

### Exercise WS8.1

1.
  - (a) Sketch the graph of the short run total cost function  $TC = 0.25q + 20$ , for  $q = 0$  to  $q = 50$ .
  - (b) Find the marginal and average cost functions. Sketch their graphs.
  - (c) Explain in words why average cost is greater than marginal cost at all levels of output.
  - (d) How does the difference between marginal and average cost,  $MC - AC$ , behave as  $q$  increases without limit?
  
2.
  - (a) Sketch the graph of the short run total cost function  $TC = 3q^2 + 5q + 300$ , for  $q = 0$  to  $q = 15$ .
  - (b) Find the marginal and average cost functions.
  - (c) Show that  $AC$  is at its minimum when  $q = 10$ , and that  $MC = AC$  at this output.
  - (d) Sketch the graphs of the  $MC$  and  $AC$  functions, on the same axes.
  
3. Consider the short run total cost function  $TC = 2.2q^3 - 16q^2 + 48q + 150$ 
  - (a) Show that average cost is at its minimum when  $q = 5$ .
  - (b) Find the output at which marginal cost is at its minimum.
  - (c) Show that  $MC = AC$  when  $AC$  is at its minimum.
  - (d) Sketch the graphs of the  $MC$  and  $AC$  functions, on the same axes.
  - (e) Use the information obtained above to sketch the graph of the  $TC$  function.
  - (f) What economic interpretation can we give to the point of minimum  $MC$ ?

### Exercise WS8.2

1. The inverse demand function for a good is  $p = -0.1q + 50$ 
  - (a) Find the total revenue function and the marginal revenue functions.
  - (b) Find the price and quantity at which total revenue is maximised.

- (c) Sketch the graphs of the inverse demand function and the marginal revenue functions on the same axes, indicating your solution on the graph. Link these graphs to the graph of the total revenue function.
2. Consider the demand function  $q = -3p^2 + 324$
- (a) Find the total and marginal revenue functions.
- (b) Find the price and quantity at which  $TR$  is at its maximum. (Hint: It's easier to start by finding the price rather than the quantity at which  $TR$  is maximised).
- (c) Sketch the graph of the demand function, the  $MR$  and the  $TR$  functions (the last is a little difficult!)
3. The inverse demand function for a good is  $p = \frac{54}{q+3} - 2$
- (a) Sketch the inverse demand function, indicating clearly the intercepts on the  $p$  and  $q$  axes (if they exist) and/or the asymptotes (if they exist). (Hint: You should recognise this function as a rectangular hyperbola.)
- (b) Find the total and marginal revenue functions, and the values of  $p$  and  $q$  that maximise total revenue.
- (c) Sketch the total and marginal revenue functions, indicating clearly the relationship between the two and their relationship with the inverse demand function.
4. "As output varies, marginal revenue can remain constant only if marginal revenue and price are equal."
- (a) State the conditions (if they exist) under which this statement is true, and give a proof (if true).
- (b) Can you find and prove a counter-example; that is, assumptions under which marginal revenue is constant as output varies, yet is not equal to price?

### Exercise WS8.3

1. A firm's short run total cost function is  $TC = 0.5q^2 + q + 500$ . The firm sells in a perfectly competitive market and the ruling price is  $p = 61$ .

- (a) Find the most profitable level of output, and the profits at that output.
- (b) Does the firm produce at minimum average cost? Explain your answer.
- (c) Sketch the graphs of total cost and total revenue with the same axes, and do the same with marginal cost and marginal revenue. Sketch the graph of the profit function. Indicate in all diagrams the equilibrium values of the variables.
2. Continuing with the firm in question 1,
- (a) Suppose the ruling market price drops to 45 (due to intensified competition, perhaps). What is the firm's response to this price reduction?
- (b) Show the new equilibrium on your graphs from question 1.
3. Continuing with question 2,
- (a) Suppose the price falls still further, to 33. What are the new equilibrium output and profits?
- (b) What price is so low that the firm would choose to cease production immediately? (Hint: what are the firm's profits if it produces nothing?)
4. Continuing with the firm in question 1, where the ruling market price is  $p = 61$ ,
- (a) Suppose the firm's short run  $TC$  function shifts to  $TC = 1.5q^2 + 10q + 500$ . Find the new equilibrium level of output and profits, and compare them with their values in question 1.
- (b) Illustrate the new equilibrium on the graphs that you drew in question 1.
5. A firm's short run total cost function is  $TC = 0.75q^2 + 8q + 300$ . The firm is a monopolist and the inverse demand function for its product is  $p = 201 - 2q$ .
- (a) Find the most profitable level of output, and the profits at that output.
- (b) When maximising profit, does the firm produce at minimum average cost?
- (c) Find the break-even points.

Chapter 8: Economic applications of functions and derivatives  
Further student exercises

- (d) Sketch the graphs of the inverse demand function, marginal revenue and marginal cost function with the same axes, showing the equilibrium price,  $MR$  and  $MC$ , and output.
- (e) Sketch the graphs of total cost and total revenue with the same axes, showing the equilibrium output and profits. Sketch the graph of the profit function, showing the equilibrium output and profits. Show the break-even points on both graphs.