

**Exercise WS3.1**

1. (a) Let  $C_p$  and  $C_d$  denote fuel costs in £ of petrol and diesel cars respectively. Then we know that, for the petrol car:

$C_p = 3.6G_p$  (where  $G_p$  = number of gallons of petrol consumed by the petrol car). Also we know that:

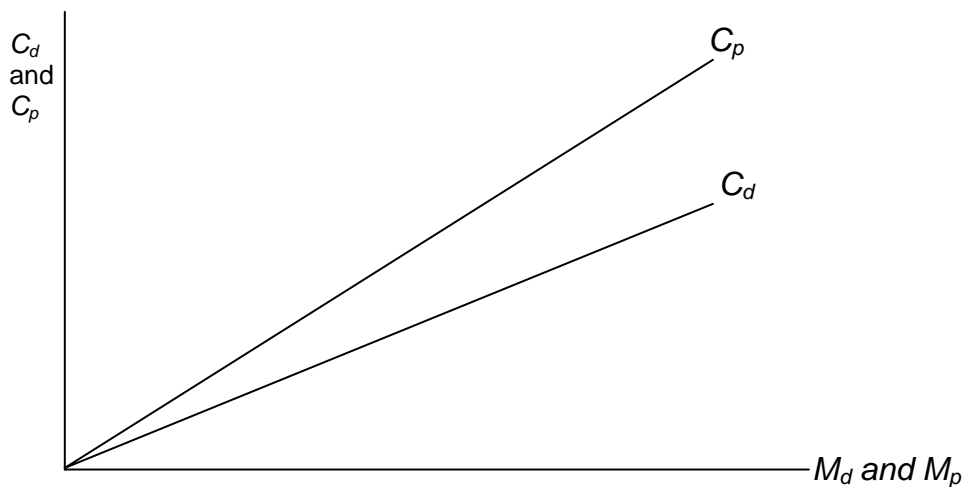
$$G_p = \frac{M_p}{35} \text{ (where } M_p = \text{distance travelled by petrol car, in miles.)}$$

Combining these two equations, we get:  $C_p = 3.6 \frac{M_p}{35}$

Applying the same reasoning to the diesel car, we get:

$$C_d = 3.6 \frac{M_d}{50} \text{ (where } M_d = \text{distance travelled by diesel car, in miles). See}$$

below for sketch graphs of these two relationships. Note that the graph of  $C_p$  is steeper than that of  $C_d$  because each additional mile travelled in the petrol car adds more to total fuel costs than in the diesel car.



- (b) We want to find the number of miles travelled by both cars at which the petrol car's total fuel costs are 1000 pounds more than the total fuel costs of the diesel car. That is, where:

$$C_p = C_d + 1000$$

Using the two equations above, this becomes:

$$3.6 \frac{M_p}{35} = 3.6 \frac{M_d}{50} + 1000 \text{ with } M_p = M_d.$$

We will write  $M$  for the common distance travelled by both cars. Thus we have:

$3.6 \frac{M}{35} = 3.6 \frac{M}{50} + 1000$ . From this, dividing both sides by 3.6 we get:

$\frac{M}{35} = \frac{M}{50} + \frac{1000}{3.6}$ . From this we get:

$$\frac{M}{35} - \frac{M}{50} = \frac{1000}{3.6}$$

→  $M \left( \frac{1}{35} - \frac{1}{50} \right) = \frac{1000}{3.6}$  (the arrow means "this implies")

→  $M \left( \frac{50 - 35}{1750} \right) = \frac{1000}{3.6}$

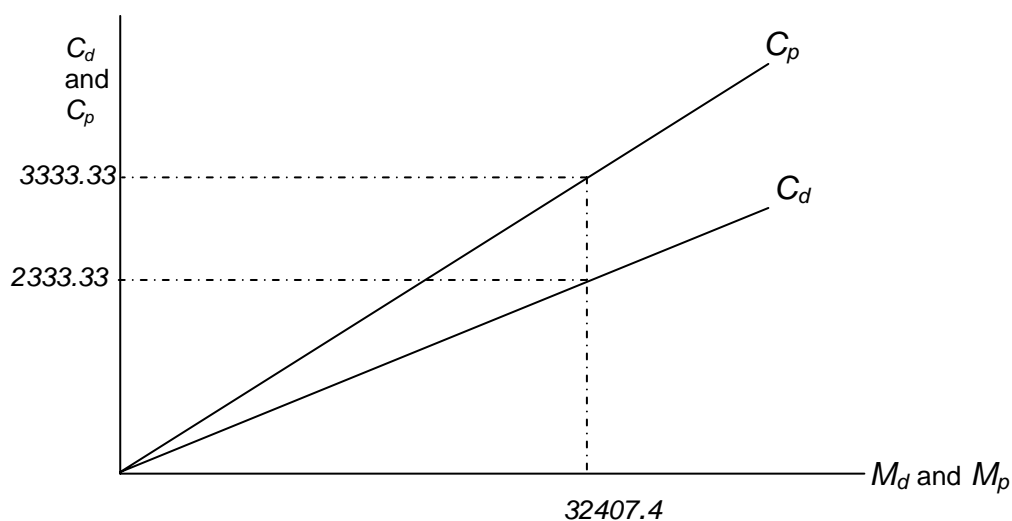
→  $M = \frac{1000}{3.6} \left( \frac{1750}{15} \right) = 32407.4$

After 32407.4 miles the fuel costs are  $C_p = 3.6 \frac{M_p}{35} = 3.6 \frac{32407.4}{35} = 3333.33$  pounds

and  $C_d = 3.6 \frac{M_p}{50} = 3.6 \frac{32407.4}{50} = 2333.33$  pounds for the petrol and diesel cars

respectively. At this distance, the savings on fuel costs exactly offset the additional purchase price of the diesel car.

See sketch below for graphical illustration of solution.



2. (a) (i)  $C_A = 12m$  (where  $m$  is the number of minutes of calls in the month).  
 (ii) There are two mathematical expressions for  $C_B$ , depending on whether  $m$  is less or greater than 300:

$$C_B = 3000 \text{ (if } m \text{ is less than or equal to 300) and}$$

$$C_B = 3000 + 15(m - 300) \text{ if } m > 300. \text{ (Note that costs here are measured in pence).}$$

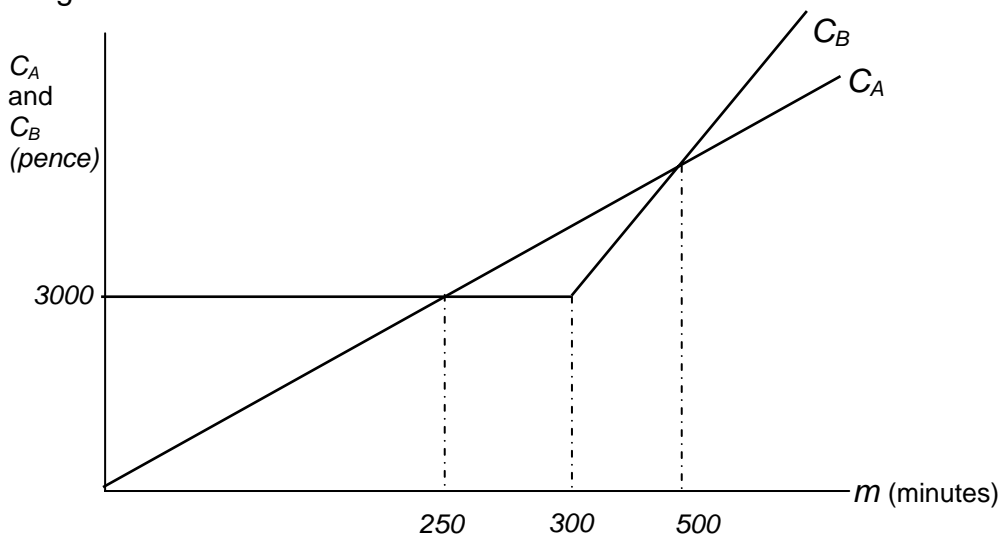
- (b) Costs are the same for both tariffs when  $C_A = C_B$ . When  $m > 300$ , from (a) above this implies  $12m = 3000 + 15(m - 300)$ . By elementary operations, we can solve this linear equation to get  $m = 500$ . That is, in the question,  $x = 500$ .
- (c) I should choose tariff A because each additional minute of calls, after 500 minutes, costs 12 pence under tariff A but 15 pence under tariff B.
- (d) If I make  $m$  minutes of calls per month, with  $m < 300$ , then I should choose tariff A if  $C_A < C_B$ . From (a) above, this is true when:

$$12m < 3000, \text{ which implies } m < 250.$$

Conversely I should choose tariff B if  $12m > 3000$ , which implies  $m > 250$  (but less than 300, by assumption).

The boundary between these two cases is when  $m = 250$ , for then my costs are the same whichever tariff I sign up for.

- (e) From (d), tariff B is cheaper when  $m$  is greater than 250 but less than 500.
- (f) See figure below.



3.

(a)  $y = 4x - 2$

(b)  $y = \frac{a}{c}x - b$

(c)  $y = \frac{x}{2} - \frac{5}{2}$  or  $y = \frac{1}{2}x - \frac{5}{2}$

(d)  $y = \frac{5}{2}x + \frac{1}{16}$

(e)  $y = \frac{10}{x+3} - 2$

(f)  $y = \frac{2}{x-4}$

4.

(a)  $x = -\frac{3}{4}$

(b)  $x = -\frac{b}{a}$

(c)  $x = 60$

(d)  $x = -\frac{90}{11}$  or  $x = \frac{-90}{11}$  or  $x = -8\frac{2}{11}$

(e)  $x = \frac{25}{4}$  or  $x = 6\frac{1}{4}$

(f)  $x = -\frac{20}{3}$  or  $x = -6\frac{2}{3}$

### Exercise WS3.2

1.

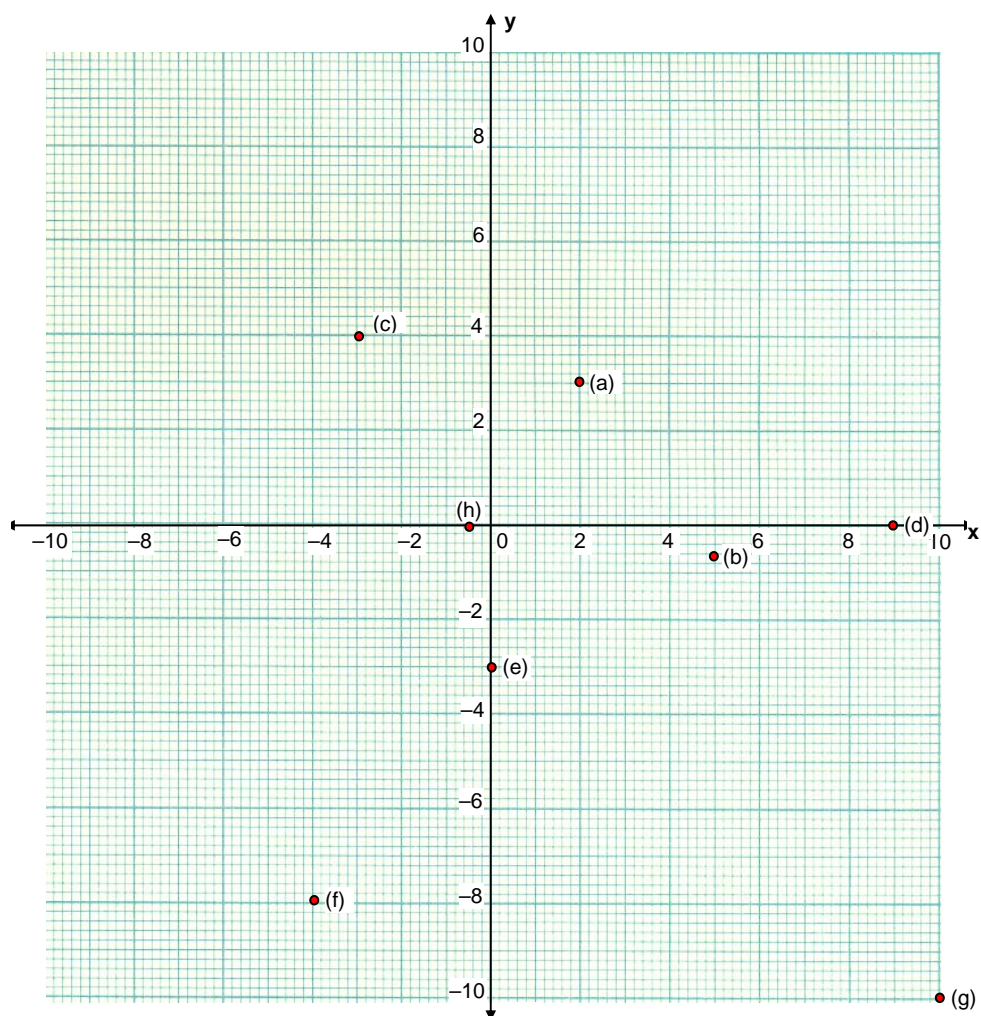
(a) 6, 8

(b) -1, 7

(c) -4, 4

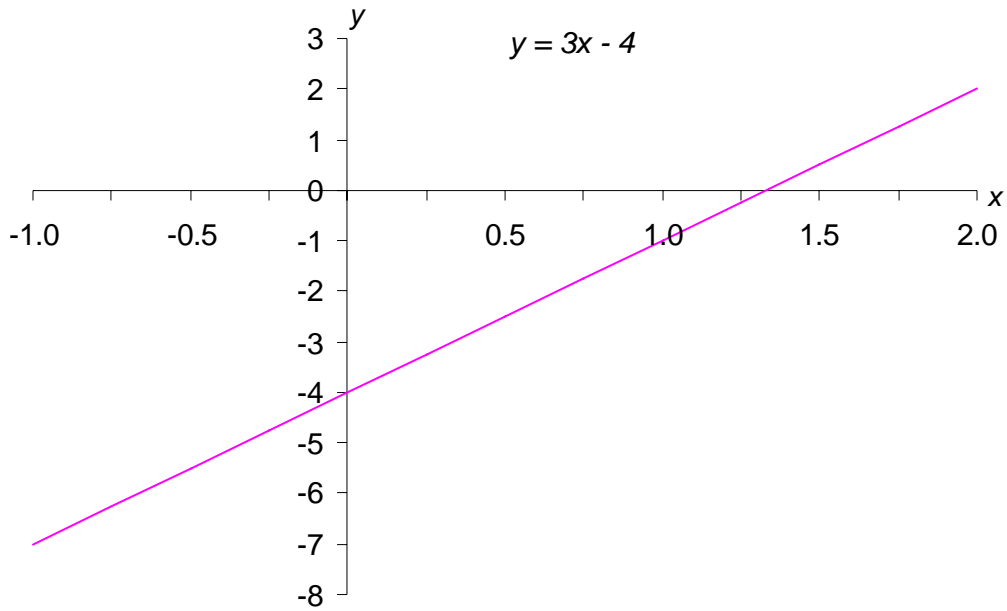
- (d) 1, 2.6
- (e) -2, -5
- (f) 8, -9
- (g) 3, -1
- (h) 4, 1

2.

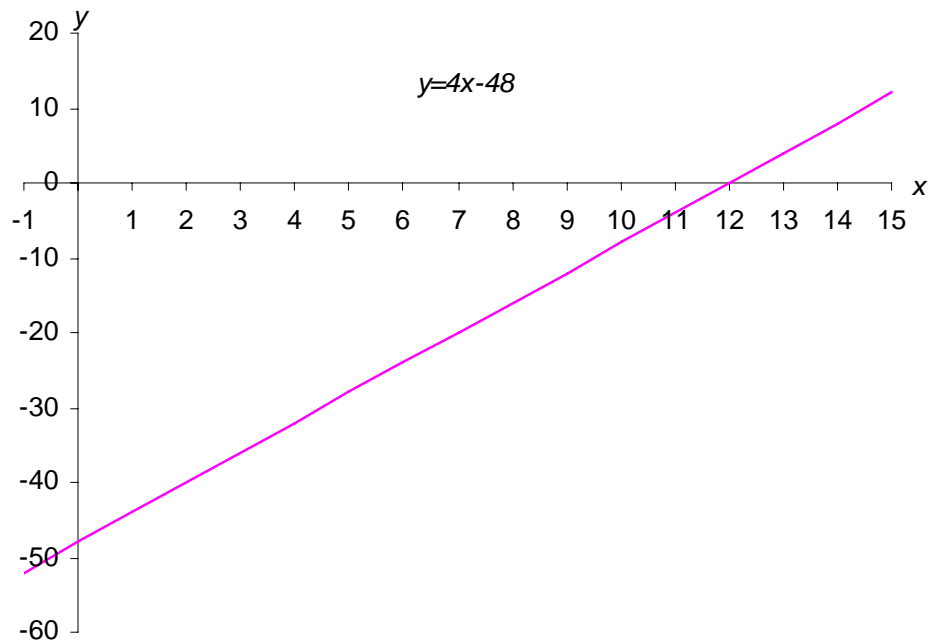


3.

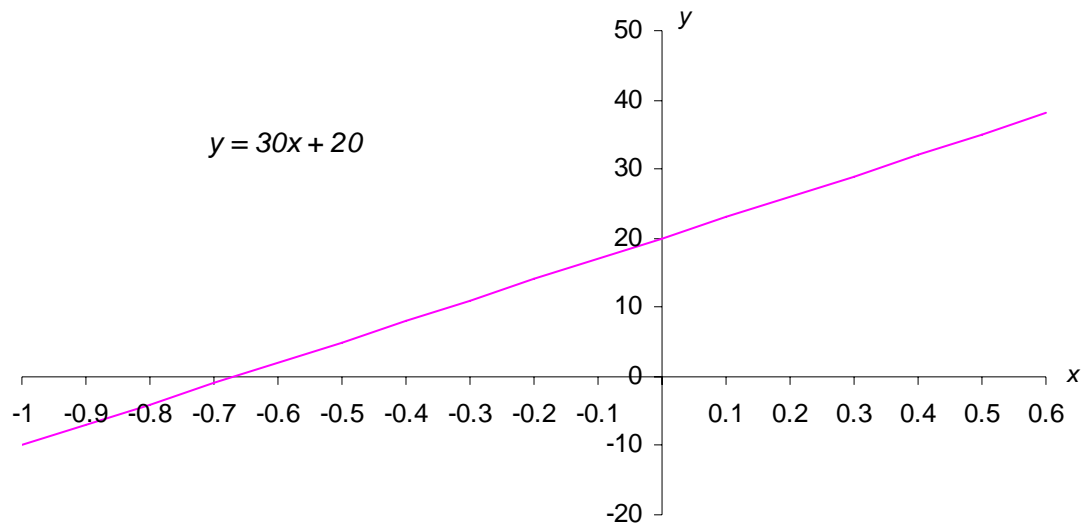
(a)  $y = 3x - 4$



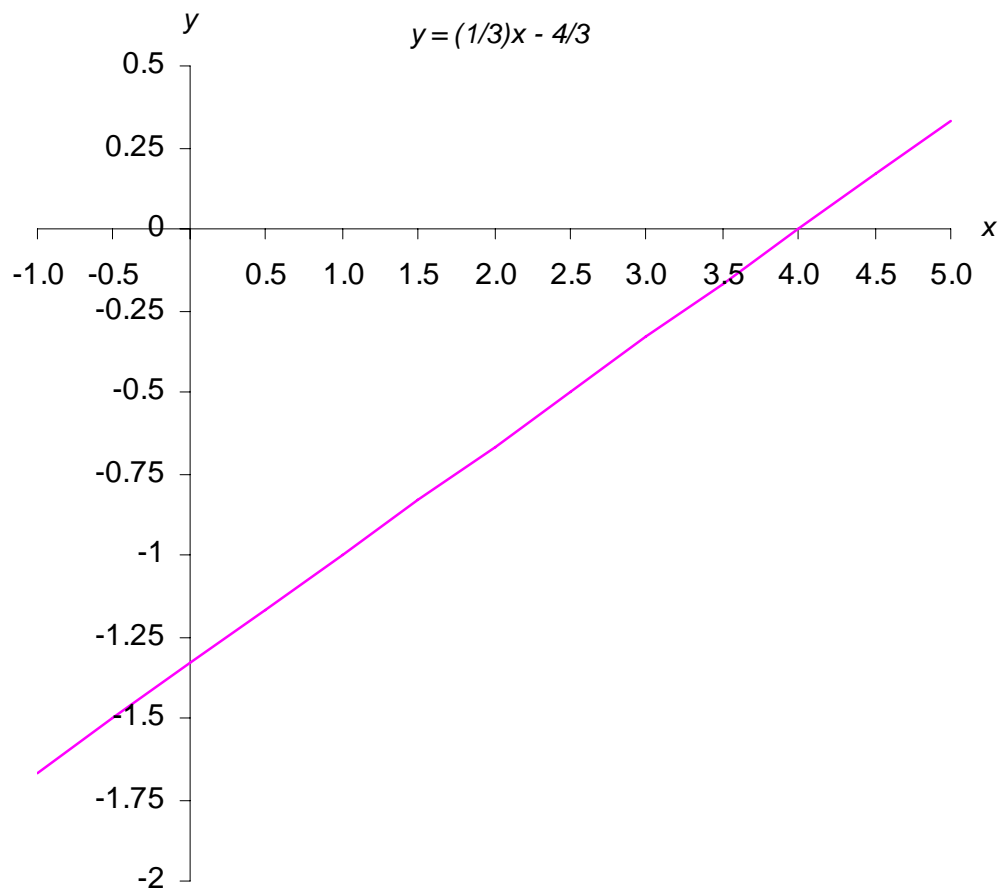
(b)  $y = 4x - 48$



(c)  $y = 30x + 20$



(d)  $y = \frac{x}{3} - \frac{4}{3}$



4. The solutions to these equations are given by the  $x$ -intercepts of the corresponding graphs in question 3. The solutions are:

(a)  $x = \frac{4}{3}$

(b)  $x = 12$

(c)  $x = -\frac{2}{3}$

(d)  $x = 4$

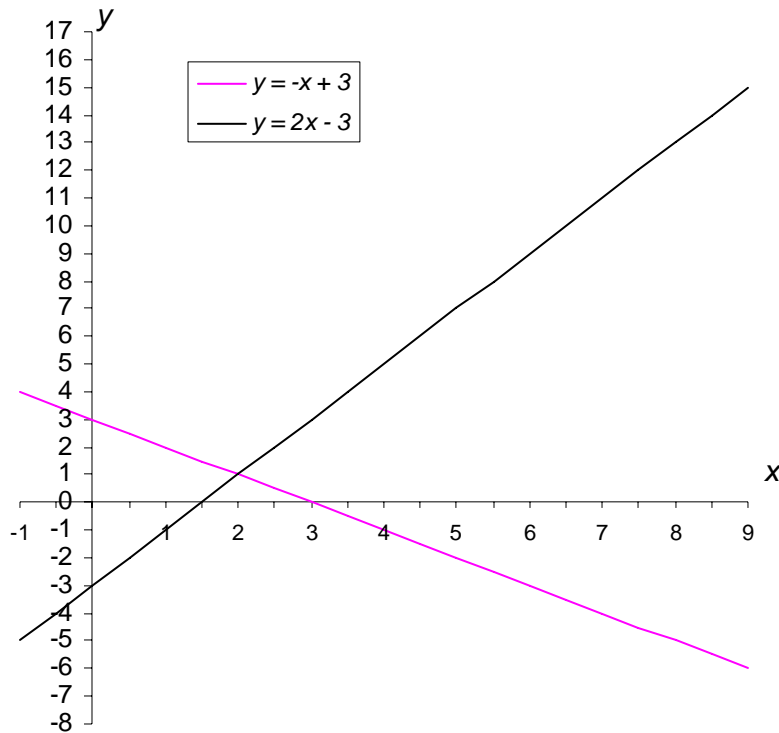
5.

	<u>slope</u>	<u>y-intercept</u>	<u>x-intercept</u>
(a)	3	$y = -\frac{1}{2}$	$x = \frac{1}{6}$
(b)	-20	$y = 120$	$x = 6$
(c)	0.125	$y = -100$	$x = 800$

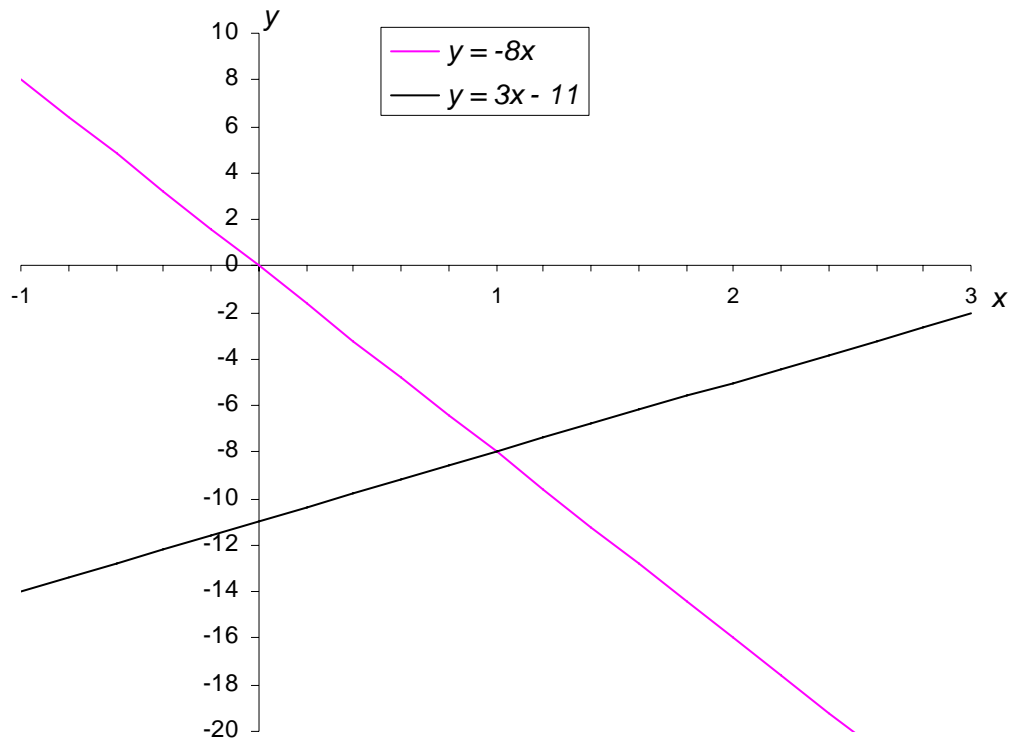
**Exercise WS3.3**

1.

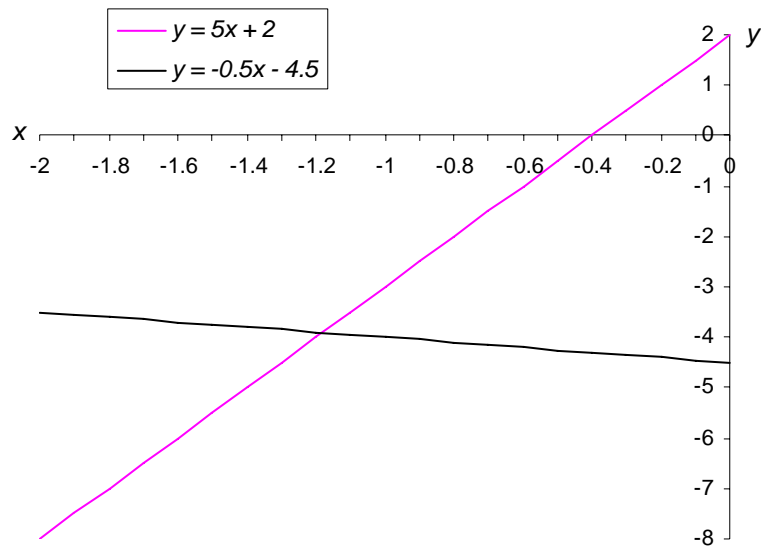
- (a) Setting  $-x + 3$  equal to  $2x - 3$ , we get the linear equation  $-x + 3 = 2x - 3$ . Solving this linear equation gives  $x = 2$ . Therefore from the first equation  $y = -2 + 3 = 1$ , and from the second equation  $y = 2 - 3 = 1$  also.



- (b) Using the same method as in (a) above, we get  $-8x = 3x - 11$ . Solving this linear equation gives  $x = 1$ . Therefore from the first equation  $y = -8$ , and from the second equation  $y = 3 - 11 = -8$  also.

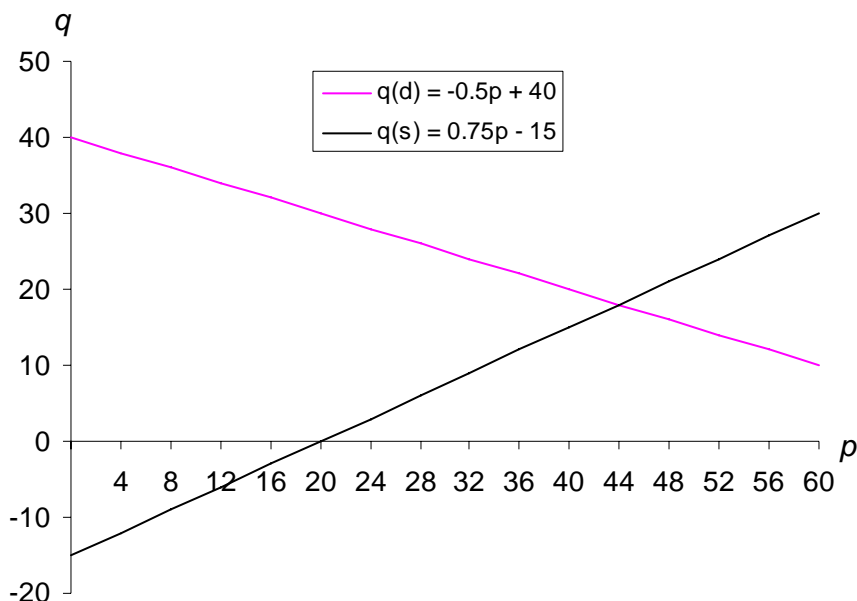


- (c) The numbers make this slightly more difficult to solve. Using the method of (a) and (b) above, the solution is  $x = -1.1818$  (to 4 d.p.). From this,  $y = -3.909$ . This seems to be confirmed by the graph, where we can see that  $x = -1.2$  approximately and  $y = -4$  approximately.

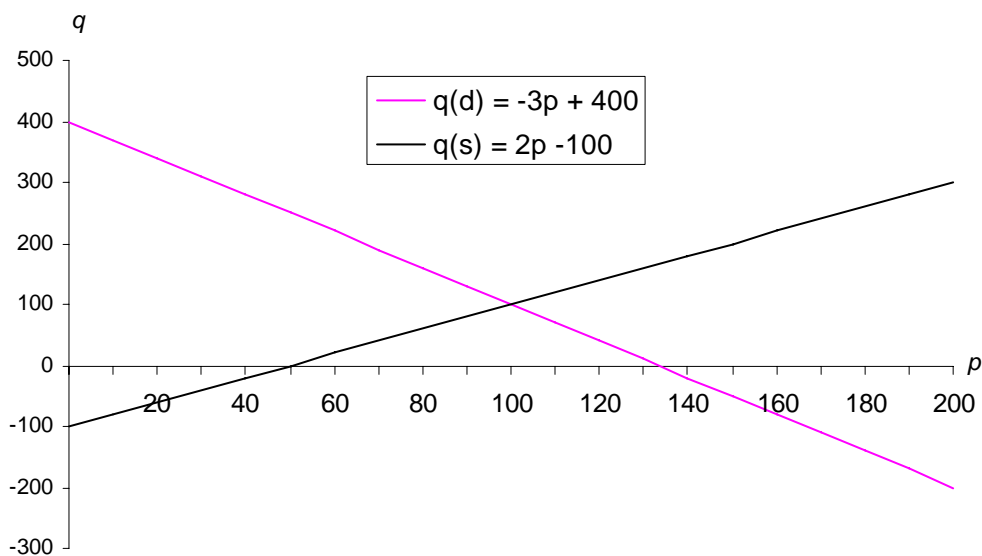


**Exercise WS3.4**

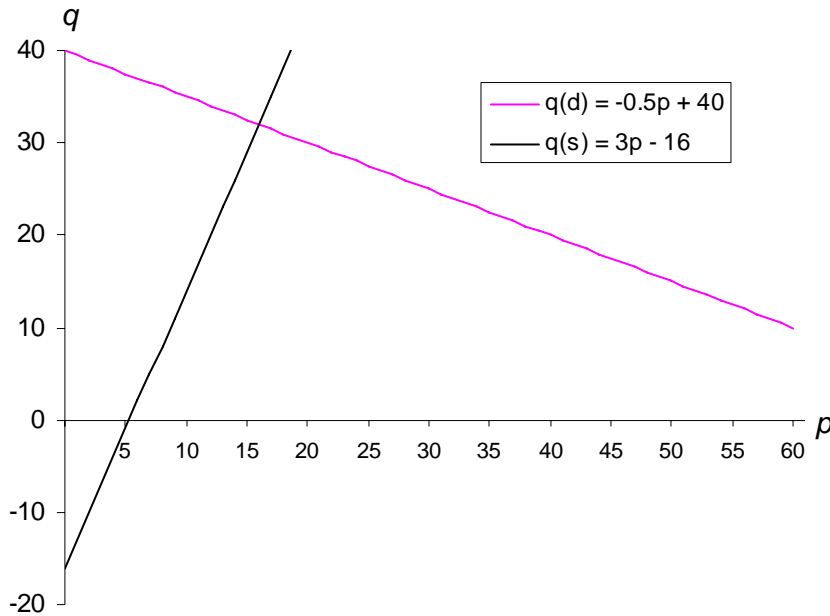
1. (a) We require  $q(d) = q(s)$  for equilibrium. Proceeding as in Exercise WS3.3 above, we set  $q(d) = q(s)$ , which implies  $-0.5p + 40 = 0.75p - 15$ . Solving this linear equation for  $p$  gives  $p = 44$ , from which  $q(d) = q(s) = 18$ .



- (b) Using the method of (a) above gives  $p = 100$ , from which  $q(d) = q(s) = 100$  also. The graph confirms this.



- (c) Using the method of (a) and (b) above gives  $p = 16$ , from which  $q(d) = q(s) = 32$ . The graph confirms this.



2. Assuming that the price paid by buyers is the same as the price received by sellers, the  $p$  in the demand function must be the same as the  $p$  in the supply function. This means that the two left hand sides of the two equations are equal, and therefore so too are the two right hand sides. So we can write:

$$-\frac{3}{2}q^D + 36 = 3q^S + 9$$

However we cannot yet solve this equation as it contains two variables. To solve it we have to add the equilibrium condition,  $q^D = q^S$ ; that is, quantity supplied equals quantity demanded. Let us write this common quantity simply as  $q$ . Then we can replace both  $q^D$  and  $q^S$  with  $q$ , and thereby get:

$$-\frac{3}{2}q + 36 = 3q + 9$$

Solving this linear equation for  $q$  gives  $q = 6$ . Thus  $q^D = q^S = 6$ . Substituting this into either the inverse supply function or the inverse demand function (or both, to be safe) gives  $p = 27$ .

3.(a) Using the equilibrium condition,  $q^D = q^S$ , we can write:

$$-0.5p + 20 = 0.75p - 15 \quad \text{or} \quad -\frac{1}{2}p + 20 = \frac{3}{4}p - 15$$

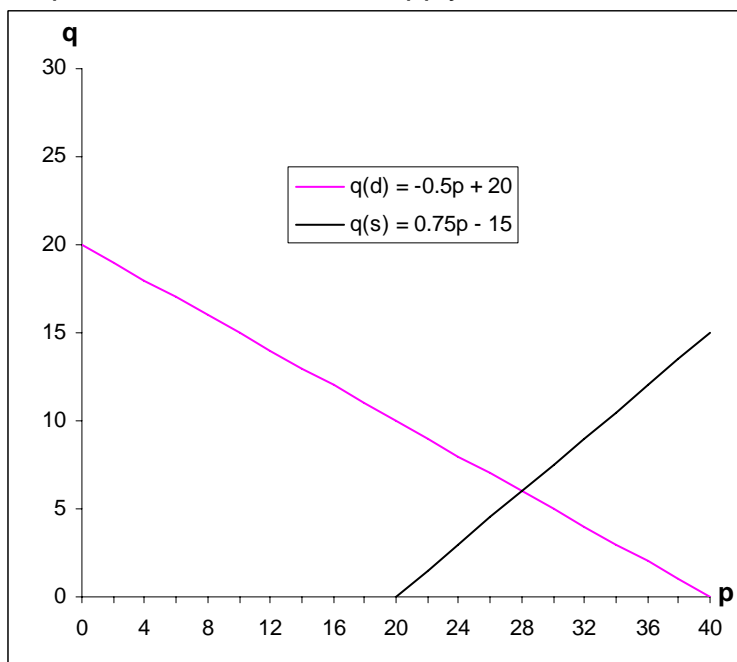
The solution to the linear equation is  $p = 28$ . Substituting this into either the supply function or the demand function (or both, to be safe) gives  $q = 6$ .

(b) By carrying out elementary operations on the demand function we can re-arrange it in inverse form as:  $p = -2q^D + 40$

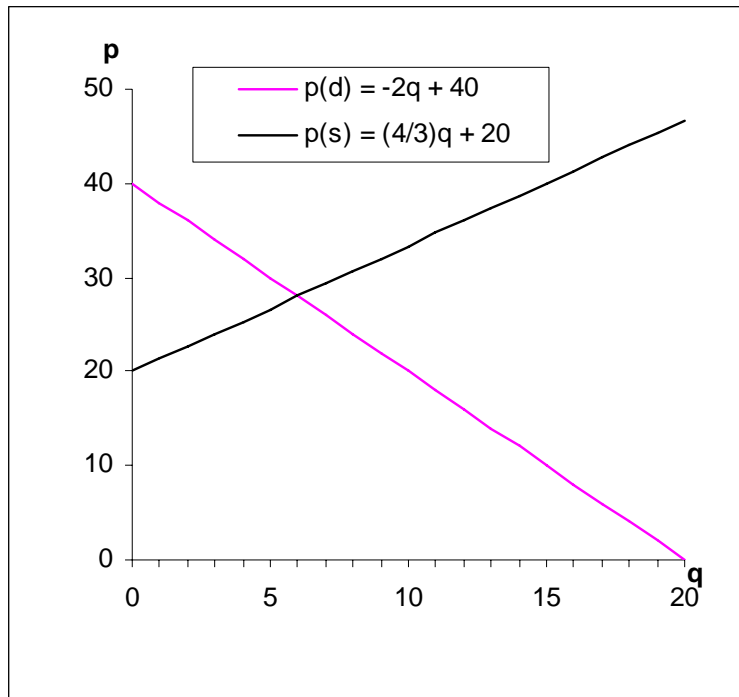
Similarly the supply function can be re-arranged as:  $p = \frac{4}{3}q^S + 20$

These can then be solved using the method of question 2 above, giving  $q = 6$ ,  $p = 28$  as in (a) above.

(c) Graph of the demand and supply functions:



Graph of the inverse demand and supply functions:

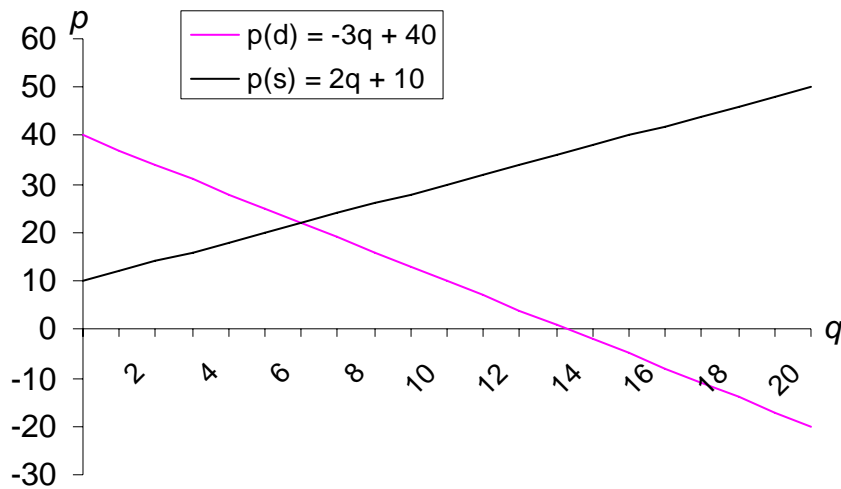


4.(a) Using the same methods as in question 3 above, the equilibrium price and quantity are  $p = 22$ ,  $q = 6$ .

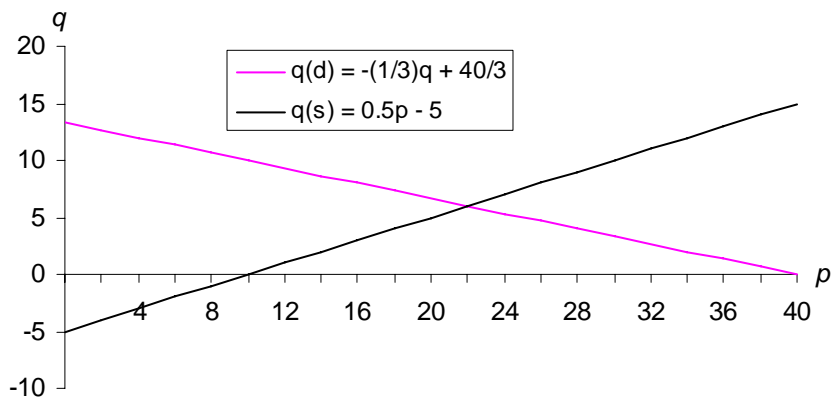
(b) Here we are given the inverse supply and inverse demand functions. Inverting these, we find the supply and demand functions as:

$$q^D = -\frac{1}{3}p + \frac{40}{3} \text{ and } q^S = \frac{1}{2}p - 5$$

(c) Graph of the inverse supply and inverse demand functions:



Graph of the supply and demand functions:



5. Combining the first two equations we get

$$Y = C + I$$

Combining the third and fifth equations we get

$$C = aY + 100 = 0.9Y + 100$$

Combining the two equations above, and using the 4th given equation ( $I = 500$ ), we get:

$$Y = C + I = 0.9Y + 100 + 500$$

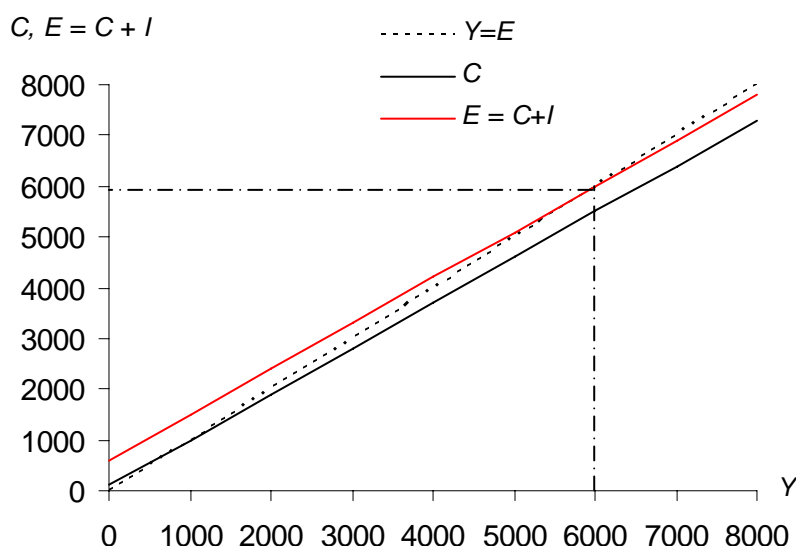
Re-arranging this, we get:

$$Y = \frac{600}{1-0.9} = 6000$$

To find consumption, we substitute  $Y = 6000$  into the consumption function  $C = 0.9Y + 100$ , giving  $C = 5500$ .

Although not asked to do so, we can also find savings,  $S$ , defined as  $Y - C$  (that is, income minus consumption). Thus  $S \equiv Y - C = 6000 - 5500 = 500$ . Note that  $S = I$ . Do you understand why?

Graphical illustration:



The equilibrium point is at the intersection of the  $Y = E$  and the  $E = C + I$  lines. From the algebra above, we know that this is where  $Y = C + I = 6000$ . At this point, consumption is found from the consumption function  $C = 0.9Y + 100$ , with  $Y = 6000$  as  $C = 5500$ .

- (b) If savings rise by 50 at every level of income, then consumption must fall by 50 at every level of income. Thus the consumption function becomes  $C = 0.9Y + 50$ . If we re-solve the model in (a) above, with  $C = 0.9Y + 50$ , we get  $Y = 5500$ ,  $C = 5000$ . Thus the attempt by consumers to save 50 more at every level of income results in both income and consumption falling by 500. Note that the level of savings,  $S$ , remains unchanged at  $S = 500$ . Do you understand why? (This feature of the basic Keynesian model is known as "the paradox of thrift".)
- (c) Given that  $C$  is now 50 lower at every level of income, we require investment to be 50 higher at every level of income in order to maintain the same level of demand,  $C + I$ . Thus  $I$  must rise by 50 to 550. We can check this by re-solving the model of (a) with the consumption function  $C = 0.9Y + 50$ , and  $I = 550$ .

6.(a) Using the method of part (a) of the previous question, we obtain:

$$Y = \frac{200 + 1800}{1 - 0.8} = 10000 \quad \text{From this, } C = 0.8Y + 200 = 8200. \text{ The graphical}$$

illustration is the same as the graph above, except that the slope of the  $C$  function and the  $C + I$  functions are less steep and the intersection with the  $Y = E$  line is consequently further to the right.

(b)(i) Following the method of (a) above, we obtain:

$$Y = C + I + G = 0.8(Y - 500) + 200 + 1800 + 500$$

Simplifying and re-arranging this, we get:

$$Y - 0.8Y = -400 + 200 + 1800 + 500 = 2100$$

$$\text{From which: } Y = \frac{2100}{1 - 0.8} = 10500$$

(b)(ii)  $C = 0.8(Y - T) + 200 = 8200$  (the same as in (a) above)

Check for mistakes: we require  $Y = C + I + G$ ; that is,  $10500 = 8200 + 1800 + 500$ , which is correct.

(b)(iii) We have  $C = 0.8Y + 200$  without the tax, and  $C = 0.8Y - 0.8T + 200$  with the tax. Comparing these, we see that when the tax is introduced,  $C$  falls by  $0.8T$ ; that is, by 80% of the tax. Aggregate demand,  $C + I + G$ , therefore increases because the fall in aggregate demand due to the fall in consumption is less than the increase in demand by the government,  $G$ . (Recall,  $T = G$ ).

This increase in aggregate demand causes  $Y$  and therefore  $C$  to increase, raising aggregate demand still further. This process continues until aggregate demand is once more equal to the level of output, which from (b)(ii) above occurs when  $Y = 10500$ . Income increases by 500, the amount of government expenditure and taxes. (This result is known as the "balanced budget multiplier theorem".)