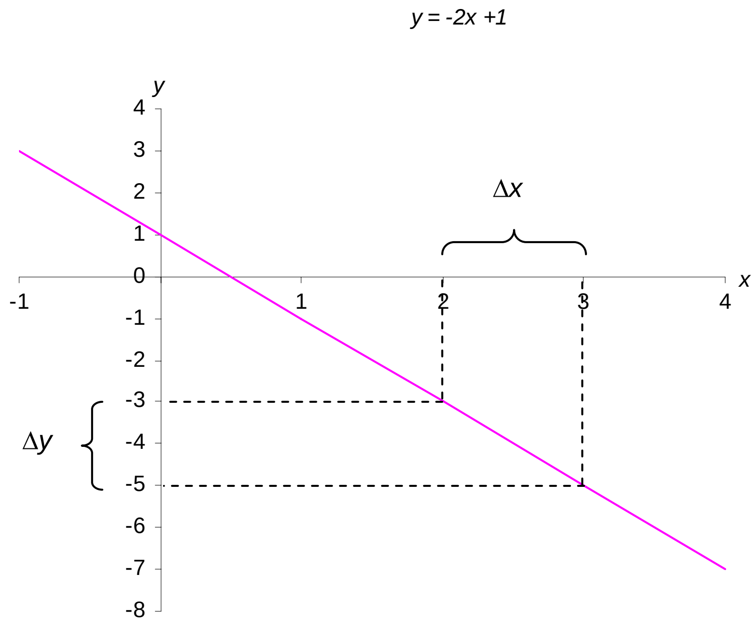
Exercise WS6.1

1. (a) By definition, the difference quotient for any function $y = f(x)$ is

$$\frac{\Delta y}{\Delta x} \equiv \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

Here $f(x) = -2x + 1$ and $\Delta x \equiv x_1 - x_0 = 3 - 2 = 1$

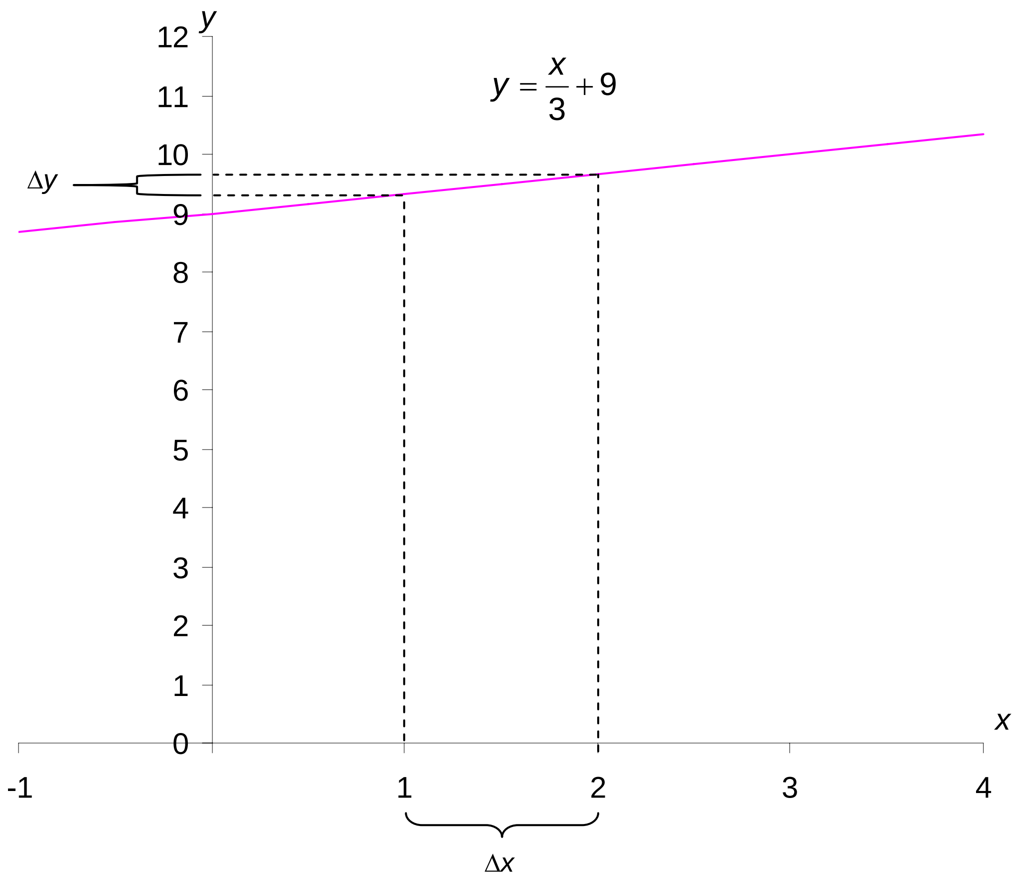
So $\frac{\Delta y}{\Delta x} \equiv \frac{[-2(3)+1]-[-2(2)+1]}{1} = -2$



1. (b) Following (a) above, we have $f(x) = \frac{x}{3}x+9$ and $\Delta x = x_1 - x_0 = 2 - 1 = 1$

So $\frac{\Delta y}{\Delta x} \equiv \frac{[\frac{2}{3}+9]-[\frac{1}{3}+9]}{1} = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

When x increases from 1 to 2, y increases from $9\frac{1}{3}$ to $9\frac{2}{3}$.



2. By definition, the difference quotient for any function $y = f(x)$ is

$$\frac{\Delta y}{\Delta x} \equiv \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

Here $f(x) = ax + b$ with $\Delta x \equiv x_1 - x_0$ with $x_0 = x_0^*$ and $x_1 = x_1^*$ (that is, the values of x_0 and x_1 are unspecified).

So $\frac{\Delta y}{\Delta x} = \frac{a(x_0^* + \Delta x) + b - [ax_0^* + b]}{\Delta x} = \frac{a\Delta x}{\Delta x} = a$

From the above we can see that we will always find that $\frac{\Delta y}{\Delta x} = a$, whatever the values of x_0 and x_1 (and hence of Δx).

Exercise WS6.2

1. $\frac{dy}{dx} = 3x^2$

2. $\frac{dy}{dx} = 3$

3. $\frac{dy}{dx} = 24x^2 + 10x + 12$

4. Since $\frac{1}{x} \equiv x^{-1}$ (an identity, true for all x), by power rule $\frac{dy}{dx} = -x^{-2} \equiv -\frac{1}{x^2}$

5. Since $\frac{1}{p^{0.5}} \equiv p^{-0.5}$ (an identity, true for all p), we have $q = p^{-0.5} + 2$, so by power rule

$$\frac{dq}{dp} = -0.5p^{-1.5} \equiv -\frac{0.5}{p^{1.5}}$$

Note that the rules apply in exactly the same way when the variables are labelled p and q as when they are labelled x and y.

6. $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{dy}{dx} = 0.5x^{-0.5}$

7. $\frac{dy}{dx} = 2ax + b$ (Note, mistake in question: c is also a parameter)

8. $\frac{dy}{dx} = 1 - x^{-2} - 0.5x^{-0.5}$ (using power rule on each term).

9. $\frac{dz}{dy} = 15y^2 - 6y$

10. $\frac{dy}{du} = u + 5$

(In questions 9 and 10, note that the rules apply in exactly the same way whatever the variables are labelled.)

Exercise WS6.3

1. $\frac{dy}{dx} = 2(x^2 + 9x)(2x + 9)$

2. $\frac{dy}{dx} = (x^2 + 3)(6x^2 + 5) + (2x^3 + 5x)(2x)$

3. $\frac{dy}{dx} = \frac{(2x-1)(3x^2 + 8x) - (x^3 + 4x^2)(2)}{(2x-1)^2}$ (using quotient rule)

4 Let

$u = (x^3 - 1)$ and $v = (x^2 + x)^{-1}$. Then $\frac{du}{dx} = 3x^2$

and (using function of a function rule) $\frac{dv}{dx} = -(x^2 + 1)^{-2}(2x)$. So, using product rule,

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= (x^3 - 1)\left[-(x^2 + 1)^{-2}(2x)\right] + (x^2 + x)^{-1}(3x^2)$$

5. We know that: $-\frac{1}{1-x} \equiv -(1-x)^{-1}$. So, using power rule and function of a function rule,

$$\frac{dy}{dx} = (-1)\left[-(1-x)^{-2}\right](-1) \equiv -(1-x)^{-2}$$

(This question and others like it can also be answered using quotient rule.)