

Exercise WS7.1

By examining the sign of $\frac{dy}{dx}$, show that:

1. $y = 3x - 4$ is monotonic increasing
2. $y = x^2 + 4$ is increasing when $x > 0$ and decreasing when $x < 0$
3. $y = x^3$ is increasing provided $x \neq 0$

Exercise WS7.2

Find any local maximum and/or minimum value(s) of the following functions. In each case, draw a sketch graph of the function showing these points.

1. $y = -x^2 + 6x - 12$
2. $y = 3x^2 - 6x - 5$
3. $y = -2x^3 + 3x^2 + 36x - 5$

Exercise WS7.3

Find the stationary points (SP) of each of the following functions, and determine whether each SP is a local maximum, local minimum or point of inflection. In each case, draw a sketch graph of the function showing the SPs.

1. $y = 2x^3 - 27x^2 + 84x + 10$
2. $y = 2x^3 + 21x^2 + 60x + 30$
3. $y = x^3 + 9x^2 + 15x + 7$

Exercise WS7.4

Find all local maximum and minimum points, and points of inflection, in the following functions. Use your findings to draw a sketch graph of each function. (Hint: Don't forget that there must be a point of inflection between a maximum and minimum value.)

1. $y = \frac{1}{3}x^3 - 2\frac{1}{4}x^2 - 5\frac{1}{2}x + 50$

2. $y = 4x^3 - 3x + 10$

3. $y = x^3 + 15x^2 - 225x$

Exercise WS7.5

1. For each of the following functions, find the slope (positive or negative) and the curvature (convex or concave) for the range of values of x specified. Draw a rough sketch of each function for the specified range of values.

(a) $y = x^2 - 6x + 12, \quad x < 3$

(b) $y = 2x^3 + 3x^2 - 36x + 25, \quad \text{for } -0.5 < x < 3$

(c) $y = -2x^3 - 12x^2 - 24x + 10, \quad x > -2 \text{ and } x < -2$

2. Show that the function $y = x^3 + x$ is always positively sloped and has a point of inflection (which is not a stationary point) at $x = 0$.
3. Given $y = 2x^2 - 6x + 15$, suppose x increases from -10 to -9 . Find the absolute and percentage error that result if we approximate the resulting change in y using the differential formula $dy = \frac{dy}{dx}dx$.