Progress exercise 4.1

1. For answer, see figure 4.4 in book.

2.

(a) $x^2 + 5x + 6$

(b) $x^2 + x - 20$

(c) $2x^2 + \frac{3}{2}x + \frac{1}{4}$

(d) $x^2 - 8x + 16$

(e) $x^2 - 25$ (note that the term involving x disappears in this case)

Progress exercise 4.2

1.

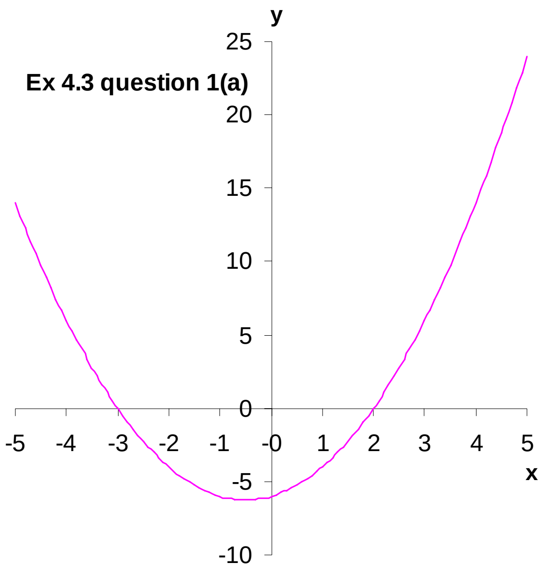
(a) soln x = 1 or 2
(b) soln x = -4 or -1
(c) soln x = -5 or 1
(d) soln x = -0.5 or -0.25
(e) soln x = 4 or –4
(f) soln x = –3 or –1
(g) soln x = 1.5 or –2
(h) soln x = –4 or –4 (a perfect square)
(i) soln x = –1 or 2
(j) no real roots because $b^2 = 9$ and $4ac = 12$, so $b^2 < 4ac$

What makes the answers to (h) and (j) different from the others? Answer: (h) is a perfect square ($b^2 = 4ac$) and (j) has no real roots ($b^2 < 4ac$). All the others have $b^2 > 4ac$, hence two distinct roots (solutions).

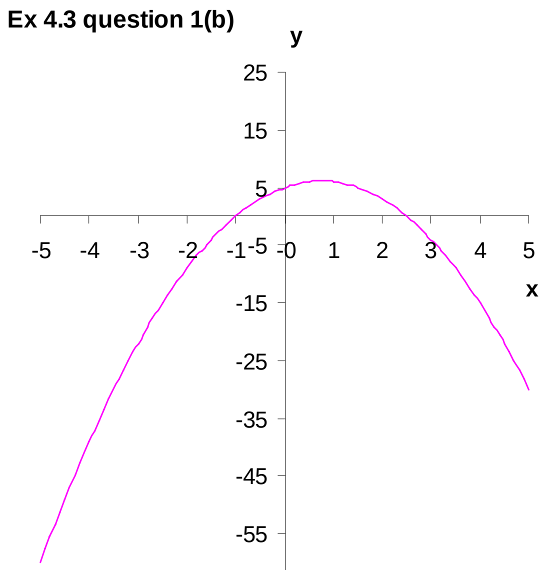
Progress exercise 4.3

1. The reason for factorizing the right hand side of each equation is that you then know the values of x at which the graph cuts the x axis. This helps you draw the sketch without having to compile a lengthy table of values. Your sketches will be much cruder than the graphs below, which were created in Excel®.

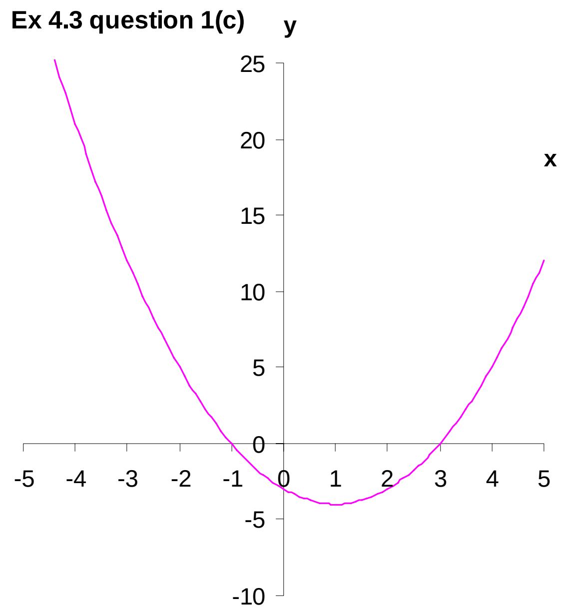
(a) $y = x^2 + x - 6$



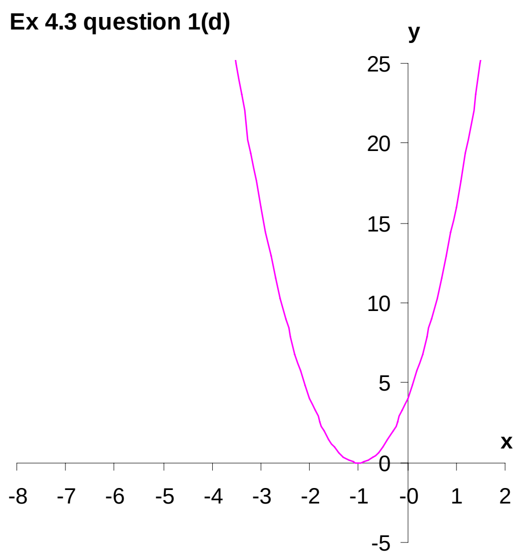
(b) $y = -2x^2 + 3x + 5$



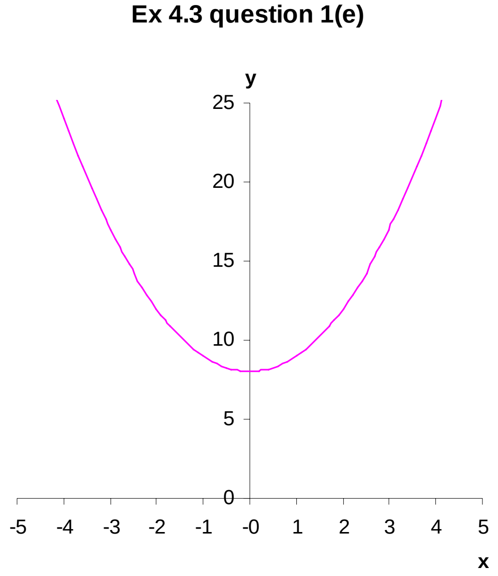
(c) $y = x^2 - 2x - 3$



(d) $y = 4x^2 + 8x + 4$ between $x = -8$ and $x = 2$



(e) $y = x^2 + 8$



2. Exact solutions are:

(a) $x^2 + x - 6 = 0$ soln 2, –3

(b) $-2x^2 + 3x + 5 = 0$ soln 2.5, –1

(c) $x^2 - 2x - 3 = 0$ soln –1, 3

(d) $4x^2 + 8x + 4 = 0$ soln –1, –1

(e) $x^2 + 8 = 0$. This equation has no (real) solutions because $b^2 = 0$ and $4ac = 32$, so $b^2 < 4ac$. The graph in 1(e) never cuts *x*-axis.

What makes (d) and (e) different from the others? In (d), $b^2 = 64$ and $4ac = 64$, so $b^2 = 4ac$. This is a perfect square. Case (e) is explained under (e) above.

3. You are given the following information about a quadratic function $y = ax^2 + bx + c$. In each case, find *a, b* and *c* and sketch the graph.

(a) Answer: We know from example 4.5 that if $-A$ and $-B$ are the roots of the quadratic equation $ax^2 + bx + c = 0$, then

$$a(x+A)(x+B) \equiv ax^2 + bx + c$$

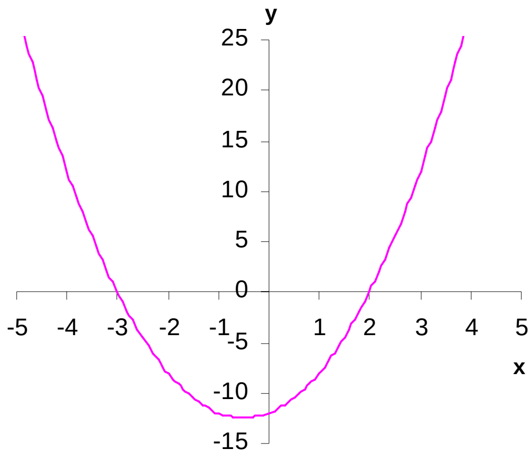
We are told that $y = ax^2 + bx + c = 0$ when $x = 2$ or -3, so therefore $a(x+A)(x+B) = 0$ when $x = 2$ or -3, which is true if $A = -2$, $B = 3$. So we have

$$a(x-2)(x+3) = a(x^2 + x - 6) = ax^2 + ax - 6a \equiv ax^2 + bx + c$$

So the quadratic function we are seeking is $y = ax^2 + ax - 6a$. Here $y = -6a$ when $x = 0$. But we are also told that $y = -12$ when $x = 0$, which implies that $y = -6a = -12$ and therefore $a = 2$.

So the quadratic function we are seeking is $y = ax^2 + ax - 6a = 2x^2 + 2x - 12$.

Ex 4.3 question 3(a)

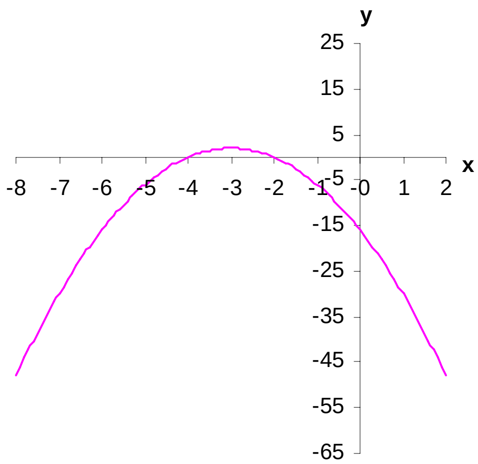


(b) Answer: Using the same method as in (a), we have

$y = a(x + A)(x + B) \equiv ax^2 + bx + c$ with *a* = –2, –*A* = –4, –*B* = –2. Therefore

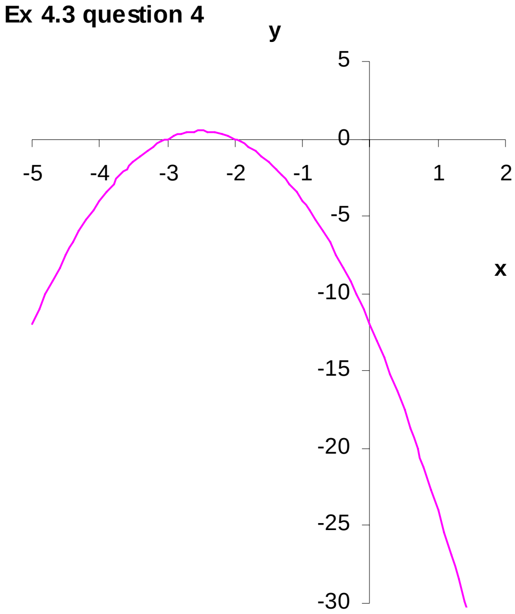
$y = -2(x + 4)(x + 2) = -2x^2 - 12x - 16$

Ex 4.3 question 3(b)



4. A quadratic function $y = ax^2 + bx + c$ has the following properties: $a = -2$, $y = 0$ when $x = -3$ or -2. Find b and c and sketch the graph.

Answer: Using the same method as in question 3, solution is: $y = -2x^2 - 10x - 12$



5. A quadratic function $y = ax^2 + bx + c$ has the following properties: $a = -3$; $y = 0$ when $x = -2$ or x_0; and $y = 6$ when $x = 0$. Find b, c and x_0 and sketch the graph.

Answer: Using the same method as in 1(a), we have

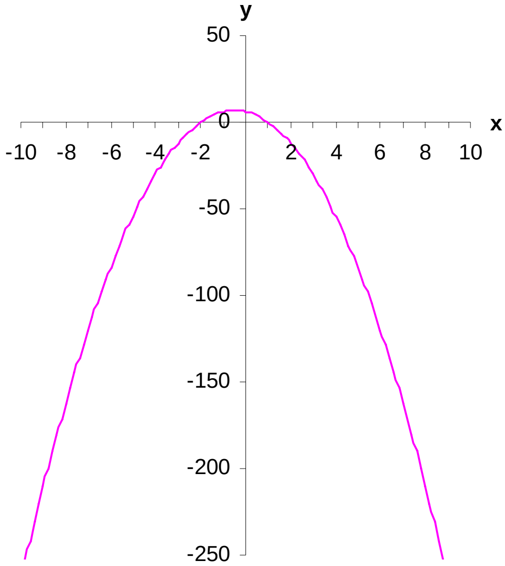
$y = a(x + A)(x + B) \equiv ax^2 + bx + c$ with $a = -3$, $-A = -2$, $-B = x_0$. Therefore

$y = -3(x + 2)(x - x_0) = -3x^2 - 3(2 + x_0)x + 6x_0$

Given $y = 6$ when $x = 0$, we must have $6x_0 = 6$ and therefore $x_0 = 1$. So

$y = -3x^2 - 3(2 - 1)x + 6(1) = -3x^2 - 3x + 6$

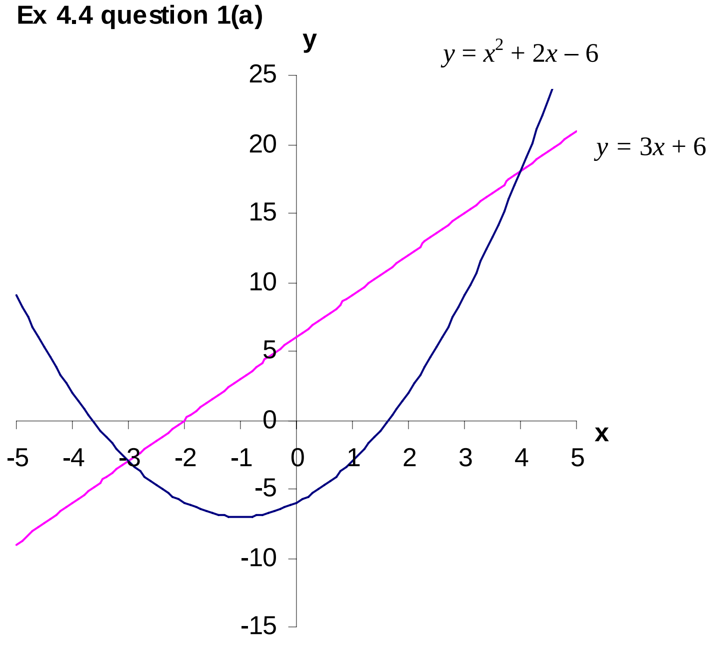
Ex 4.3 question 5



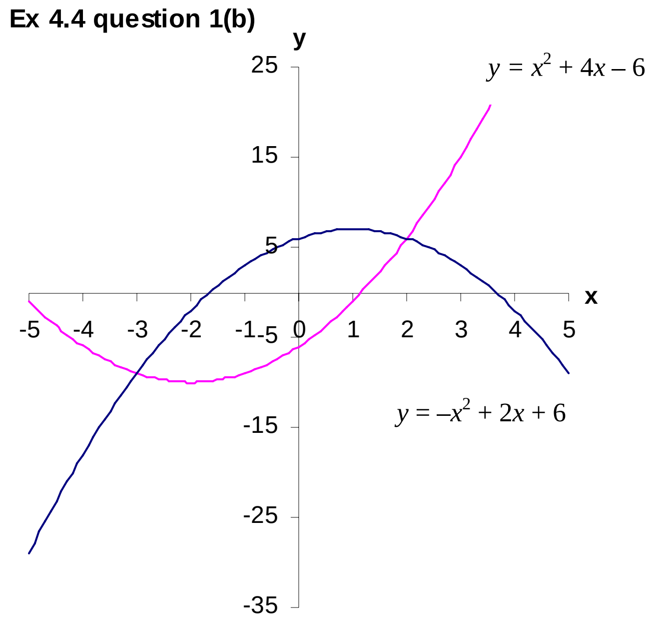
Progress exercise 4.4

1. Solve the following simultaneous equations, and draw sketch graphs of the functions, indicating your solutions.

(a) $y = 3x + 6$; $y = x^2 + 2x - 6$ solns: $x = -3$, $y = -3$; or $x = 4$, $y = 18$.
Solutions are at intersection of the two graphs



(b) $y = x^2 + 4x - 6$; $y = -x^2 + 2x + 6$ solns $x = 2$, $y = 6$; or $x = -3$, $y = 9$
Solutions are at intersection of the two graphs



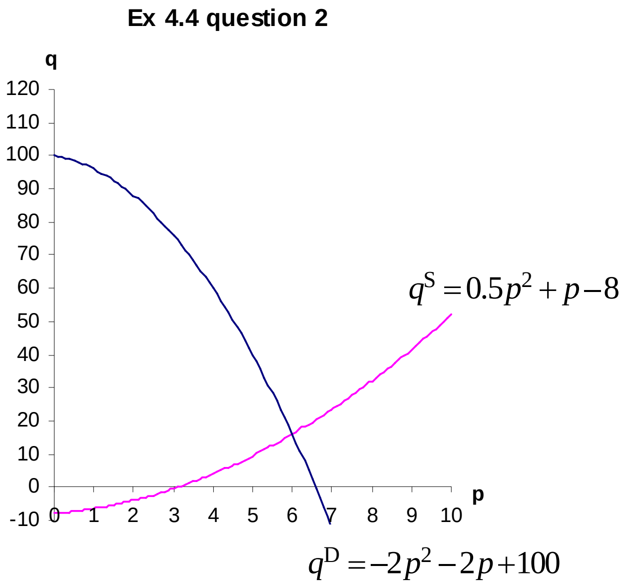
2. Given the following supply and demand functions for a good, find the equilibrium price and quantity, and draw sketch graphs of the functions, indicating your solution.

$$q^{S} = 0.5p^2 + p - 8 \quad ; \quad q^{D} = -2p^2 - 2p + 100$$

Answer: Impose the equilibrium condition $q^S = q^D$, then solve as simultaneous equations, giving:

$$0.5p^2 + p - 8 = -2p^2 - 2p + 100$$

Soln p = 6, q = 16



3. Given the following inverse supply and demand functions for a good, find the equilibrium price and quantity, and draw sketch graphs of the functions, indicating your solution.

$$p^{S} = 10q - 30 \quad ; \quad p^{D} = -0.5q^2 - 8q + 200$$

Answer: Impose the equilibrium condition $p^S = p^D$, then solve as simultaneous equations, giving soln q=10, p=70