Progress exercise 6.1

1.

(a) $\frac{\Delta y}{\Delta x} = 2$

(b) $\frac{\Delta y}{\Delta x} = \frac{1}{2}$

(c) $\frac{\Delta y}{\Delta x} = 50$

(d) $\frac{\Delta y}{\Delta x} = -3$

(e) $\frac{\Delta y}{\Delta x} = 0$

(f) $\frac{\Delta y}{\Delta x} = -\frac{1}{2}$

2. (a) For unspecified values x_1 and x_0, question 1(a) above becomes:

$$\frac{\Delta y}{\Delta x} = \frac{(2x_1 + 4) - 2(x_0 + 4)}{x_1 - x_0} = \frac{2(x_1 - x_0)}{x_1 - x_0} = 2$$

Similarly, 1(b) becomes:

$$\frac{\Delta y}{\Delta x} = \frac{\left(\frac{1}{2}x_1 - 2\right) - \left(\frac{1}{2}x_0 - 2\right)}{x_1 - x_0} = \frac{\frac{1}{2}(x_1 - x_0)}{x_1 - x_0} = \frac{1}{2}$$

And 1(c) becomes:

$$\frac{\Delta y}{\Delta z} = \frac{50x_1 - 50x_0}{x_1 - x_0} = 50$$

(b) If $y = ax+b$, then with unspecified values x_1 and x_0 we get:

$$\frac{\Delta y}{\Delta x} = \frac{(ax_1 + b) - (ax_0 + b)}{x_1 - x_0} = \frac{a(x_1 - x_0)}{x_1 - x_0} = a$$

Progress exercise 6.2

$\frac{dy}{dx} =$

(a) $10x$
(b) $4x^3$
(c) $36x^2$
(d) $0.5x^{-0.5}$
(e) $3x^2 + 2x$
(f) $12x^3 + 20x$
(g) $12x^2 - 16$
(h) $30x^2 + 10x - 9 - 5x^{-2}$
(i) $\frac{3}{5}x^2 + x + 5$
(j) $\frac{1}{2}x^{-\frac{1}{2}} + 2x^{-3}$
(k) $\frac{1}{2}x^{-\frac{1}{2}} + 2x^{-3}$
(l) $4x^{-3} - 0.5x^{-0.5} + 1$
(m) $-x^{-2} + 0.25x^{-1.25}$
(n) $1 - 2x^{-3}$ $\left(\text{since } \frac{1}{x^2} = x^{-2}\right)$
(o) $-10x^{-4} + 0.09x^{-1.3} + \frac{1}{4}x^{-\frac{1}{2}}$
(p) a
(q) $2ax + b$
(r) $\frac{dq}{dp} = 6p^2 + 10p$
(s) $\frac{dq}{dp} = -\alpha A p^{-\alpha - 1}$
(t) $\frac{dz}{dt} = 1.5t^2 + 4$

Exercise 6.3

(a) $\frac{dy}{dx} = 4(1 + 2x)$
(b) $\frac{dy}{dx} - x\left(1 - x^2\right)^{-\frac{1}{2}}$
(c) $\frac{dy}{dx} = 5\left(x^2 - 2x^3\right)^4\left(2x - 6x^2\right)$
(d) $\frac{dy}{dx} = 0.5\left(x^2 - x\right)^{-0.5}(2x - 1)$

(e) $\dfrac{dy}{dx} = 12x - 1$

(f) $\dfrac{dy}{dx} = \left(4x^3 - 3\right)\left(10x^4 + 1\right) + \left(2x^5 + x\right)\left(8x\right)$

(g) $\dfrac{dy}{dx} = \dfrac{4x}{\left(1 - x^2\right)^2}$

(h) $\dfrac{dy}{dx} = \dfrac{x\left(x^2 + 3\right)}{\left(1 - x^2\right)^2\left(x^2 + 1\right)^{\frac{1}{2}}}$

(i) Using fn of a fn rule, $\dfrac{dx}{dy} = 0.5\left(y + 1\right)^{-0.5}$

By inverse function rule, $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}} = \dfrac{1}{0.5}\left(y + 1\right)^{0.5}$

$= 2(y + 1)^{0.5} = 2x$ (since $x = (y + 1)^{0.5}$).

(j) $\dfrac{dy}{dx} = \dfrac{1}{\left(1 - x\right)^2}$.