Exercise WS21.1

1. Expand the following functions around $x = 0$, up to the second (second derivative) term. Calculate the error from using your expansion to approximate the value of y when $x = 1$.

(a) $y = \frac{x+2}{x+4}$

(b) $y = e^{2x}$

(c) $y = \ln(x^3 + 1)$

2. Expand the following functions around $x = 0$, up to the third (third derivative) term. Calculate the error from using your expansion to approximate the value of y when $x = 1$.

(a) $y = \frac{x+2}{x+4})$

(b) $y = 2^x$

(c) $y = x \ln(x + 2)$

3. Find and classify the stationary points for the functions given below.

(a) $z = x^2 + 2y^2 - 10x - 12y$

(b) $z = 10x - x^2 - 2y^2 + 8y$

(c) $y = (1 - x)^3$

(d) $y = (1 - x)^4$

4. Find and classify the stationary points of the function

$$z = 4y^2 - 4y - 2x^2y - x^3$$

5. Which of the following functions are concave for x>0?

(a) $y = -x^{-2}$

(b) $y = x^{-1/2}$

(c) $y = -x(1-2x)$

(d) $y = -(x^{-2} + x^{-1/2})$

(e) $y = -x^{-1/2} + x(1-2x)$

(f) $y = x + (x-a)^2$

6. "A linear function is neither strictly concave nor strictly convex." True or false?

7. Draw a function which is strictly concave for $x > 0$ and strictly convex for $x < 0$.

8. (a) What is the maximum of $x - 2(x-a)^2$ (where x is constrained to be non-negative) for any value of a ?

   (b) What is the maximum of $x - 2(x-a)^2$ (where x is constrained by $x \geq 1$) for any value of a?

Exercise WS21.2

1. If $z = bx - xy - x^2 + cy - 2y^2$, find the maximum value of z and determine how the optimising values of x and y change (a) as b changes and (b) as c changes.

2. If the number of cigarettes sold (in millions, and denoted by q) depends on the price $p + t$, where t is a tax, according to the demand function $q = 100 - 2(p+t)$, and if p is fixed by the international price of cigarettes, find the tax t which maximises the government's tax revenue. Find how t responds to a change in p.

3. A monopolistic producer of two goods has a joint total cost function

$$TC = 10Q_1 + Q_1Q_2 + 10Q_2$$

Where Q_1 and Q_2 denote the quantities of good 1 and good 2 respectively. If P_1 and P_2 denote the corresponding prices, then the demand equations are

$$P_1 = 50 - 2Q_1 + Q_2 \qquad \text{and} \qquad P_2 = 30 + 2Q_1 - 4Q_2$$

(a) Determine the total revenue function $TR = P_1Q_1 + P_2Q_2$ in terms of Q_1 and Q_2 only.

(b) Find the profit function in terms of Q_1 and Q_2.

(c) Find the profit-maximising levels of Q_1 and Q_2.

(d) Find the maximum profit.

(e) Show that second-order conditions hold.

4. Repeat question 3 with the following difference: an amount t of the price received for each unit of good 1 is paid in tax to the government, and the firm maximises profit paid net of tax. Suppose t changes by a small amount dt. Find the changes in optimal Q_1 and Q_2. Find the change in maximum profit. Show that this change is equal to the optimal level of Q_1.

Exercise WS21.3

1. Solve each of the following second-order homogeneous difference equations. In each case, determine whether the solution is convergent or divergent, oscillatory or otherwise.

(a) $Y_t - 11Y_{t-1} + 10Y_{t-2} = 0$

(b) $9Y_t - Y_{t-2} = 0$

(c) $2Y_t - 2Y_{t-1} + Y_{t-2} = 0$

2 Find the general solution of each of the following second order non-homogeneous difference equations, and comment on their time paths. (Note that the equations in parts (a) – (c) have the same left hand sides as in question (1) above.)

(a) $Y_t - 11Y_{t-1} + 10Y_{t-2} = 4$

(b) $9Y_t - Y_{t-2} = 1$

(c) $Y_t - 10Y_{t-1} + 21Y_{t-2} = 8$

(d) $2Y_t - 2Y_{t-1} + Y_{t-2} = 6$

3. Find the solutions to parts (b) – (d) of question 2, subject to the given initial conditions:

   (b) $Y_0 = 0, \quad Y_1 = 1$

   (c) $Y_0 = \frac{1}{2}, \quad Y_1 = 1$

   (d) $Y_0 = 2, \quad Y_1 = 3$