

### Exercise WS10.1

1. Find the 15th and 25th terms of the series: 3, 6, 9, 12, ..., ...
2. Find the sum of the first ten terms of the series:  $\frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}, \dots$
3. Find the sum to infinity of the series:  $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$
4. Given the following reduced form macroeconomic model:

$$Y = C + I$$

$$C = 0.8Y_{-1} + 100$$

where  $Y$  = national output (= national income);  $I$  = aggregate investment;  $C$  = aggregate consumption (all in the current time period);  $Y_{-1}$  = national income in the previous time period.

- (a) What condition must be satisfied for this economy to be in equilibrium? (This condition, in the form of an equation, is known as an equilibrium condition.)
- (b) Find the equilibrium levels of  $Y$  and  $C$ , if  $I = 900$ .
- (c) If  $I$  then rises to 1000 and remains at this level, trace the paths of  $C$  and  $Y$  over the next 6 time periods.
- (d) What are the levels of  $C$  and  $Y$  in the new equilibrium? How long does it take to reach this new equilibrium? How near to equilibrium is the economy after 12 time periods have passed?
- (e) Do you regard this model as reasonable in its assumptions? Give reasons for your answer.

### Exercise WS10.2

1. A firm wishes to increase its output from 1000 to 2000 units per week. Due to labour shortages and other supply side constraints, it can increase output by a maximum of 100 units per week. After 7 weeks, how near to its target will the level of output be?
2. Suppose in question 1 that the supply side constraints limited the growth of output to 10% per week. After 7 weeks, how near to its target will the level of output be? Explain the difference between your answers to questions 1 and 2.
3. The price level in the UK is currently increasing by about 2.5% per year. Show that if this inflation rate continues for 10 years, the price level will have

risen by 28%. (Hint: Give the initial price level an arbitrary value, such as 100.)

4. (a) If I deposit 500 euros in a bank account paying interest of 6% per year, how much will I get back after 10 years, including compounded interest? (Assume interest is added once per year.)
- (b) Repeat (a) above with the assumption that interest is added quarterly (that is, every 3 months). What is the effective annual interest rate (EAR) in this case?
- (c) Repeat (b) above, assuming that interest is added monthly.
5. I deposit 10,000 euros in a bank account that pays interest of 5% per year (credited annually in arrears; that is, at the end of each year). At the end of each year I withdraw and spend  $x$  percent of my bank balance at that time. After 10 years I find that I have only 100 euros left in the bank. What is  $x$ ?

**Exercise WS10.3**

1. Assuming that I can borrow or lend any amount at 10% per annum, with interest paid or received annually in arrears, what is the present discounted value (PV) to me of £10 000 due 20 years from now?
2. In the context of a British government policy entitled the Private Finance Initiative, the National Health Service can acquire a new hospital either by buying it now for a single payment of £100 million, or by instalments of £7 million per year for the next 30 years. If the government can borrow or lend at 6% per year, and all risks and uncertainty are ignored, which is the cheaper method of acquiring the hospital?
3. How is your answer to question 2 changed if the relevant discount rate falls from 6% to 5%? Explain your answer.
4. Calculate the present value (PV) of the following income series, discounted once-yearly at 4% per year.

Years fom now	1	2	3	4	5	6	7	8
Income series	200	200	150	150	100	100	50	50

5. (a) A 'perpetual' bond issued by the government pays interest of 10 euros per year, forever, the first payment due one year from now. Assuming that there is no risk that the government will default on these payments, what will be the market value of the bond if the market rate of interest on riskless loans is 5% per year?

- (b) If the market rate of interest doubled or halved, calculate the resulting percentage change in the market value of the bond. Derive the equation giving the relationship between the market rate of interest and the market value of the bond, and sketch its graph.
- (c) Repeat (a) with the assumption that the first payment of interest is due tomorrow. Compare and explain your two answers.

#### Exercise WS10.4

1. I borrow 10 000 euros to buy a car, the loan to be repaid in 60 equal monthly instalments consisting of interest and capital repayment. Interest will be charged every month at 1.5% of the outstanding debt. Calculate (a) the amount of each monthly instalment; (b) the total amount of interest that I will pay; (c) the effective annual interest rate (EAR).
2. (a) In question 1, by how much would my total interest payments be reduced if I increased my initial deposit by 1 000 euros and thus borrowed only 9 000 instead of 10 000 euros?  
  
(b) In question 1, what would be the EAR if the monthly rate rose to 2%? Does the EAR rise in the same proportion as the rise in the monthly rate?
3. Repeat question 5(a) from exercise WS10.3 assuming that the bond, instead of being perpetual, will be redeemed (= repurchased) at its nominal or face value of 100 euros by the government in 10 years' time. What would the bond be worth if it were due to be redeemed tomorrow?