

Exercise WS13.1

1. Find $\frac{dy}{dx}$ for each of the following functions, and in cases (a) and (b) draw a sketch graph of the function:

(a) $y = e^{3x}$

(b) $y = -\frac{1}{3}(\ln x)$

(c) $y = e^{x^2+3x}$

(d) $y = e^{0.5x^2+3x}$

2. The table below gives an index of UK labour productivity (defined as output per worker for the whole economy) for 1980 to 2003, with 2001 = 100.

- (a) Calculate the year-to-year growth of labour productivity for the years 1980-90 and 1997-2003.
- (b) Calculate the average annual growth rate of labour productivity for the periods (i) 1980-90; (ii) 1990-97; (iii) 1997-2003.

Year	Productivity index	Year	Productivity index
1980	64.4	1991	79.5
1981	65.8	1992	82.4
1982	68.6	1993	85.2
1983	71.3	1994	88.4
1984	71.3	1995	89.7
1985	73.1	1996	91.4
1986	75.6	1997	92.7
1987	77.3	1998	95.0
1988	78.2	1999	96.1
1989	77.6	2000	98.8
1990	77.9	2001	100.0

Chapter 13: Derivatives of exponential and logarithmic functions and their applications

Further student exercises

		2002	100.7
		2003	101.9
Source: ONS Time Series Data			

Exercise WS13.2

1. (a) A variable with an initial value of 200 is growing through time at a constant nominal rate of 5% per year. Write down the equation that describes this growth, assuming (i) continuous growth; (ii) growth in annual jumps.
- (b) If in (a) the actual growth of the variable is continuous, but we choose to model the growth as occurring in annual jumps, what is the resulting error in (i) our calculation of the effective annual growth rate; (ii) the level of the variable after 10 years?
2. (a) Demonstrate algebraically that if a variable, Y , is growing continuously at a constant (proportionate) rate, the graph of $\ln Y$ as a function of time will be linear, and with a slope equal to the growth rate of Y .
- (b) Illustrate (a) when Y has an initial value of 100 and a nominal growth rate of 2.5% per year.
- (c) Sketch the graph of $\ln Y$ as a function of time, if the growth rate is (i) increasing; (ii) decreasing.
3. The table below gives money supply data for the Democratic Republic of Agraria for 1980-2000, with 1980 = 100.
 - (a) Plot a reasonably accurate graph of the money supply time series. (Hint: You can format your graph in either of two ways: (i) by joining up the data points to form a continuous curve, which means you are treating the money supply as a continuous variable; or (ii) by creating a bar chart in which the height of each bar gives the money supply for the year in question, which means you are treating the money supply as a discrete variable that varies in annual jumps.) From inspection of the graph, try to assess whether the annual growth rate of the money supply is constant, increasing or decreasing through time.
 - (b) Repeat (a) above, but this time plotting the natural log of the money supply.

- (c) Explain in words (preferably supported by the relevant maths) why assessing the growth rate is easier in (b) than in (a).

Year	Money supply index	Year	Money supply index
1980	100.00	1991	1768.93
1981	140.00	1992	2087.34
1982	193.20	1993	2421.32
1983	262.75	1994	2760.30
1984	352.09	1995	3091.54
1985	464.76	1996	3400.69
1986	604.18	1997	3672.74
1987	773.35	1998	3893.11
1988	974.43	1999	4048.83
1989	1208.29	2000	4129.81
1990	1474.11		

4. Given the demand function $q = 100p^{-1.2}$
- (a) Find an expression for the price elasticity of demand.
- (b) Take natural logs on both sides and hence show that the function is "log-log linear".
- (c) Differentiate your expression in (b) above to find $\frac{d(\ln q)}{d(\ln p)}$
 (Hint: Just apply the normal rules of differentiation.)
 Why does it matter whether the exponent of p is greater or less than 1 in absolute value?
- Hence verify that $\frac{d(\ln q)}{d(\ln p)} = \frac{p}{q} \frac{dq}{dp}$ and explain this result.
- (d) Sketch the graph of the demand function, (i) with q and p on the axes, and (ii) with $\ln q$ and $\ln p$ on the axes. What presentational advantage does (ii) have over (i)?