the marginal cost curve at  $q = 27$ , so this is the most profitable output both before and after the advertising campaign. However the most profitable price rises from 78 to 105, with a corresponding increase in profits due to increased total revenue.

Note: the "old" demand function has an intercept of 105 on the  $p$  axis, meaning that when  $p = 105$ , quantity demanded is zero. When the "new" demand function comes into operation, the profit maximising price equals 105, because this is the price at which the marginal cost curve cuts the "new" marginal revenue curve. That both of these numbers are 105 is entirely an accident of this particular example, and has no analytical significance.

**Progress exercise 8.3 question 6**

(a)  $TC = 20q + 500$ . So  $MC = 20$ . Given  $p = -3q + 146$ , we have

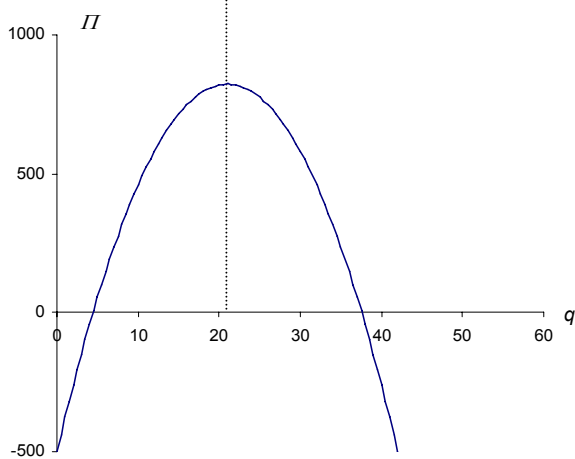
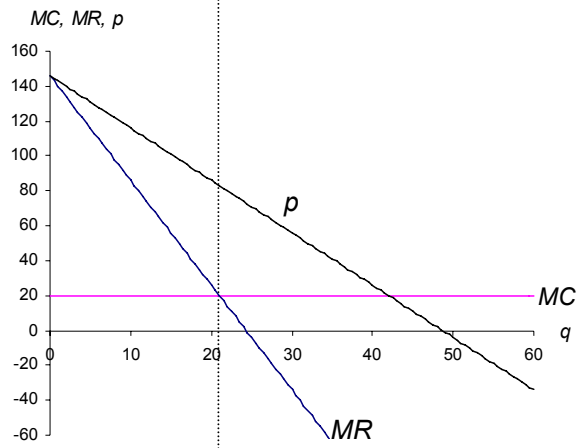
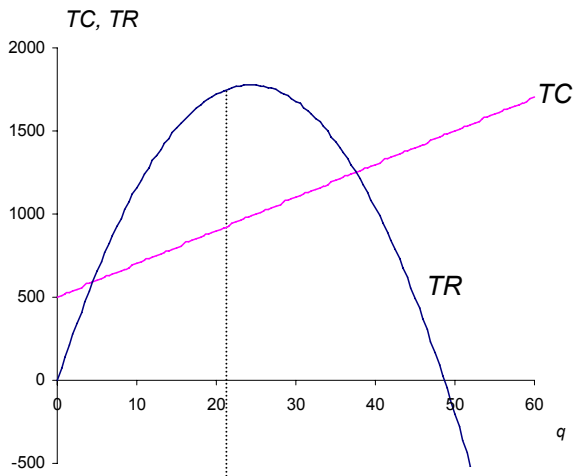
$TR \equiv pq = (-3q + 146)q = -3q^2 + 146q$ . So  $MR = -6q + 146$ . For maximum profit we set  $MC = MR \Rightarrow 20 = -6q + 146 \Rightarrow q = 21$ . We then have

$$II = TR - TC = -3q^2 + 146q - (20q + 500) = 823 \text{ when } q = 21$$

(b) No, because  $AC = 20 + \frac{500}{q}$  which falls continuously as  $q$  increases. (That is, the  $AC$  curve has no minimum.)

(c), (d) & (e) See next page.

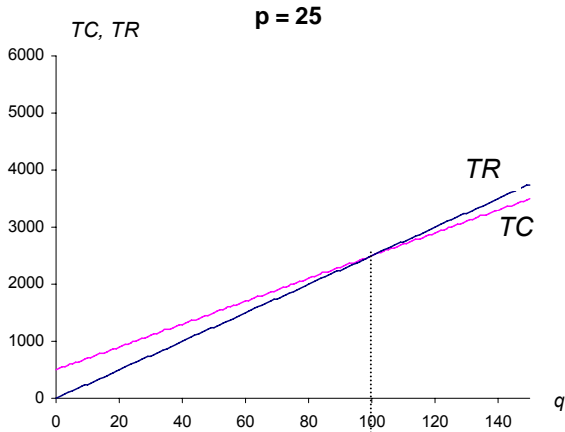
Figures for Ex 8.3 question 6



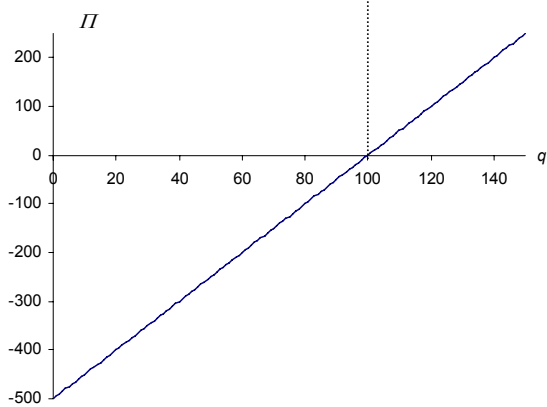
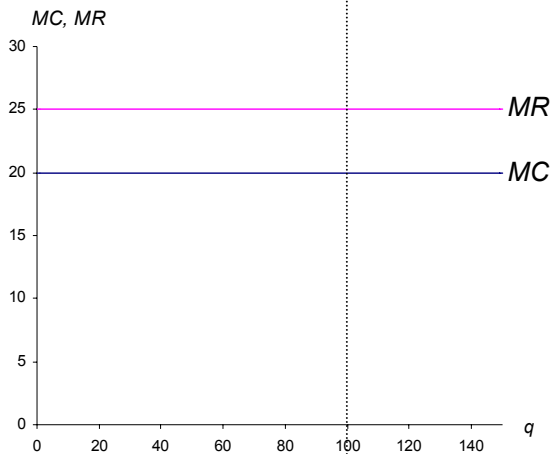
**Progress exercise 8.3 question 7**

- (a) As in 6 above,  $MC = 20$ . But now  $p = 25$ , so  $TR = 25q$ , from which  $MR = 25$ . Thus  $MR > MC$  at all output levels. Therefore the firm can always increase its profits by increasing its output, hence would wish to increase output without limit. There is no equilibrium.
- (b) Break-even output is where  $TR = TC$  and profits are therefore zero. We have  $TR = 25q$  and  $TC = 20q + 500$ . Setting  $TR = TC$  and solving the resulting equation gives  $q = 100$ . (Note, dividing both sides of  $TR = TC$  by  $q$  gives  $AR = AC$ , so break-even can also be found by setting  $AR = AC$ .)
- (c) If  $p = 15$ , then  $MR = p = 15$ . And  $MC = 20$  (unchanged). Again there is no equilibrium, but this time because  $MC > MR$  at all output levels. Therefore the firm can always increase its profits (= reduce its losses) by reducing output, hence would reduce output to zero. (This is sometimes known as a “corner solution”; see chapter W21.)
- (d) No, because  $MC$  is the only cost measure that is relevant in choosing the most profitable output, and  $MC$  is independent of fixed costs. (See also answer to 3, above). A change in fixed costs changes profits but does not change the profit maximising output.
- (e) See algebra above. For figures see next page.

Figures for question 7(a);  $p = 25$

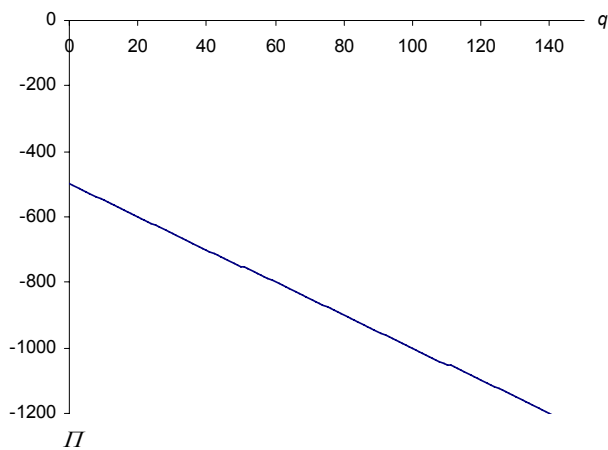
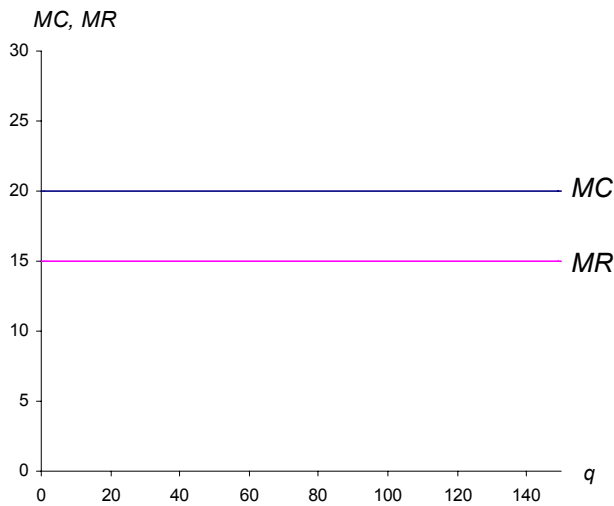
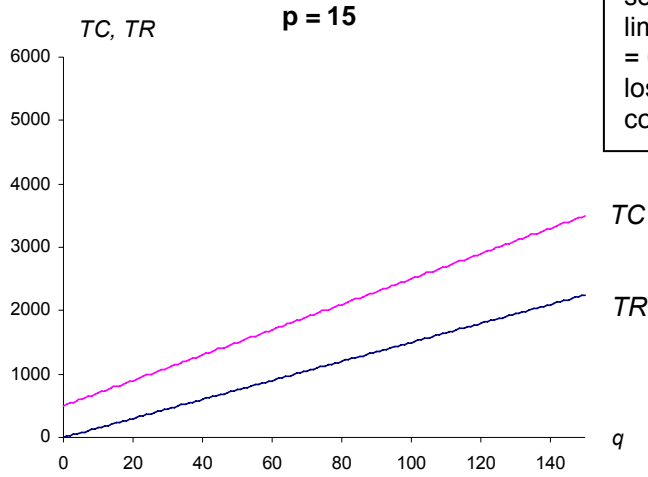


When  $p = 25$ , any increase in output adds more to revenue than to costs, so the firm wishes to expand without limit. Break-even is at  $q = 100$ .



**Figures for exercise 8.3 question 7(c);  $p = 15$**

When  $p = 15$ , any increase in output adds more to costs than to revenue, so the firm wishes to contract without limit. Its losses are minimised when  $q = 0$ ; that is, it produces nothing. Its losses are then given by its fixed costs.



**Progress exercise 8.3 question 8**

(a) We have  $MC = 6q^2 - 96q + 700$ .

Given  $p = 532$  we have  $TR = 532q$ , so  $MR = 532$ .

(a) (i) Setting  $MR = MC \Rightarrow 532 = 6q^2 - 96q + 700 \Rightarrow 6q^2 - 96q + 168 = 0$ . Roots of this quadratic equation are  $q = 2$  and  $q = 14$  (that is, two SPs). To determine which of these is a maximum of  $\Pi$  we must examine the second order condition.

Second order condition for a maximum is  $\frac{dMC}{dq} > \frac{dMR}{dq}$  and for a minimum

$\frac{dMC}{dq} < \frac{dMR}{dq}$  (see equation 8.8 in section 8.20 in the book). Here,

$$\frac{dMC}{dq} = 12q - 96 \text{ and } \frac{dMR}{dq} = 0.$$

When  $q = 2$ ,  $\frac{dMC}{dq} = 24 - 96 < 0$  so  $\frac{dMC}{dq} < \frac{dMR}{dq}$ . Therefore  $q = 2$  a min of  $\Pi$ .

When  $q = 14$ ,  $\frac{dMC}{dq} = 12(14) - 96 > 0$  so  $\frac{dMC}{dq} > \frac{dMR}{dq}$ . Therefore  $q = 14$  a max of  $\Pi$ .

(a) (ii)  $\Pi = TR - TC = -2q^3 + 48q^2 + 168q - 100$ . Therefore

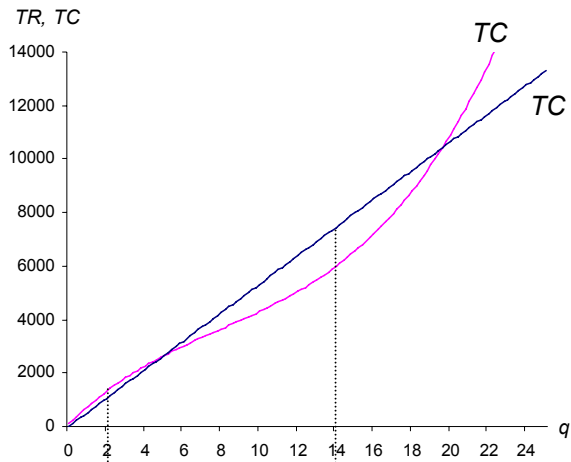
$$\frac{d\Pi}{dq} = -6q^2 + 96q + 168 = 0 \text{ for SP. Roots of this quadratic equation are } q = 2$$

and  $q = 14$ . For max profits we need  $\frac{d^2\Pi}{dq^2} < 0$ .

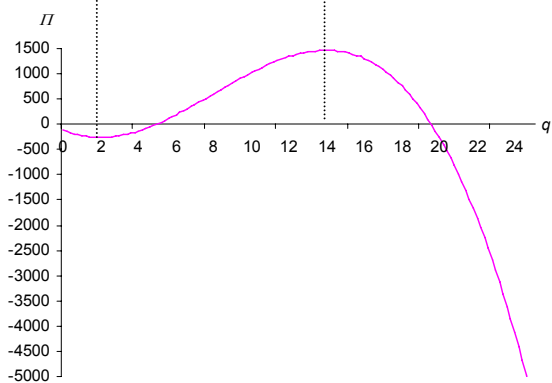
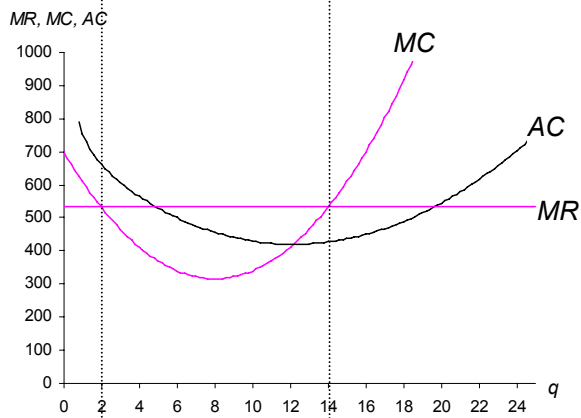
Here  $\frac{d^2\Pi}{dq^2} = -12q + 96$ . This is positive when  $q = 2$ , so this is a min of  $\Pi$ ; and

negative when  $q = 14$  so this is a maximum of  $\Pi$ . At  $q = 2$ ,  $\Pi = -260$  (loss). At  $q = 14$ ,  $\Pi = 1468$

(b) See next page.



Notes: (a) At  $q = 14$ , the second order condition for a maximum of profits is satisfied, because the slope of the  $MC$  curve is steeper than the slope of the  $MR$  curve. This condition is not satisfied at  $q = 2$ .  
 (b)  $q = 2$  is a local minimum of profits. But it is not a global minimum, since from the graph we can see that profits are lower when  $q$  is greater than approx. 20.



- (c) When  $q = 2$ ,  $MC$  is falling while  $MR$  is constant. Therefore the second order condition for a minimum of  $\Pi$ ,  $\frac{dMC}{dq} < \frac{dMR}{dq}$  is satisfied, and any change in output (increase or decrease) will increase profits. When  $q = 14$ ,  $MC$  is now rising so  $\frac{dMC}{dq} > \frac{dMR}{dq}$ . Thus the second order condition for a maximum of  $\Pi$  is satisfied, and any change in output reduces profits.
- (d) A firm would only choose to produce at an output at which  $MC$  was decreasing if  $MR$  at that output was decreasing even faster, for only then would the second order condition for maximum profit be satisfied. Graphically, both the  $MC$  and  $MR$  curves would be negatively sloped, with  $MR$  steeper.

**Progress exercise 8.3 question 9**

- (a) (i) We have  $MC = 12q^2 - 370q + 4500$ . Given  $q = -\frac{1}{5}p + 600$  (demand function)  $\Rightarrow p = -5q + 3000$  (inverse demand function). Therefore  $TR = -5q^2 + 3000q$  and  $MR = -10q + 3000$ . Setting  $MR = MC \Rightarrow 12q^2 - 360q + 1500 = 0$
- Solving this quadratic equation gives  $q = 5$  or  $25$
- Second order condition. Here  $\frac{dMC}{dq} = 24q - 370$ , and  $\frac{dMR}{dq} = -10$ . When  $q = 5$ ,  $\frac{dMC}{dq} < \frac{dMR}{dq}$  so minimum of  $\Pi$ . When  $q = 25$ ,  $\frac{dMC}{dq} > \frac{dMR}{dq}$  so maximum of  $\Pi$ .
- Profit levels: we have  $\Pi = TR - TC = -4q^3 + 180q^2 - 1500q + 500$ . So when  $q = 5$ ,  $\Pi = -4000$ ; when  $q = 25$ ,  $\Pi = 12000$
- (a) (ii) From (i) above,  $\Pi = -4q^3 + 180q^2 - 1500q - 500$ . So for max profit we want  $\frac{d\Pi}{dq} = -12q^2 + 360q - 1500 = 0 \Rightarrow -12(q^2 + 30q - 125) = 0$

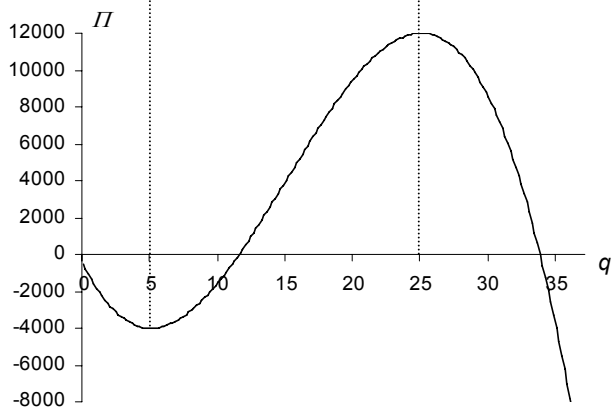
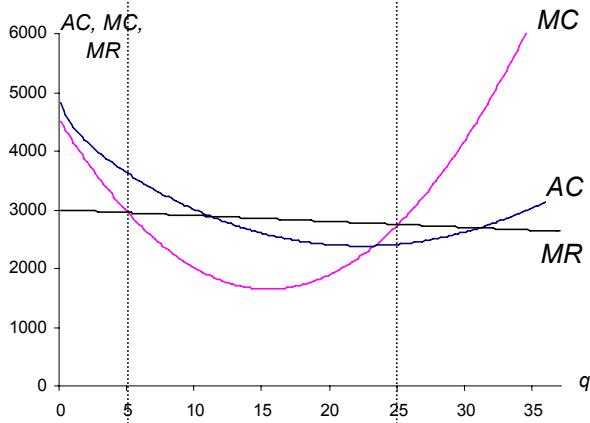
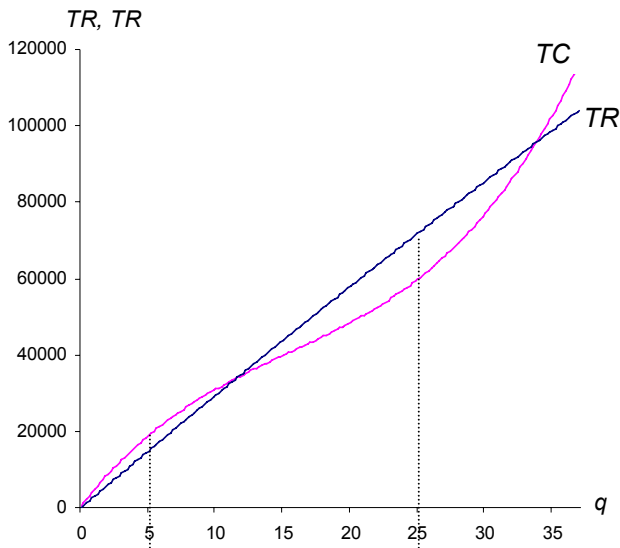
From (i) above we know that the solution to this quadratic equation is  $q = 5$  or 25.

Second order condition:  $\frac{d^2\Pi}{dq^2} = -12(2q + 30) = 24(15 - q)$ . This is positive at  $q = 5$  so minimum; and negative at  $q = 25$  so maximum. Profits are obviously the same as in (i).

(b) See next page

(c) As in 8(c) above.

Notes: see question 8.



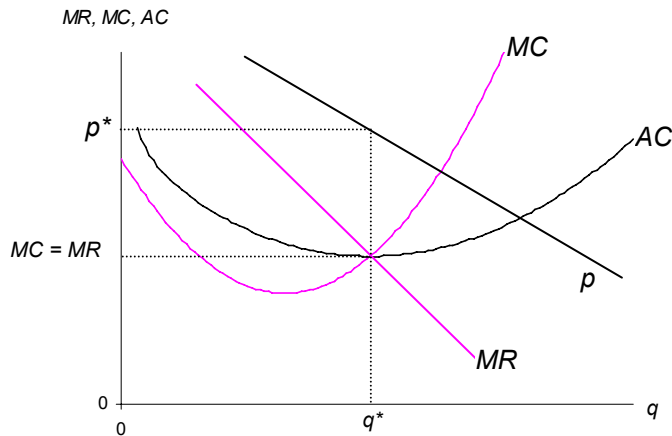
**Progress exercise 8.3 question 10**

We know that when  $AC$  is at its minimum,  $MC = AC$ . We also know that when profit is at its maximum,  $MC = MR$ . For both conditions to be satisfied simultaneously therefore requires  $MR = MC = AC$ . (In both the monopoly case and the perfect comp. case, we will assume that the necessary second order conditions for a minimum of  $AC$  and a maximum of  $\Pi$  are satisfied.)

- (a) Monopoly. For a monopolist, the  $MR$  curve is downward sloping, so  $MR = MC = AC$  implies that the  $MR$  curve cuts the  $AC$  curve at the latter's minimum.
- (b) Perfect competition. For a perfectly competitive firm,  $MR = p =$  a constant (independent of output) so the  $MR$  curve is a horizontal straight line. Then if  $MR = MC = AC$  this implies that the horizontal  $MR$  curve is tangent to the minimum point on the  $AC$  curve.

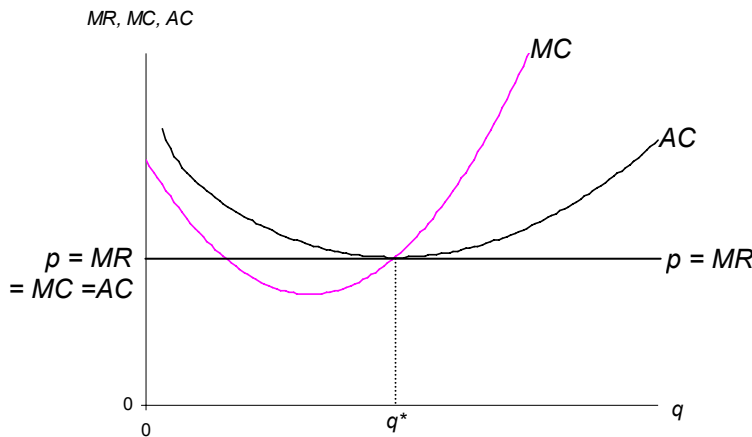
For graphical treatment, see next page.

**(a) A firm with monopoly power**



The most profitable output is where  $MC = MR$ ; that is,  $q = q^*$  with  $p = p^*$ . If the  $AC$  curve also passes through the point of intersection of the  $MC$  and  $MR$  curves, then we also have  $MC = AC$  and average cost will therefore be minimised at this output. Further economic analysis is required to determine the likelihood of this. Note that when the firm has monopoly power,  $MR < p$ .

**(b) A perfectly competitive firm**



The analysis of this case is the same as that of (a) above. The most profitable output is where  $MC = MR$ . If, at this point, we also have  $MC = AC$ , then average cost will be minimised at this output. Further economic analysis is required to determine the likelihood of this. Under perfect competition we also have  $p = MR$ . Note that in this case profits are zero because price equals average cost.