Exercise WS9.1

1. Consider the supply function $q = 0.25p - 10$ (consider only positive values of p and q).

(a) Find the arc elasticity of supply when p increases (i) from 44 to 45; (ii) from 64 to 65.

(b) Find the point elasticity (i) when $p = 44$; (ii) when $p = 64$, and compare your answers with (a) above.

(c) Show that the point elasticity is always greater than 1, but approaches a limiting value of 1 as p approaches infinity.

(d) Sketch the graph of the supply function, showing how the elasticities may be measured geometrically.

2. Given the supply function $q = -0.1p^2 + 10p + 10$ (consider only $0 < p < 50$, since when $p > 50$ the function is negatively sloped, which of course is inappropriate for a supply function).

(a) Find the arc elasticity when p increases from (i) 5 to 6 and (ii) from 10 to 11, and compare your answers with the point elasticities when $p = 5$ and $p = 10$. Why are the arc and point elasticities so similar in this case?

(b) Sketch the graph of the supply function, showing how the elasticities may be measured geometrically. (It is necessary to restrict p to being less than 50, since the curve is negatively sloped when $p > 50$, which is inappropriate for a supply function).

Exercise WS9.2

1. Consider the demand function $q = -3p + 150$.

(a) Find the arc elasticity of demand when p increases (i) from 10 to 11; (ii) from 40 to 41.

(b) Find the point elasticity (i) when $p = 10$; (ii) when $p = 40$.

(c) Sketch the demand function, indicating the solutions to (a) and (b) above.

(d) Why is demand elastic when p is high and inelastic when p is low, in this example?

2. Given the demand function $q = -3p^2 + 2p + 1965$.

(a) Find the arc elasticity of demand when *p* increases (i) from 10 to 11; (ii) from 24 to 25.

(b) Find the point elasticity (i) when $p = 10$; (ii) when $p = 24$.

(c) Sketch the demand function, indicating the solutions to (a) and (b) above.

(d) For what <u>mathematical</u> reason does demand become more elastic (that is, E^D decreases or $| E^D |$ increases) as *p* increases, in this example?

3. Consider the inverse demand function $p = \frac{30.625}{q+2} - 5$.

(a) Sketch the inverse demand function. Show that the intercepts are $p = 10.3125$ and (approx.) $q = 4.15$.

(b) Find the demand function (that is, the inverse of the inverse demand function).

(c) Find the point elasticity of demand (either as a function of *p* or as a function of *q*) and show that demand is elastic when $p > 3.75$ and inelastic when $p < 3.75$.

Exercise WS9.3.

These questions are a continuation of exercise WS9.2.

1. Using the demand function from exercise WS9.2, question 1; that is, $q = -3p + 150$.

(a) Find the total revenue and marginal revenue functions (as functions of *q*).

(b) Find the price and quantity, p^* and q^*, at which total revenue is at its maximum.

(c) Show that, when $p = p^*$ and $q = q^*$, the demand elasticity is unity.

(d) Show that, when $p > p^*$ and $q < q^*$, demand is elastic (that is, $E^D < -1$ or $| E^D | > 1$).

(e) Similarly show that, when $p < p^*$ and $q > q^*$, demand is inelastic (that is, $E^D > -1$ or $| E^D | < 1$).

(f) Sketch the graphs of the inverse demand function, total revenue and marginal revenue functions.

(g) Explain, using words and diagrams only, the relationship between marginal revenue and the elasticity of demand.

2. Consider the demand function from exercise WS9.2, question 2; that is, $q = -3p^2 + 2p + 1965$.

(a) Find total revenue as a function of *p*. (Hint: Don't attempt to find total revenue as a function of *q*, as to do so requires use of the inverse demand function which is difficult to find in this case.)

(b) Show that when $-9p^2 + 4p + 1965 = 0$, total revenue is at its maximum. Find the price and quantity which maximise total revenue. (Ignore the negative solution to this quadratic equation.)

(c) Show that the demand elasticity, as a function of *p*, is given by $E^D = \frac{-6p^2+2p}{-3p^2+2p+1965}$. Show that when demand has unit elasticity (that is, $E^D = -1$ or $|E^D| = 1$), total revenue is at its maximum.

(d) For what values of *p* is demand (i) elastic; (ii) inelastic? What is the effect of a small price reduction on total revenue, when demand is (i) elastic; (ii) inelastic?

(e) Sketch the graphs of the demand function, total revenue and marginal revenue functions.

(f) Explain, using words and diagrams only, the relationship between marginal revenue and the elasticity of demand.

3. Consider the inverse demand function $p = \frac{30.625}{q+2} - 5$ from exercise WS9.2, question 3.

(a) Find total revenue as a function of *q*.

(b) Find marginal revenue as a function of *q*. Use this equation to find the price and quantity which maximise total revenue. (Ignore the negative solution to this quadratic equation.)

(c) Show that the demand elasticity, as a function of *q*, is given by $E^D = \frac{-(q+2)(20.625-5q)}{30.625q}$. Show that when demand has unit elasticity (that is, $E^D = -1$ or $|E^D| = 1$), total revenue is at its maximum.

(d) For what values of p is demand (i) elastic; (ii) inelastic? What is the effect of a small price reduction on total revenue, when demand is (i) elastic; (ii) inelastic?

(e) Sketch the graphs of the inverse demand function, total revenue and marginal revenue functions.

(f) Explain, using words and diagrams only, the relationship between marginal revenue and the elasticity of demand.

Exercise WS9.4

1. Consider the total cost function $TC = 3q^2 - q + 4800$.

(a) Find marginal cost (MC) and average cost (AC) as functions of q.

(b) Find the output at which average cost is minimised, and show that $MC = AC$ at this output.

(c) Show that when $MC < AC$, AC is falling; and when $MC > AC$, AC is rising.

(d) Find the elasticity of total cost, E^{TC}, with respect to output, and show that it equals $\frac{MC}{AC}$.

(e) Show that when $E^{TC} < 1$, AC is falling; and when $E^{TC} > 1$, AC is rising.

(f) Illustrate this graphically.

2. Consider the non-linear aggregate consumption function $C = 0.05Y^2 + Y + 80$ (where C = aggregate consumption and Y = aggregate income).

(a) Find the marginal propensity to consume (MPC) and the average propensity to consume (APC).

(b) What is the relationship between MPC and APC in this case?

(c) What happens to the MPC as Y becomes larger and larger? Is it likely that an aggregate consumption function would have this functional form?

(d) Find the elasticity of the consumption function and show that it equals the MPC divided by the APC.

(e) Illustrate your solutions to (b) and (c) graphically, showing the consumption function, the MPC and APC.