

Exercise WS12.1

1. (a) Without using a calculator, make a rough estimate of the value of each of the following. Then check your answers using a calculator.
 - (i) e^2
 - (ii) $e^{0.5}$
 - (iii) $\ln 0.5$
- (b) Write down the inverses of each of the above.

2. (a) Sketch the graph of $y = e^x$ for values of x between -3 and $+3$. (Don't worry too much about the scale on the y axis).
- (b) Use your graph to estimate the values of e^2 , e^{-1} , $e^{0.5}$, $\ln 20$, and $\ln 0.5$.

3. Sketch the graph of $y = 100e^{0.05x}$.

Exercise W12.2

1. Repeat question 3 of exercise WS11.2, assuming now that traffic grows continuously rather than in annual jumps. By how much are your answers changed by this change in assumption about the growth process? Which assumption about growth seems more appropriate in analysing these data?

Exercise WS12.3

1. Between 1955 and 2003, the volume of UK exports of goods (in index number form, with 2001 = 100) grew from 13 to 97.8. (The source of these trade data is the ONS Time Series Data set.)
 - (a) Calculate the average annual growth rate, assuming (i) continuous growth; (ii) growth in annual jumps.
 - (b) Assuming continuous growth, sketch the graph of the average export volume, X , (also known as the trend value) as a function of time. Also sketch the graph of $\ln X$ against (= as a function of) time.

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2. The table below gives the price level, P , of the Democratic Republic of Ruritania, in index number form with 1980 = 100.

Year	Price index, P	Year	Price index, P
1980	100.00	1990	643.63
1981	125.00	1991	740.17
1982	155.00	1992	858.60
1983	190.65	1993	1004.56
1984	232.59	1994	1185.39
1985	281.44	1995	1410.61
1986	337.73	1996	1692.73
1987	401.89	1997	2048.20
1988	474.23	1998	2498.81
1989	554.85	1999	3073.54
		2000	3811.18

- Draw a reasonably accurate graph of P as a function of time, for 1980-2000
- From inspection of the graph, what can you infer about the inflation rate; that is, the growth rate of P ? Does it appear to be constant, increasing, or declining?
- Draw another reasonably accurate graph, this time taking the natural log of P as the dependent variable. (You can instead take the common log if you wish.)
- From inspection of this graph, what can you infer about the growth rate of P ? Does it appear to be constant, increasing, or declining?
- Use your calculator to check the actual year-to-year inflation rate, from the table. (See chapter 9.2 of the book if you need to review how to do this.)
- Hence explain briefly why we often use a logarithmic scale to graph a variable growing through time.

Hint: It doesn't really matter whether you treat time as a continuous or discrete variable (that is, whether you assume that the variable, P , grows continuously or in annual jumps).

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3. Find the present value of 1000 euros due in 10 years' time, discounted continuously at the nominal rate of 5% per year. Compare this with the present value if discounting occurs (a) once per year; (b) four times per year.

4. In the Kingdom of Valetudinaria, expenditure on government provision of health services is currently 5% of GDP. The government wishes to increase this ratio to 8%, the regional average, and accordingly plans to increase health spending by 6% per year until this target is achieved. If GDP grows at 2.5% per year, after how many years will the government's target be reached?