Exercise WS3.1

1. I am planning to buy a new car which will have either a petrol or diesel engine. The petrol model averages 35 miles per gallon of fuel. The diesel version averages 50 miles per gallon but its purchase price is £1000 higher. Petrol and diesel fuel both cost £3.60 per gallon.

(a) Draw a sketch graph showing fuel costs as a function of miles travelled for the two types of car.

(b) Assuming I buy the diesel car, calculate how many miles I must drive before I have recovered its higher initial cost, and show this point on your graph.

2. I have a choice of two mobile phone tariffs. (A 'tariff' means a charging or payment scheme.) Under tariff A I simply pay 12 pence per minute for every call I make. Under tariff B I pay £30 per month. The first 300 minutes of calls per month are then free of charge, and minutes over 300 per month are charged at 15 pence per minute.

(a) Write down two equations; (i) the total cost (C_{A}) per month of calls under tariff A as a function of the number of minutes of calls in the month; and (ii) the total cost (C_{B}) under tariff B as a function of the number of minutes of calls in the month. (Hint: In case (ii) you will need to distinguish between the sub-case in which I make more than 300 minutes of calls per month, and the sub-case in which I make 300 or fewer minutes of calls per month.)

(b) Assuming I make more than 300 minutes of calls per month, what is the number of monthly minutes of calls, *x*, such that the total cost would be the same under both tariff A and tariff B?

(c) In (b), if I make more than *x* minutes of calls per month, should I choose tariff A or B?

(d) If I make fewer than 300 minutes of calls per month, should I choose A or B?

(e) For what range of values of *x* is tariff B cheaper than A?

(f) Illustrate your answers graphically.

3. Find *y* in terms of the other variables and parameters, given:

(a) $y + 2 = 4x$

(b) $ax = c(y + b)$

(c) $\frac{x+1}{2} = y + 3$

(d) $\frac{10}{x} = 4y - \frac{1}{4}$

(e) $y + 2 = \frac{10}{x+3}$

(f) $x = \frac{1}{y} + \frac{1}{2}x + 2$

4. Solve the following equations for x. (In case (b) the values of the parameters are unspecified, so you will not be able to find a numeric solution.)

(a) $4x + 3 = 0$

(b) $0 = ax + b$

(c) $\frac{x}{5} = 12$

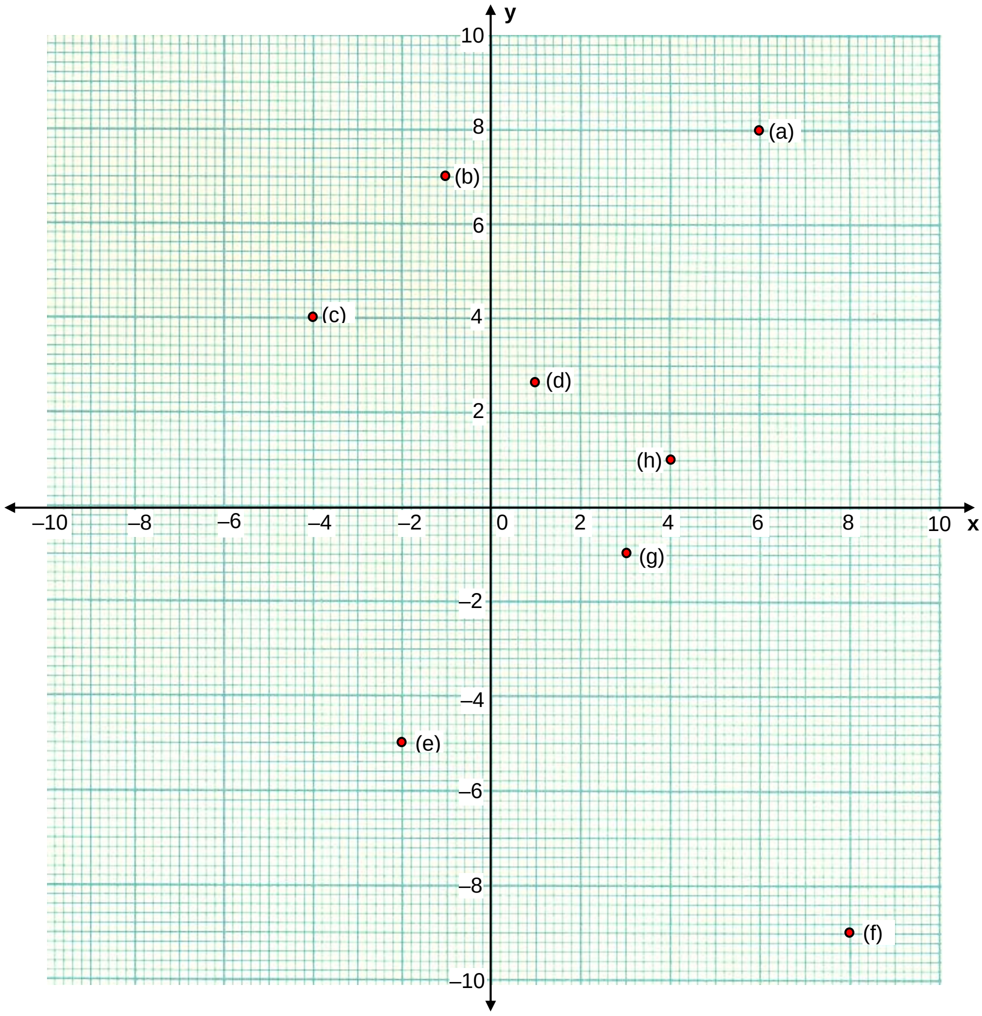
(d) $x + \frac{1}{10}x = -9$

(e) $\frac{x+5}{x-4} = 5$

(f) $\frac{1}{4}x + 3 = 8 + x$

Exercise WS3.2

1. Write down the coordinates of points (a) to (h) in the figure below.



2. On a sheet of graph paper draw the x and y axes and choose an appropriate scale. Then mark the points in the x, y plane with the following coordinates:

(a) (2, 3)

(b) (5, –½)

(c) (–3, 4)

(d) (9, 0)

(e) (0, –3)

(f) (–4, –8)

(g) (10, –10)

(h) (–0.5, 0.25)

3. For each of the following implicit functions, express *y* as an explicit function of *x*, and plot its graph:

(a) $3x - y = 4$

(b) $\frac{1}{3}x - \frac{1}{12}y = 4$

(c) $0.1y = 3x + 2$

(d) $x = 3y + 4$

4. Using your answers to question 3 above, find from the relevant graphs approximate solutions to the following equations. Then check your answers by finding the algebraic solutions.

(a) $3x - 4 = 0$

(b) $4x - 48 = 0$

(c) $30x + 20 = 0$

(d) $\frac{1}{3}x - \frac{4}{3} = 0$

5. For each of the following, write down the slope and the y intercept, and calculate the x intercept:

(a) $y = 3x - \frac{1}{2}$

(b) $y = -20x + 120$

(c) $y = 0.125x - 100$

Exercise WS3.3

1. Solve the following pairs of linear simultaneous equations by algebraic methods. In each case, sketch the graphs and thus verify your answers graphically.

(a) $y = -x + 3,\ y = 2x - 3$

(b) $y = -8x,\ y = 3x - 11$

(c) $y = 5x + 2,\ y = -0.5x - 4.5$

Exercise WS3.4

1. Given the following supply and demand functions, find the equilibrium price and quantity. In each case, show the solution graphically.

(a) $q^{D} = -0.5p + 40,\ q^{S} = 0.75p - 15;$ for values of p between 0 and 60

(b) $q^{D} = -3p + 400,\ q^{S} = 2p - 100;$ for values of p between 0 and 200

(c) $q^{D} = -0.5p + 40,\ q^{S} = 3p - 16;$ for values of p between 0 and 60

2. Find the equilibrium price and quantity, given the inverse demand and supply functions:

$$p = -\tfrac{3}{2}q^{D} + 36;\ p = 3q^{S} + 9$$

3. Given the following supply and demand functions:

$q^D = -0.5p + 20; \; q^S = 0.75p - 15$

(a) Find the equilibrium price and quantity

(b) Find the inverse demand and supply functions, and verify that solving these simultaneously gives the same price and quantity.

(c) Illustrate (a) and (b) graphically.

4. Repeat question 3 for the following inverse supply and demand functions:

$p^{\mathrm{D}} = -3q + 40; \; p^{\mathrm{S}} = 2q + 10$

5. Given the following macroeconomic model:

$Y \equiv E$	(aggregate income/expenditure identity)
$E \equiv C + I$	(components of expenditure)
$\hat{C} = aY + 100$	(consumption function; behavioural relationship)
$I = \bar{I} = 900$	(exogenous investment)
$C = \hat{C}$	(equilibrium condition)

where $a = 0.9$, $\bar{I} = 500$, and the other variables are as defined in chapter 3.

(a) Find the equilibrium levels of income and consumption, and illustrate diagrammatically.

(b) Suppose consumers become more pessimistic about the future and decide to save 50 more at every level of income. Find the new level of *Y,* the change in *Y*, the change in *C,* and the multiplier in this case. (Hint: an increase in savings implies a decrease in consumption.)

(c) By how much would investment need to rise to restore *Y* to its previous level?

6. Given the following macroeconomic model (where the variables are as defined in chapter 3):

$Y \equiv E$	(aggregate income/expenditure identity)

$E \equiv C + I$ (components of expenditure)

$\hat{C} = 0.8Y + 200$ (consumption function; behavioural relationship)

$I = 1800$ (exogenous investment)

$C = \hat{C}$ (equilibrium condition)

(a) Find the equilibrium levels of income and consumption, and illustrate diagrammatically.

(b) Suppose the government now levies a tax, T, on household income, thereby reducing household income to $Y - T$. When the government spends the tax revenue this becomes a new source of aggregate expenditure or demand, G, with $G \equiv T$. Our model then becomes

$Y \equiv E$

$E \equiv C + I + G$

$\hat{C} = 0.8(Y - T) + 200$

$I = 1800$

$C = \hat{C}$

$G \equiv T$

(i) Find the new equilibrium level of income, if $T = 500$.

(ii) Find the new equilibrium level of consumption.

(iii) Does it seem surprising that the imposition of an income tax increases the level of aggregate income and leaves aggregate consumption unchanged? Can you explain why this happens?

Note that $Y - T$, household income after income tax has been deducted, is often referred to as *disposable income*.