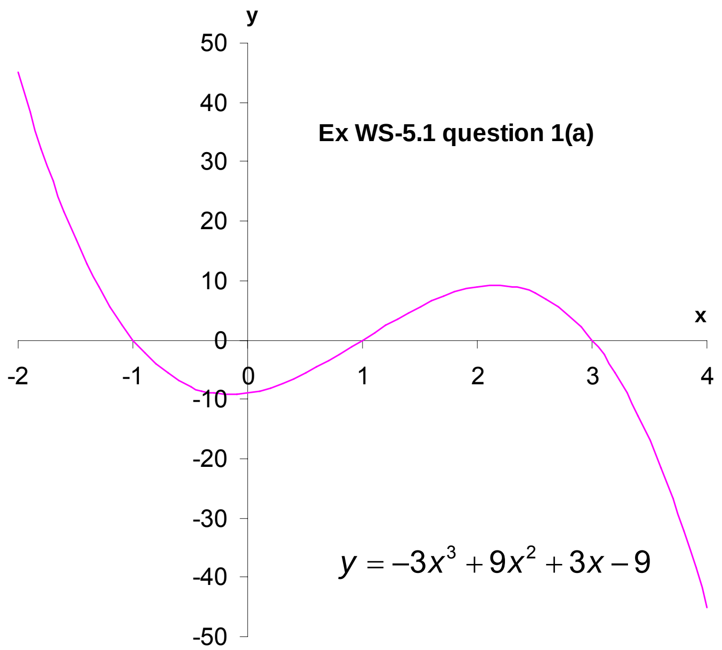
Exercise WS5.1

1.(a)



Although not part of the question, note that this cubic equation cuts the *x*-axis at –1, 1 and 3. These are therefore the roots of the cubic equation $-3x^3 + 9x^2 + 3x - 9 = 0$. Therefore the factors of $-3x^3 + 9x^2 + 3x - 9$ are $a(x + 1)$, $(x – 1)$ and $(x – 3)$, where *a* is some constant yet to be determined. You can check this by multiplying out

$$a(x + 1)(x - 1)(x - 3)$$

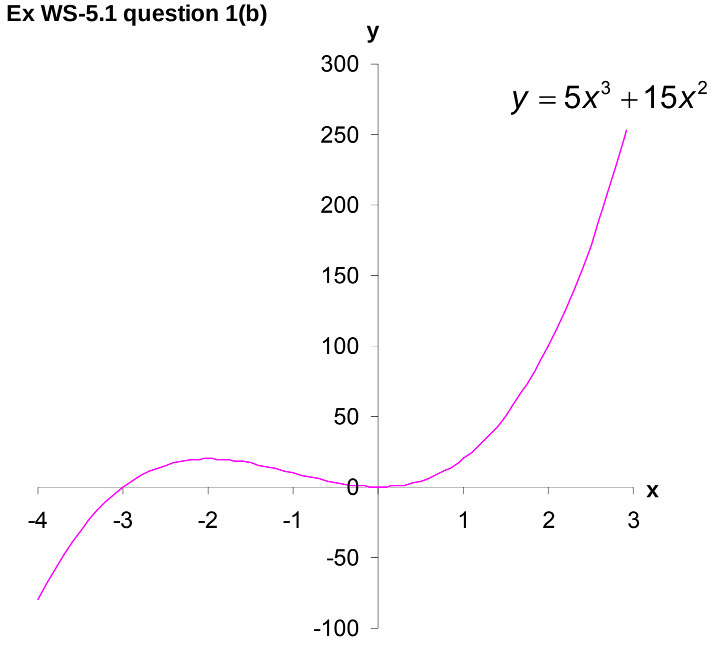
You should get

$$a(x^3 - 3x^2 - x + 3)$$

and therefore *a* must equal –3, for then we have

$$-3(x^3 - 3x^2 - x + 3) = -3x^3 + 9x^2 + 3x - 9$$

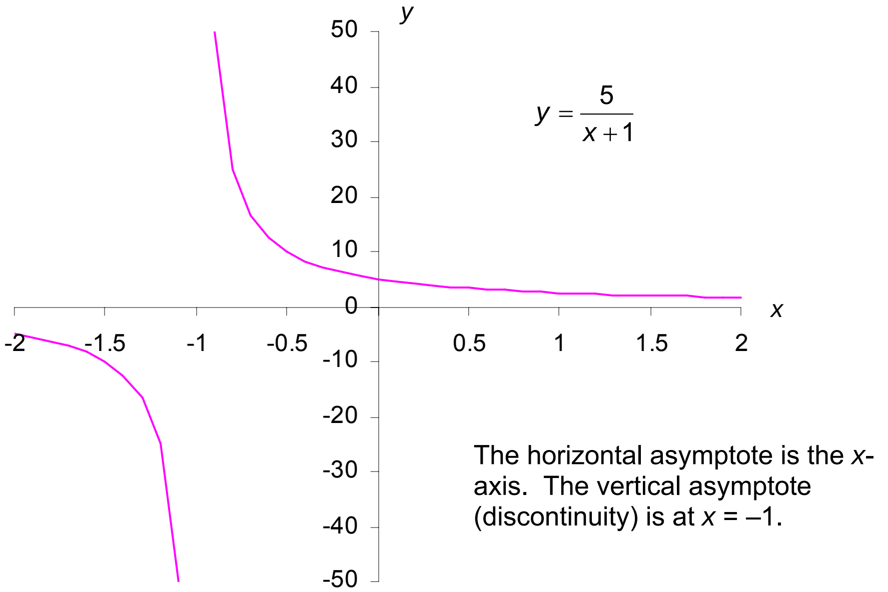
1.(b)



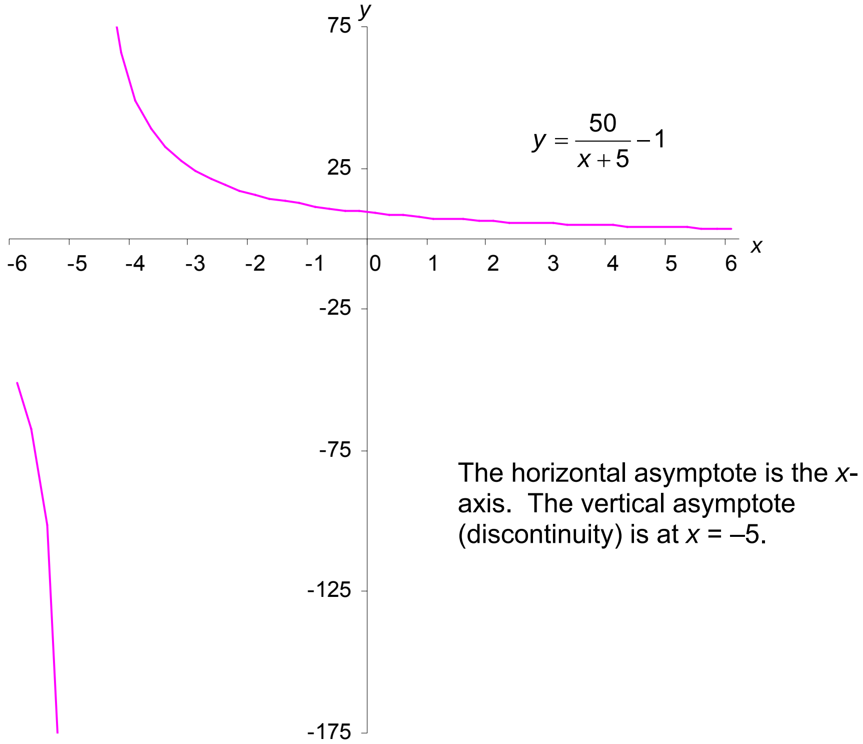
In this case $y = 5x^3 + 15x^2$ factorises to $y = 5x^2(x+3)$. Since $5x^2(x+3) = 0$ when x = 0 or x = –3, we see immediately that these are the intercepts on the x-axis. The graph confirms this.

Exercise WS5.2

1.(a)



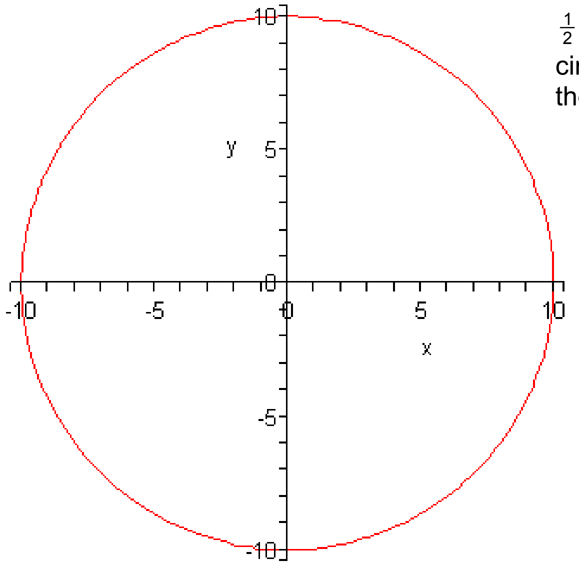
1.(b)



Using only positive values of x and y, this function would be plausible as a demand function as it has the negative slope we normally expect of a demand function. If *x* denotes price and *y* denotes quantity demanded, the demand function shows relatively small changes in quantity (*y*) in response to relatively large changes in price (*x*). If *x* denotes quantity and *y* denotes price, the opposite is true.

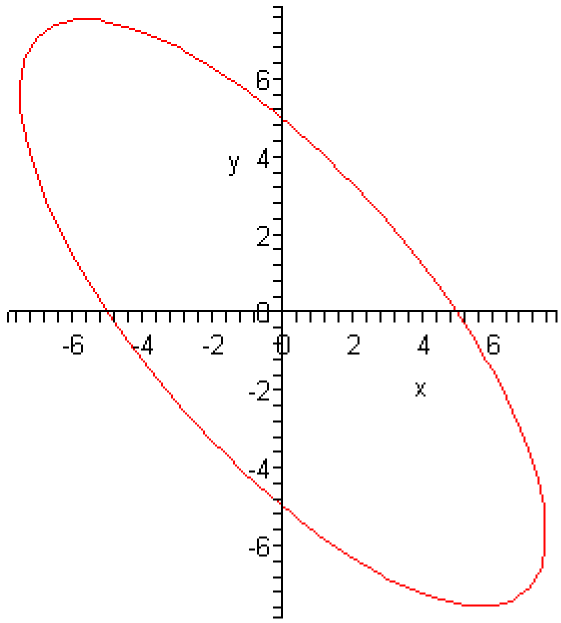
Exercise WS5.3

1.(a)



$\frac{1}{2}x^2 + \frac{1}{2}y^2 = 50$ is a circle with its centre at the origin and radius 10.

1.(b)



$x^2 + \frac{3}{2}xy + y^2 = 25$ is an ellipse. Without the $\frac{3}{2}xy$ term, it would be a circle centred at the origin with radius 5. The effect of the $\frac{3}{2}xy$ is to stretch the curve away from the origin when x and y have opposite signs, and to squeeze the curve closer to the origin when x and y are both positive or both negative. This stretching and squeezing turns the circle into an ellipse.

Exercise WS5.4

1. Given $-10 < x < 10$, multiply through by $-\frac{1}{2}$. This gives:

$$5 > -\tfrac{1}{2}x > -5$$

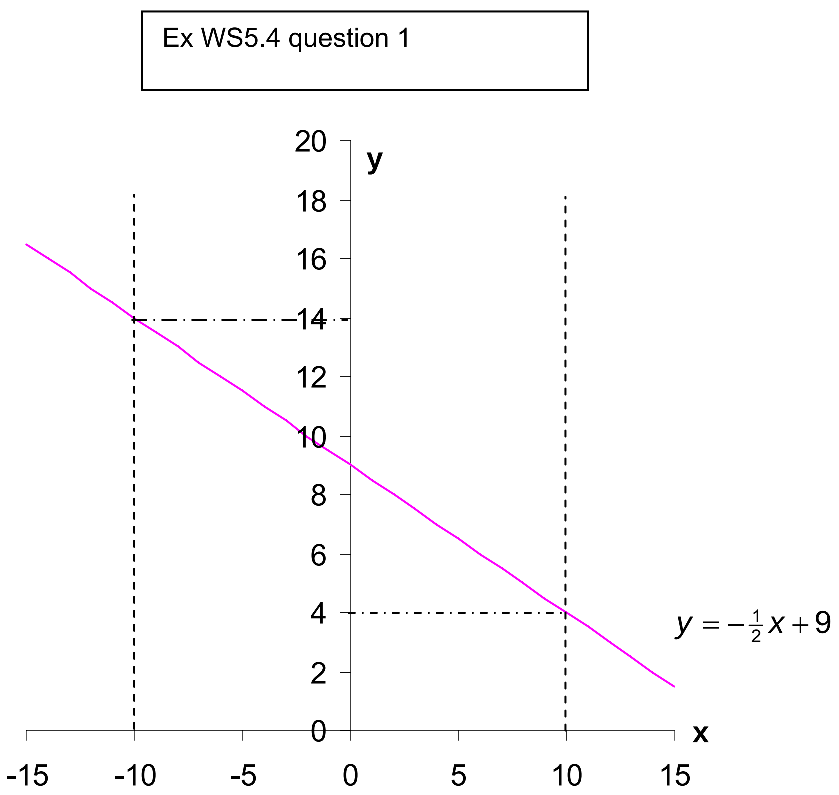
(Note, direction of inequality reversed because we have multiplied by a negative number). If desired, this inequality can be re-written in the reverse direction as:

$$-5 < -\tfrac{1}{2}x < 5$$

Now add 9 to all three elements, giving:

$$4 < -\tfrac{1}{2}x + 9 < 14$$

This is our solution; if *x* lies between –10 and +10, $-\frac{1}{2}x + 9$ lies between 4 and 14. We can also see this by graphing $y = -\frac{1}{2}x + 9$ (below). The vertical dotted lines show the limits on *x*, and we read off the corresponding upper and lower limits on *y* as 14 and 4.

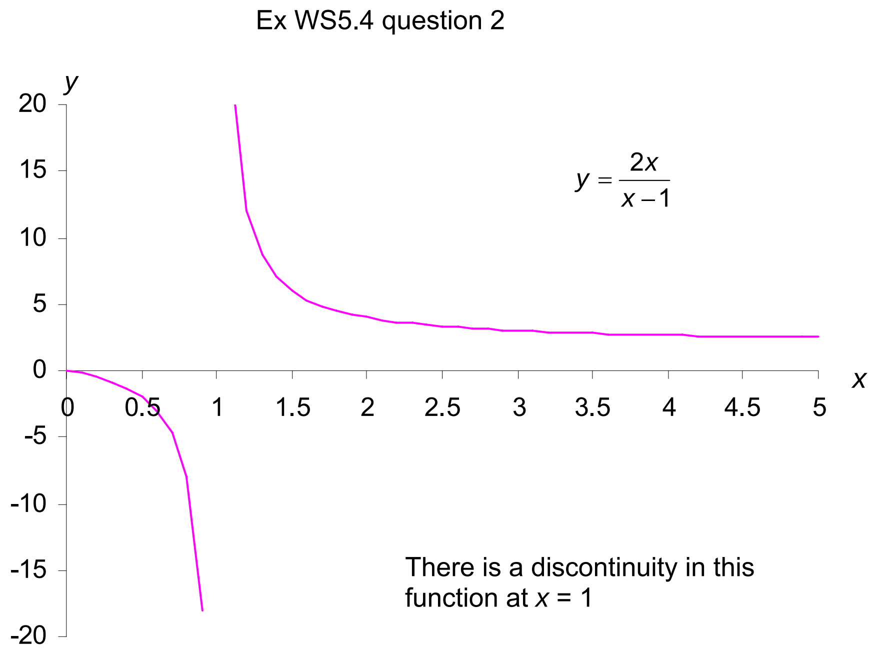


2. This question requires a little thought. Note that the inequality condition $1 < x < 5$ is "strict"; that is, *x* must not actually equal 1 or 5 but must always be greater than 1 but less than 5.

Suppose *x* is very slightly greater than its lower limit of 1. Then the numerator of $y = \frac{2x}{x-1}$ is slightly greater than 2, and the denominator is slightly greater than zero. Thus *y* is positive and very large. In fact, as *x* approaches 1 (while remaining greater than 1), *y* approaches positive infinity.

At the other extreme, suppose *x* is very slightly below its upper limit of 5; say *x* = 4.9. Then $y = \frac{2x}{x-1} = \frac{9.8}{3.9} = 2.5128$ (to 4 d.p.). If *x* = 5 (which is not permitted) then *y* would equal 2.5. So as long as *x* is less than 5, *y* is greater than 2.5.

Collecting results, $1 < x < 5$ implies $y = \frac{2x}{x-1}$ lies between 2.5 and +∞. This is confirmed by the graph (below).



3.(a)(i) $3x + 4y = 1200$

(a)(ii) $y = 300 - \frac{3}{4}x$

(b) Opportunity cost of x is $\frac{P_x}{P_y} = \frac{3}{4}$. This means that by reducing purchases of good y by three-quarters of a unit, the consumer releases enough money to increase purchases of good x by 1 unit.

(c) (i) 300; (ii) 400. This means that the consumer's money income will buy either 300 units of y or 400 units of x.

(d) 80 units of x cost 240 euros (or whatever is the currency unit). This leaves 1200 – 240 = 960 euros to spend on y. For 960 euros the consumer can buy 240 units of y.

Expenditure on x as a percentage of the budget is $\frac{240}{1200} \times 100 = 20\%$.

Expenditure on y as a percentage of the budget is $\frac{960}{1200} \times 100 = 80\%$.

(e) Equation of budget constraint is $3x + 4y = 1200$ as an implicit function (that is, with both x and y on the same side of the equation). As an explicit function with y as the dependent variable, the equation of the budget constraint is $y = 300 - \frac{3}{4}x$ (see (a) above). The latter is graphed below.

Ex WS5.4 question 3