2

Population growth and decline

> Adam and Eve in 5610 years might have, by the ordinary proportion of Procreation, begotten more people, then are now probably upon the face of the earth.
>
> (Graunt 1662: 14)

OUTLINE

2.1 Concepts of growth
2.2 Analysing growth
2.3 Geometric growth
2.4 Exponential growth
2.5 Growth and replacement
2.6 Conclusion
Study resources

LEARNING OBJECTIVES

To understand:

- the main concepts used in the study of population growth
- basic approaches to the study of population growth
- applications of the geometric growth formula
- applications of the exponential growth formula
- the nature of processes of population turnover and replacement

COMPUTER APPLICATIONS

Excel modules:	Population Clocks.xls (Box 2.2) Growth.xls (Box 2.3)
Spreadsheet exercise 2:	Geometric growth rates (See Study resources)

The methods of analysing changes in population numbers are applicable to all scales of demographic investigation, from local to global. Applying them, and interpreting the results, calls for some acquaintance with the concepts upon which they are founded, such as geometric and exponential change. Understanding the concepts also facilitates informed reading of the literature on population trends, including authors' arguments about the nature of world population growth and its probable future.

In the twentieth century, dire warnings and apocalyptic forecasts for the planet followed from the assumption that a particular pattern of increase, such as exponential growth, would persist in the future. The twentieth century also brought forth national population decline, or at least a fear of it, as a recurring theme in some more developed countries. The continuation of low birth rates seems destined to give decline greater prominence during the twenty-first century. Assumptions about constant rates leading to extinction, however, will be as untenable as those about constant rates creating ever burgeoning numbers.

This chapter begins with a summary of the principal concepts of growth. Their main uses are in the everyday analysis of population change at local, regional and national scales. After introducing the concepts, much of the chapter is concerned with the principal measures of growth and decline, together with their applications.

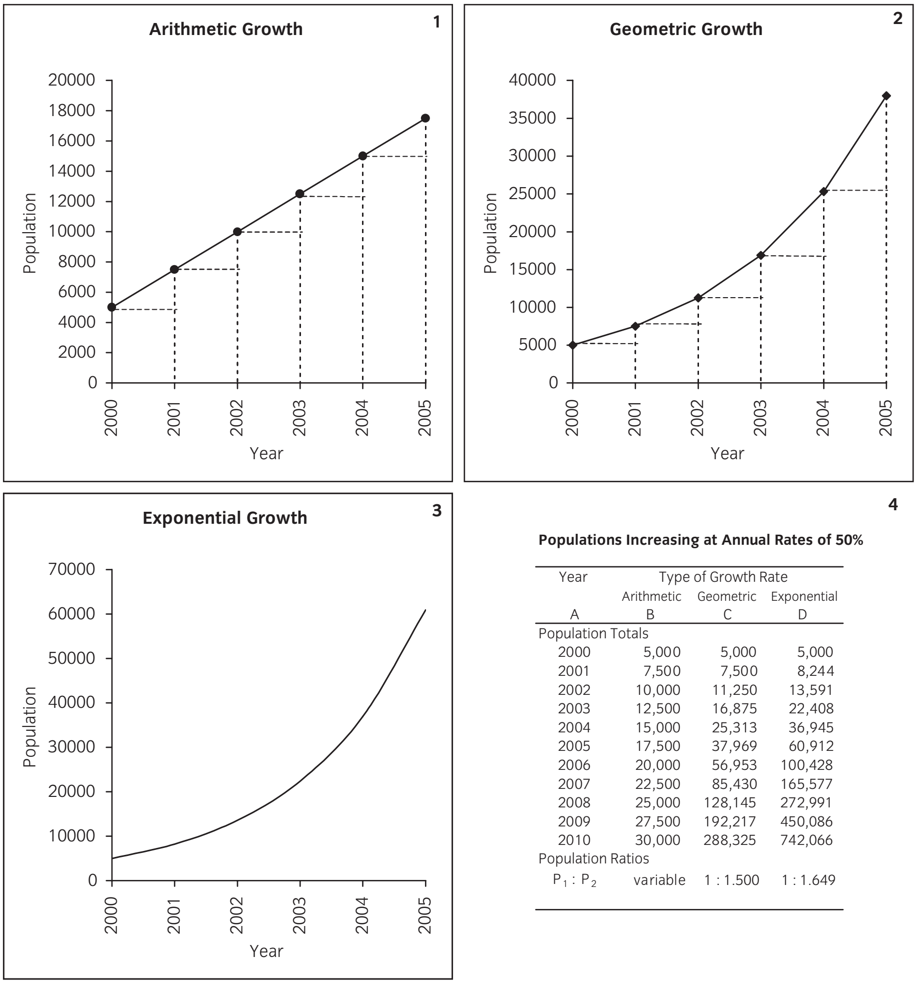
2.1 Concepts of growth

Since the late eighteenth century, various concepts have had their heyday in the writings of the prophets of population. Each has different implications for the nature of the relationship between population size, resource use and environmental impacts. The concepts are mathematically based and authors have employed them to make predictions through extrapolating past developments. This assumes that the concepts actually have theoretical significance: if past trends in population growth conformed to a mathematical principle there would be solid justification for making predictions on the basis of that principle. Unfortunately, contemporary population growth shows no such conformity. The ensuing discussion briefly introduces the best-known mathematical patterns of growth: arithmetic, geometric, exponential and logistic. Particular authors have seen all but the first of these as representing an underlying principle of population growth. The computer modules for this chapter – *Population Clocks.xls* and *Growth.xls* – illustrate the divergence resulting from different types and rates of growth. *Growth.xls* also calculates population sizes and rates from entered data, while *Population Clocks.xls* – which displays functioning population clocks – provides the option of viewing long-run outcomes in accelerated time.

Arithmetic growth

A population growing arithmetically would increase by a constant number of people in each period. If a population of 5000 grows by 100 annually, its size over successive years will be: 5100, 5200, 5300, . . . Since the annual increases are the same, these numbers form an arithmetic progression or series; the size of the population in the example is also increasing at a constant *arithmetic rate* of 2 per cent (100/5000 = 0.02 or 2 per cent). Arithmetic growth is analogous to 'simple interest', whereby interest is paid only on the initial sum deposited, the principal, rather than on accumulating savings. Five per cent simple interest on $100 merely returns a constant $5 interest every year.

Arithmetic change produces a linear trend in population growth – following a straight line rather than a curve. This is illustrated in Figure 2.1 (panel 1) where the



Populations Increasing at Annual Rates of 50%

Year	Type of Growth Rate		
	Arithmetic	Geometric	Exponential
A	B	C	D
Population Totals			
2000	5,000	5,000	5,000
2001	7,500	7,500	8,244
2002	10,000	11,250	13,591
2003	12,500	16,875	22,408
2004	15,000	25,313	36,945
2005	17,500	37,969	60,912
2006	20,000	56,953	100,428
2007	22,500	85,430	165,577
2008	25,000	128,145	272,991
2009	27,500	192,217	450,086
2010	30,000	288,325	742,066
Population Ratios			
$P_1 : P_2$	variable	1 : 1.500	1 : 1.649

Figure 2.1 Comparison of arithmetic, geometric and exponential growth.

dotted lines show the constant increases added at the end of every year: the increments or steps are of equal size. Figure 2.1 depicts unusually high annual growth rates of 50 per cent in order to illustrate contrasting outcomes from arithmetic, geometric and exponential growth in a short time span. Normally, these differences are evident only in the long term, and may be minimal over a five year period.

Real examples of arithmetic population growth are uncommon and necessarily limited to settings where administrators have the option of establishing targets to increase or decrease numbers by a constant amount. For instance during a phase of expansion, budgets could support a linear increase in numbers – such as of employees in an organization, enrolments at a university or soldiers in an army. Despite its limitations for describing actual population changes, arithmetic growth is the basis for a widely used measure in demography, namely *average annual increase*. It was also an important concept in Malthus's (1798) population theory (Figure 2.2).

AN

ESSAY

ON THE

PRINCIPLE OF POPULATION,

AS IT AFFECTS

THE FUTURE IMPROVEMENT OF SOCIETY

WITH REMARKS

ON THE SPECULATIONS OF MR. GODWIN,

M. CONDORCET

AND OTHER WRITERS

LONDON

PRINTED FOR J. JOHNSON, IN ST. PAUL'S

CHURCHYARD

1798

Figure 2.2 Title page text from the first and second editions of Malthus's Essay.

First edition (published anonymously 1798).
Second edition 1803.
Source: Castles and Miller (1998: 4).

AN

ESSAY

ON THE

PRINCIPLE OF POPULATION;

A VIEW ON ITS PAST AND PRESENT EFFECTS

ON

HUMAN HAPPINESS;

WITH AN INQUIRY INTO OUR PROSPECTS RESPECTING THE FUTURE REMOVAL

OR MITIGATION OF THE EVILS WHICH IT OCCASSIONS.

A NEW EDITION, VERY MUCH ENLARGED

BY T.R. MALTHUS, A.M

FELLOW OF JESUS COLLEGE, CAMBRIDGE

LONDON

PRINTED FOR J. JOHNSON, IN ST. PAUL'S CHURCHYARD,

BY T. BENSLEY, HOLT COURT, FLEET STREET,

1803

Figure 2.2 *continued*

Thomas Robert Malthus (1766–1834), an English clergyman, and professor of history and political economy, is the best known of demography's pioneers. His *Essay on the Principle of Population* (1798) is one of the most important and influential books ever written on the subject of human numbers (Grebenik 1989: 1); it was the first to stimulate interest in the relationship between population and resources (Keyfitz 1972). Malthus wrote to oppose prevailing ideas about the benefits of population growth. His 'gloomy thesis' was that population growth threatened prosperity because it inevitably outran increases in food supplies. He reasoned that, at best, agricultural production grew arithmetically, by the same amount every 25 years, while population grew geometrically, doubling every 25 years (Malthus 1798: 16). Thus the implications of his 'principle of population' arose from the differences between arithmetic and geometric growth.

Geometric growth

Geometric growth quickly leads to greater numbers. Comparison of panels 1 and 2 in Figure 2.1 indicates that, whereas arithmetic growth entails constant increments, geometric growth entails ever larger increments. In geometric growth, population increments become larger because increases are self-reinforcing. This is illustrated in developing countries with high birth rates, where expanding generations of children eventually become expanding new generations of parents who, in turn, beget even larger generations of children.

Geometric population growth is analogous to the growth of a bank balance receiving compound interest. According to this, the interest is calculated each year with reference to the principal plus previous interest payments, thereby yielding a far greater return over time than simple interest. The geometric growth rate in demography is calculated using the 'compound interest formula'.

Under arithmetic growth, successive population totals differ from one another by a constant amount. Under geometric growth they differ by a constant ratio (Table 2.1). In other words, the population totals for successive years form a geometric progression in which the ratio of adjacent totals remains constant. In panel 4 of Figure 2.1, which presents the data for the charts, this ratio is 1 : 1.50. In the example, successive populations are 50 per cent larger, because the annual growth rate is 50 per cent.

The main problem with describing population growth as a geometric progression is that actual populations seldom increase at constant rates. Populations typically have different (geometric) growth rates from year to year. Nevertheless, the misleading assumption of population growth as tending towards a geometric

Table 2.1 Summary of concepts of population growth

Type of growth	Description of trend	Growth rates	Absolute increments	Ratio of adjacent populations
Arithmetic	growth through constant increments at constant intervals	constant	constant	changing
Geometric	growth compounding at constant intervals	constant	changing	constant
Exponential	growth compounding continuously	constant	changing	constant
Logistic	growth rates changing in relation to population size	changing	changing	changing

progression is important in Malthus's 'principle of population'. The same assumption is present also in some contemporary writing – where recent growth rates are held constant to show extreme outcomes. The imbalance resulting from the assumed arithmetic expansion of food production and geometric expansion of human numbers led Malthus to conclude that populations tend to outgrow their resources. His 'principle of population' was that human numbers ultimately press upon the available means of subsistence.

The notion of diverging trends from arithmetic and geometric growth proved untenable as a description of the course of change in the relationship between population and food supplies. Population growth does not conform to a geometric progression, nor do populations actually grow geometrically, adding increments at constant intervals. In small populations, changes occur at irregular intervals depending on the timing of births, deaths and migrations. In larger populations, changes may occur almost continuously – not just at yearly intervals. Recognition of this led to a focus on exponential growth, which more accurately describes the continuous and cumulative nature of population growth.

Exponential growth

Exponential growth refers to the situation where growth compounds continuously – at every instant of time. Accordingly, it is sometimes called 'instantaneous growth' (Krebs 1978). Geometric growth is a special case of exponential growth (Pressat 1985: 74), because compounding occurs at intervals much longer than an instant. The shorter the interval over which increments occur, the faster the population increases – just as the balance in a bank account with daily interest grows more quickly than one with yearly interest.

Figure 2.1 (panel 3) shows that exponential growth produces a smooth curve with no steps between increments, because change is continuous. By 2010, exponential growth at an annual rate of 50 per cent results in a total of nearly three-quarters of a million, compared with 288 325 from geometric growth – compounding annually – and 30 000 from arithmetic growth. As in geometric growth, the ratio between adjacent populations is constant: 5000 : 8244, and 8244 : 13 591, for example, are equivalent to ratios of 1 : 1.649. In other words, numbers increase by about 65 per cent annually. Continuous compounding explains why a 50 per cent annual growth rate actually creates a 65 per cent increase (see Box 2.1).

Exponential growth was prominent in one of the late twentieth century statements of a mathematical principle of population growth, namely the book *The Limits to Growth* (Meadows et al. 1974, first published 1972) and its sequel *Beyond the Limits* (Meadows et al. 1992). Whereas Malthus believed that populations tend to grow geometrically, the authors of the *Limits to Growth* argued that the world's population is growing exponentially, and consumption of resources is also increasing exponentially (Meadows et al. 1974: 25). They went further in saying that the world's population is really increasing 'super-exponentially',

BOX 2.1 **A paradox of growth**

A seeming paradox is that a 100 per cent per annum exponential growth rate produces more than a 100 per cent increase in a year. The reason is that growth would amount to 100 per cent over a single interval of a year but, with exponential growth, there is an infinite number of intervals over which growth is compounding. For example, if a 100 per cent annual increase on an initial population of 100 was split between two intervals of six months, the rate for each interval would be 50 per cent ($r = 0.5$). Using the geometric growth formula, the population would be 150 (100×1.5) at mid-year and 225 (100×1.5^2) at the end of the year. Similarly, calculating the increments quarterly (i.e. 25 per cent growth per quarter) the total population would be 244 (100×1.25^4) at the end of the year. Thus the greater the number of points at which increments are added, the greater the growth. Exponential growth yields higher outcomes for the same reasons that daily interest payments cause a bank balance to grow faster than monthly or yearly payments (see Krebs 1978).

by which they meant that far from being constant, growth rates have increased through time such that 'the population curve is rising even faster than it would if growth were strictly exponential' (Meadows et al. 1974: 34).

To explain the implications of exponential growth for population numbers the authors quoted the following story (Meadows et al. 1974: 29; 1992: 18):

> There is an old Persian legend about a clever courtier who presented a beautiful chessboard to his king and requested that the king give him in return 1 grain of rice for the first square on the board, 2 grains of rice for the second square, 4 grains for the third, and so forth. The king readily agreed and ordered rice to be brought from his stores. By the fortieth square a million million rice grains had to be brought from the storerooms. The king's entire rice supply was exhausted long before he reached the sixty-fourth square. Exponential increase is deceptive because it generates immense numbers very quickly.

This story, however, is misleading when applied to population growth. It implies that numbers tend to keep doubling at short intervals indefinitely. Population growth does not occur in this way, since the rates seldom remain constant for long. Also, national population doublings normally take decades rather than years. The world's population growth rate has never exceeded 2.1 per cent (doubling time 33 years), and the growth rate has been falling since the mid-1960s (Figure 2.3); in 2001 it was estimated to be 1.3 per cent (Population Reference Bureau 2001b).

Calculating exponential growth rates gives due recognition to the continuous nature of growth in large populations in a period of time, but it is unrealistic to assume that the rates form an exponential series. In the United States and Britain, the growth rates of their populations have varied greatly over time (Figure 2.3): there is no evidence to support the notion that predictions of their future populations can assume a constant exponential growth rate. Vastly different predictions result

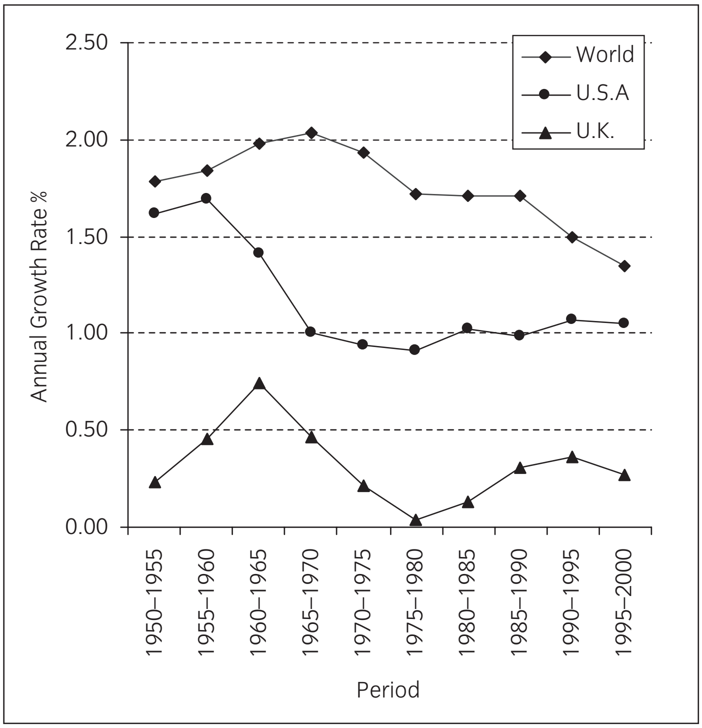


Figure 2.3 Population growth rates, 1950–2000: World, USA and UK.

Source: United Nations 2001. *World Population Prospects: the 2000 Revision* (CD).

according to the date and the rate selected as the starting point. Thus the main problem in describing population growth as an exponential series is the assumption that the rates remain constant through time.

Constant rates of decline raise the same problems as constant rates of growth. A global population extinction scenario, based on contemporary negative growth rates for Germany, shows the human race disappearing by 2400, and the population of the more developed countries vanishing about 150 years earlier. This scenario requires a constant exponential growth rate of –2 per cent to emerge in the twenty-first century (Bourgeois-Pichat 1989: 71ff). Yet declining populations may be expected to experience varying demographic rates as a result of unpredictable shifts in fertility and migration, together with variations in the size of *birth cohorts* – groups born in the same period – reaching different stages of life through time.

Despite these criticisms, constant exponential growth is consistent with constant birth and death rates; hence, the assumption that population growth forms an exponential series has had important applications in demography, especially in the development of stable population models. These have been vital in many areas, such as clarifying the role of fertility and mortality change in the evolution of age structures, projecting long-run outcomes of processes and enabling demographic estimation from incomplete or defective data (see Chapter 9).

The logistic curve

Recognition that populations cannot grow indefinitely has led to interest in other mathematical approaches to representing population growth and defining its upper limit. One of the best known is the logistic curve, first discovered in 1838, but rediscovered and popularized by an American geneticist, Raymond Pearl, and his colleagues in the 1920s (Shryock and Siegel 1973: 382).

The curve is based on Pearl's observations of the growth in the numbers of fruit flies (*Drosophila melanogaster*) under experimental conditions. Pearl later sought to demonstrate that the growth of the human population follows the logistic curve. Pearl proclaimed the logistic curve to be the universal law of population growth (Krebs 1978: 191). Application of the logistic curve to predict the United States population from 1920 to 1940 produced good results, but later events demonstrated that the United States was not simply following a logistic trend (Plane and Rogerson, 1994: 64). Even updating Pearl's work and using data to 1960, as in Figure 2.4, still produces low forecasts of population growth. The population of the United States in 2000 was 275.6 million (Population Reference Bureau 2000), a total far higher than the upper limit of the logistic curve (250.5 million).

Nevertheless the logistic curve corresponds with textbook diagrams depicting the historical course of population growth during the demographic transition

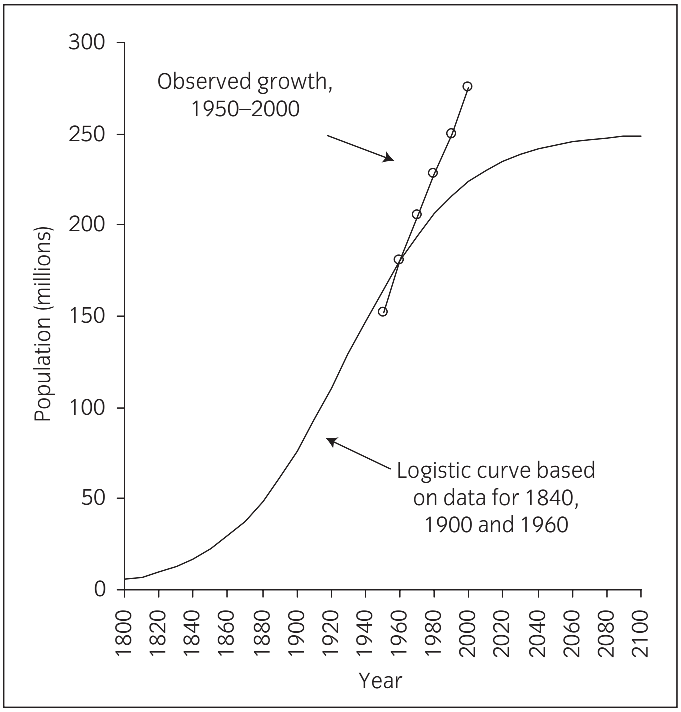


Figure 2.4 Logistic curve and observed growth, United States.

Data source: after Jain (1979: 92–4).

(Plane and Rogerson 1994: 62–3). The curve denotes an initial period of slow growth, followed by a phase of rapid growth – due to falling death rates – and finally a plateau in population numbers arising from low birth rates. The 'S-shaped' curve is also consistent with certain patterns of the diffusion through time in the numbers adopting innovations – with small increases at first, then larger increases and finally a tapering off. In population studies, potential topics for study in comparison with an S curve include changes through time in the proportion of people aware of sexually transmitted diseases, the proportion of newly married couples using contraception, and the proportion of households owning consumer goods, such as microwave ovens or computers.

Overall, an S-shaped trend (Figure 2.4) is a more realistic depiction of long-run national or global population growth than straight lines or exponential curves. It recognizes that numbers cannot increase indefinitely and that social goals and environmental constraints might be expected to bring a slowing of growth. Its main disadvantage is that, like other mathematical concepts, it cannot predict the future, because it is not founded on an explanation of changes. Moreover, despite the greater sophistication of the logistic curve in recognizing changing rates of growth

BOX 2.2 **Population clocks module (*Population Clocks.xls*)**

Population clocks, on office walls and Web pages, usually display changes in population numbers, for countries or the world, occurring in real time. This module can function in the same way, but it also provides opportunities to:

- accelerate time, to reveal long range outcomes;
- set the initial population to any total;
- change growth rates, showing the effects of lower or higher rates of change;
- compare populations growing at different exponential rates.

The module has an opening display, with menu items and instructions, and two clock displays.

Clock 1 A single clock with controls for changing the initial population, the rate of growth and the clock speed per second (in seconds, minutes, hours, days, weeks or years). An animated bar graph compares the initial and current population totals, and tables display current features of population change.

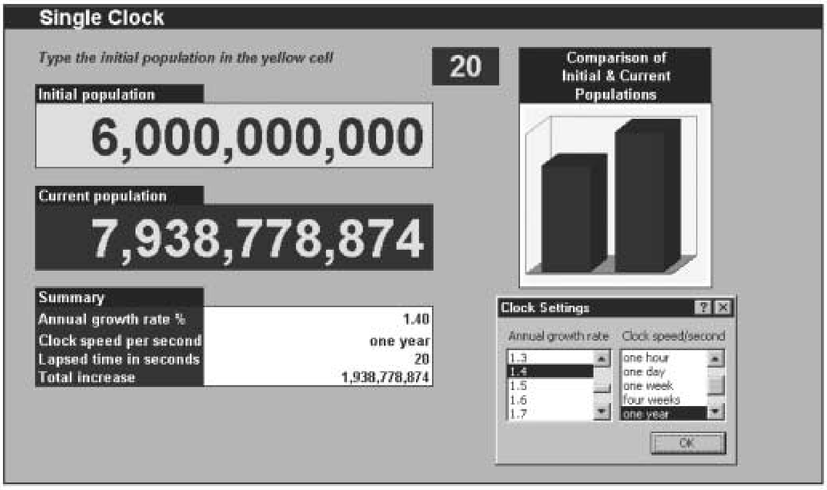
Clock 2 A dual clock for comparing two populations growing at different rates. An animated pie graph shows their changing shares of the combined totals.

Instructions

To use the module, insert the CD in the drive, start Excel and open the file *Population Clocks.xls*. Enable macros when prompted.

continued

BOX 2.2 continued



Single Clock

Type the initial population in the yellow cell

20

Initial population: 6,000,000,000

Current population: 7,938,778,874

Summary	
Annual growth rate %	1.40
Clock speed per second	one year
Lapsed time in seconds	20
Total increase	1,938,778,874

Clock Settings

Annual growth rate: 1.4

Clock speed/second: one year

- Click **Single Clock** or **Dual Clock** to open a display.
- Enter the starting population(s) in the yellow cell(s).
- Click **Menu** to set the growth rate(s) and clock speed.
- Click **Reset** to set the starting time to zero.
- Click **Start** to run the clock.
- To stop the clock, press the **Esc** key and click **End** on the menu.
- Click **Menu** to return to the opening screen.

See Appendix B for further advice on using the Excel modules.

Illustrative applications

Clock 1 (Single): World population growth

- Set the population and growth rate to the current figures (do not type commas when entering numbers).
- Set the clock speed to real time (1 second per second) and observe how the world's numbers are changing.
- Stop the clock by pressing the Esc key and clicking End on the menu box that appears. Then click reset to set the lapsed time to zero.
- Set the clock speed to 1 year per second and run the clock to show future numbers. Observe how long the population takes to double.
- Repeat the previous two steps to compare the doubling time for other growth rates, such as 1 per cent.

continued

BOX 2.2 continued

Clock 1 (Single): National population growth

- Use the clock to illustrate current national growth trends and alternative scenarios with lower or higher growth rates. Run at 1 year per second to demonstrate long-run outcomes.

Clock 1 (Single): Urban population growth

- Set the total population and growth rate to that of a rapidly growing major city and run the clock to obtain projections of future population totals. Discuss whether it is plausible to assume constant growth rates.

Clock 2 (Dual): Differential growth of ethnic groups

- When ethnic groups grow at different rates, be it tribes in African countries, indigenous and non-indigenous peoples in Fiji, or ethnic minority groups generally, the changing balance of numbers can be a source of concern, tension or conflict. To illustrate outcomes of different growth rates, set the total population to 1 million, the first growth rate to 3 per cent and the second growth rate to 1 per cent. Set the clock speed to 1 year per second, and observe the changes in the two groups' shares of the total population. The clock assumes that the two populations are equally represented at the start.

Clock 2 (Dual): Differential growth of regions

- Use a similar approach to compare outcomes of differential regional growth, such as rural versus urban or growing versus declining regions.

over time, the other simpler concepts serve as the bases for the most commonly used methods of analysing population growth.

2.2 Analysing growth

Population size

Population numbers can evoke mental images of densely packed masses of human beings. One of the most vivid is Paul Ehrlich's description of a taxi-ride in Delhi (Ehrlich 1970):

> I have understood the population explosion intellectually for a long time. I came to understand it emotionally one stinking hot night in Delhi a couple of years ago. My wife

> and daughter and I were returning to our hotel in an ancient taxi. The seats were hopping with fleas. The only functional gear was third. As we crawled through the city, we entered a crowded slum area. The temperature was well over 100, and the air was a haze of dust and smoke. The streets seemed alive with people. People eating, people washing, people sleeping. People visiting, arguing, and screaming. People thrusting their hands through the taxi window, begging. People defecating and urinating. People clinging to buses. People herding animals. People, people, people, people. As we moved through the mob, hand horn squawking, the dust, noise, heat, and cooking fires gave the scene a hellish aspect. Would we ever get to our hotel? All three of us were, frankly, frightened. It seemed that anything could happen – but, of course, nothing did. Old India hands will laugh at our reaction. We were just some over-privileged tourists, unaccustomed to the sights and sounds of India. Perhaps, but since that night I've known the *feel* of overpopulation.

Despite this image of a crowded world, the entire global population of 6 billion in 1999 would 'fit' into a megalopolis, or super-city, the size of the American state of Texas (land area =678 358 km^2, 261 914 square miles), which is larger than France (551 494 km$^{2)}$ but smaller than Pakistan (796 094 km^2). In this scenario, the world's population would be living at a density of about 9000 persons per square kilometre, a density a little below that of New York City as a whole in 1990 (9154 persons per square kilometre) and far less than that of the borough of Manhattan, which has a population density of 26 000 per square kilometre. The scenario obviously takes no account of the nature of the land and standards of living, nor of the vastly greater territorial demands of individuals in terms of their consumption of resources and impact on the environment. Yet envisioning the global population in a 'Texas City' does serve to show, vis-à-vis Ehrlich's anecdote, that population numbers are open to contrasting interpretations, depending partly how phlegmatic or emotive the writer's stance.

Consideration of population size is central to the debate about the future of the world's population, but it is also an essential aspect of the day-to-day analysis of population change. For businesses, the size of the market, or potential market, is vital, while in government administration there is ongoing concern for the numbers requiring physical infrastructure and services. Total population numbers are often the overriding consideration in planning.

Basic measures of growth and decline

Beyond population size, basic measures of change are frequently needed to describe developments. Table 2.2 provides a summary of these measures, together with examples based on World Bank projections for Mexico. The chief advantages of the measures are the ease with which they can be calculated and understood. As discussed in Chapter 1, P_0 represents the population at the start of a period and P_n the population at its end, that is after n intervals of time (e.g. days,

Table 2.2 Basic measures of population growth and decline

Formulas	Examples
Definitions	*Mexico*
P_0 = population at the start, e.g. year zero	P_0 = 98 787 000 (Year 2000)
P_n = population at the end, e.g. after n years	P_n = 162 356 000 (Year 2050) n = 50 years (mid-2000 to mid-2050)
n = number of intervals (e.g. years) between P_0 and P_n	
1 *Absolute change*	
$P_n - P_0$	162 356 000 – 98 787 000 = 63 569 000
2 *Percentage change*	
$\left(\frac{P_n - P_0}{P_0}\right) \times 100$	63 569 000/98 787 000 × 100 = 64.35%
3 *Average annual increase*	
$\frac{P_n - P_0}{n}$	63 569 000/50 = 1 271 380
4 *Arithmetic growth rate*	
$\left(\frac{P_n - P_0}{n}\right) \div P_0 \times 100$	1 271 380/98 787 000 × 100 = 1.29%

Data source: World Bank (1994: 343)

months or, more usually, years). In the set of examples for Mexico, *n* is set at 50 years.

1. *Absolute change* is simply the difference between the size of the population at the start and end of a period of time. This represents the *net growth* of the population. It is not a measure to be ignored because of its simplicity: sometimes the main implications of population changes arise from the extra numbers to be fed, housed, educated, employed or supported. Table 2.2 presents an example of absolute change in numbers in Mexico, projected for the first half of the twenty-first century. The calculation reveals that, in the 50 years, Mexico could gain the equivalent of the population of the United Kingdom or France in the year 2000.
2. *Percentage change* (absolute change/initial population × 100) provides a means of comparing developments in different populations or different periods of time

by gauging the amount of change relative to initial numbers. Percentage change is always based on numbers at the start of the period. It is a measure frequently used in census atlases and other maps of intercensal population change (see Chapter 10), because percentages are comparable and intelligible to a wide readership.

3. *Average annual increase (or decrease)* is based on the concept of arithmetic change. It assumes constant annual gains or losses – ignoring the notion that growth is self-reinforcing. The measure provides a rough-and-ready indication of the numbers being added to a population, but the longer the period of observation, or the greater the amount of change, the more likely the average is misleading. Average annual change is sometimes used to provide a quick estimate or projection of the size of a population in a particular year, but the same caveats apply. Estimates of average annual increase really have claims to accuracy only when the interval is short or where little change has occurred. The Mexico example shows an average annual increase from 2000 to 2050 about equivalent to adding, every year, the population of Hawaii.
4. The *arithmetic growth rate* is included in Table 2.2 mainly for comparison with other rates: it is seldom used in demographic calculations. The rate compares the annual average increase with the size of the initial population. The result is multiplied by 100, since population growth rates are usually expressed as rates per cent.

In calculating measures of population change there are several potential sources of error arising from the nature of the data. Firstly, boundary changes are common and may call for many adjustments to data for geographical analyses of population change within cities or across regions. For example, incorporation of new territory, such as a neighbouring town, can greatly change the population of a city. The main strategy in such situations is to use constant boundaries, typically the most recent, for establishing the size of the population at the start and the end of the period.

Secondly, changes in the dates of censuses and population estimates affect the length of the interval, which should be expressed as accurately as possible (Pollard et al. 1990: 21). Thus if the first estimate is for 30th June 2000 and the second for 10th August 2001, the length of the interval (n) is 1 year plus 41 days, or 1.11 years.

Thirdly, definition changes can have dramatic effects that may be difficult to resolve. Statistics on the growth rate of the labour force will vary greatly if the first population includes the unemployed, part-time workers and unpaid workers while the second population includes only people in full-time paid employment. Similarly, the apparent growth of ethnic minority groups will vary according to whether the source data are based on self-identification, degree of descent, birthplace, language or religion, or a combination of these. Inclusion or exclusion of children of mixed descent can also affect results considerably. Adjustments may be essential to achieve a consistent definition of the population at the start and end.

2.3 Geometric growth

Beyond the basics, more accurate measures of population growth avoid the limiting assumption of constant annual increments or decrements and facilitate better estimates of intermediate values. Measures of population change based on the geometric growth rate are widely used, as are those based on the exponential growth rate. The former are slightly easier to understand and calculate yet yield similar results where the time interval is short or the rate is within the typical range for national populations (from about –1 to 4 per cent). Also, the collection of official statistics on population size is inevitably periodic – at regular intervals such as census dates – and the intervening pattern of change need not be continuous and cumulative, especially where migration contributes substantially. Measuring growth somewhat crudely, as a geometric process, acknowledges the nature of the source data and the imprecision of estimates between dates.

Rates of change are not the only needed measures from the geometric growth formula. Analyses of population trends often require other information about the past or future size of the population, the time required to reach a given size and the population's doubling time. Table 2.3 therefore presents variations of the geometric growth formula for calculating the initial and end-of-period populations, together with time intervals. As in Table 2.2, definitions are at the top, together with the values employed in the accompanying examples. The footnotes to the table provide brief explanations of the derivation of each formula. Some of the formulas need to be calculated using logarithms; the table shows the logarithmic form of all of the formulas for comparison, as well as for use where a scientific calculator or computer is not available. Results of calculations vary according to the number of decimal places retained in intermediate steps. The examples in Table 2.3 provide sufficient detail to produce results matching the data for the United States at the top of the table.

1. *End of period population* Formula 1 in Table 2.3 has applications in estimating intercensal numbers, or projecting the future population, assuming that the growth rate remains constant. This assumption becomes increasingly untenable as n increases, because growth rates typically vary through time. In the table, example 1 estimates population numbers in 2010 from information on the initial population in 1990 together with the rate of growth and the time period. The growth rate per cent must be divided by 100. Changing the value of n would permit estimates of the size of the United States population in other years, such as 1999 ($n = 9$) and 2009 ($n = 19$).
2. *Initial population* The second formula works backwards in time to calculate what past population numbers would have been if numbers had been growing at a particular rate. The assumption of constant growth rates is too constraining to allow back-projection to proceed very far. Thus, reconstructing the past in this way is best confined to shorter intervals, all the more so in populations

Table 2.3 Measures of population change from the geometric growth rate

Formulas	Examples
Definitions	*United States*
P_0 = population at the start	$P_0 = 250.4$ (millions, mid-1990)
P_n = population at the end	$P_n = 297.2$ (millions, mid 2010)
n = number of intervals between P_0 and P_n	n = 20 years
r = annual growth rate	$r = 0.860\,41\%$
1 *End of period population*	
$P_n = P_0\,(1+r)^n$	$P_n = 250.4 \times (1.008\,604\,1)^{20}$ $= 297.2$
or	or
$\log P_n = \log P_0 + \log(1+r) \times n$	$\log P_n = 2.398\,63 + 0.074\,41$ $\therefore\ P_n = 297.2$
2 *Initial population*	
$P_0 = \dfrac{P_n}{(1+r)^n}$	$P_0 = 297.2/(1.008\,604\,1)^{20}$ $= 250.4$
or	or
$\log P_0 = \log P_n - \log(1+r) \times n$	$\log P_0 = 2.473\,05 - 0.074\,41$ $\therefore\ P_0 = 250.4$
3 *Geometric growth rate*	
$r = \sqrt[n]{\dfrac{P_n}{P_0}} - 1$	$r = (297.2/250.4)^{(1/20)} - 1$ $= 0.008\,604\,1$ or $0.860\,41\%$
or	or
$\log(1+r) = \dfrac{\log\left(\dfrac{P_n}{P_0}\right)}{n}$	$\log(1+r) = 0.003\,720\,724$ $\therefore\ r = 0.860\,41\%$
4 *Interval between two populations*	
$n = \dfrac{\log\left(\dfrac{P_n}{P_0}\right)}{\log(1+r)}$	$n = 0.074\,414\,481/0.003\,720\,724$ $= 20$ years
5 *Doubling time*	
$n = \dfrac{\log 2}{\log(1+r)}$	$n = 0.301\,03/0.003\,720\,724$ $= 80.9$ years

Derivation of formulas:

1. End of period population: ‘compound interest’ formula.
2. Initial population: divide both sides of formula 1 by $(1+r)^n$.
3. Geometric growth rate: divide both sides of formula 1 by P_0 and make r the subject.
4. Interval: make n the subject of the logarithmic form of formula 3.
5. Doubling time: in formula 4, $(P_n/P_0) = 2$, since the population doubles.

where substantial growth derives from migration – the most variable and unpredictable of all components of population change. The example calculates the 1990 population of the United States from information on the projected population in 2010, the growth rate and the time interval. As before, using other values of *n* would provide estimates for other years. Also, expressing *n* as a fraction, such as 5.5 years, can yield estimates for points in time other than the mid-year, to which the figure for 2010 refers.

3. *Growth rate* The growth rate provides an informative basis for comparisons between countries, since it gives an impression of stage of development in terms of progress through the demographic transition. However, the contribution of migration to national population growth is not known accurately for some countries, and cross-national data on rates of natural increase are sometimes published instead of figures on overall growth rates.

   In contemporary national populations, the highest growth rates occur where the demographic transition is yet to be completed, but more developed countries receiving substantial flows of international migrants, such as the United States and Australia, also have rates higher than other 'post-transition' populations. New towns and suburbs have the most spectacular growth rates on account of low initial numbers, but their growth rates typically fall as the population increases. Canberra, the capital of Australia, had an annual growth rate that peaked at 14 per cent during the 1960s, because of substantial government investment in establishing the new city. Subsequently, Canberra's growth rate declined rapidly from this peak, mainly because the larger the population, the greater the increments needed to sustain a particular rate.

   In the United States example in Table 2.3, the growth rate calculation proceeds either by finding the 20th root of P_n/P_0, or by raising P_n/P_0 to the power of 1/20 (the reciprocal of the power). Appendix A, on basic maths, describes how to calculate powers and roots of numbers.

4. *Interval between two populations* This denotes the time taken to change from one specified total to another. Thus the example in the table addresses the question of how long it will take a population to reach a particular size. Applications arise in planning when seeking to anticipate the date that numbers will reach consequential levels. For instance, population size determines the rights of communities to self-government or separate political representation. The passing of size thresholds, such as a million or a billion, are also occasions for reflection and action. The world's population probably reached 6 billion around the end of 1999; formula 4 can provide a first estimate of when it will reach 7 billion. The example shows the derivation of *n* from the United States data.

   It is important to note that finding the length of the interval, *n*, is not identical to estimating the year in which a total was reached. To find the year, add *n* to the date to which P_0 refers – including fractions of a year as necessary. For example, if *n* equals 15.6 years and P_0 refers to a census held at mid-year 1990, the required date will be 1990.5 + 15.6 = 2006.1, that is the year 2006. Ignoring the census date would lead to the error of specifying 2005 as the year. A decimal fraction can potentially be used to specify a particular day, but such precision is usually spurious.

5. *Doubling time* After the growth rate, the doubling time is the most widely mentioned measure of national and global population growth, because it is readily comprehended. In 2000, the world population doubling time was 51 years, compared with 49 years in 1999 (Population Reference Bureau 1999: 2; 2000: 2). Transitional populations in Africa have most of the fastest contemporary doubling times of 20–23 years.

   Malthus (1798: 16) made effective use of doubling time to convey the implications of population growth:

   > In the United States of America, where the means of subsistence have been more ample, the manners of the people more pure, and consequently the checks to early marriages fewer, than in any of the modern states of Europe, the population has been found to double itself in twenty-five years.

   A well-known shortcut formula for calculating doubling times is to divide 70 by the growth rate per cent. Thus annual growth rates of 2 per cent and 1 per cent give doubling times of 35 years and 70 years respectively. This approximation, which assumes an exponential, rather than geometric, growth rate, is improved slightly by using 69 instead of 70. The approximation is based on the doubling time formula from the exponential growth rate, as discussed below.

2.4 Exponential growth

Exponential growth yields higher totals than geometric growth (given P_0, r and n), especially over longer periods of time or where the rates are high (Figure 2.1). This is because exponential growth is compounding at every moment, rather than at fixed intervals. Conversely, exponential growth rates are always lower than geometric rates from the same P_0, P_n and n.

Examples of exponential change are given in Table 2.4, where most of the formulas are expressed in natural logarithms (base e). Alternatively, the formulas may be expressed using common logarithms, but are less concise. Like π ($\pi = 3.141\,59$), e is a mathematical constant ($e = 2.718\,28$). As in the calculations based on geometric growth, each term (P_0, P_n, n or r) can be calculated from the other three.

Since the natural logarithm of 2 is 0.693 15, the formula for the doubling time under exponential growth (Table 2.4, formula 5) is approximately $0.69/r$. Multiplying top and bottom by 100 gives $69/r$ per cent or, rounding to the nearest ten, $70/r$ per cent. Thus is derived the shortcut formula for the doubling time. Similarly,

Table 2.4 Measures of population change from the exponential growth rate

Formulas	Examples
Definitions	*Pakistan*
P_0 = population at the start	$P_0 = 112.4$ (millions, mid-1990)
P_n = population at the end	$P_n = 146.5$ (millions, mid-1999)
n = number of intervals between P_0 and P_n	$n = 9$ years
r = annual growth rate	$r = 2.94401\%$
ln = natural logarithm	
e is a constant (2.71828)	
1 *End of period population*	
$P_n = P_0 e^{rn}$	$P_n = 112.4 \times 1.30338$ $= 146.5$
or	or
$\ln P_n = \ln P_0 + rn$	$\ln P_n = 4.72206 + 0.26496$ $= 4.98702$ $\therefore P_n = 146.5$
2 *Initial population*	
$P_0 = \dfrac{P_n}{e^{rn}}$	$P_0 = 146.5/1.30338$ $= 112.4$
or	or
$\ln P_0 = \ln P_n - rn$	$\ln P_0 = 4.98702 - 0.26496$ $= 4.72206$ $\therefore P_0 = 112.4$
3 *Exponential growth rate*	
$r = \dfrac{\ln\left(\dfrac{P_n}{P_0}\right)}{n}$	$r = \ln(1.30338)/n$ $= 0.02944$ or 2.944%
4 *Interval between two populations*	
$n = \dfrac{\ln\left(\dfrac{P_n}{P_0}\right)}{r}$	$n = \ln(1.30338)/0.0294401$ $= 9$ years
5 *Doubling time*	
$n = \dfrac{\ln 2}{r}$	$n = 0.69315/0.0294401$ $= 23.5$ years

Data sources: World Bank (1994); Population Reference Bureau (1999).

the tripling time of a population is equal to the natural logarithm of 3 (1.098 61) divided by *r*, or approximately 110/*r* per cent.

The examples show that in the 1990s, Pakistan, one of the world's most populous countries, had a high growth rate of 2.9 per cent, with a doubling time of just 24 years. Changing the values of *n* or *r* in formulas 1 and 2 provides a straightforward approach to estimating and projecting the population. For instance, the 1996 population of Pakistan may be estimated by setting *n* to 6 in formula 1. Similarly, setting P_0 to the 1999 total and *r* to 2 per cent, would provide a projection at this lower rate of growth for any value of *n*.

Exponential growth rates are also employed in estimating past populations, but preferably only where there is evidence to support assumptions. An example of the misapplication of this technique is an estimate in 1959 of the number of people who had ever lived in the world (see Bourgeois-Pichat 1989: 81–2). The estimate assumed that the world's population had followed an exponential trend from 600 000 BC to the present, resulting in figures of between 3400 billion humans ever born, assuming one couple at the start, and 5300 billion assuming 500 couples at the start. This ignored the likely rises and falls in population growth rates over time, as illustrated in Hollingsworth's reconstruction of the population change in Egypt (Figure 2.5). An alternative estimate, based on varied sources of evidence, placed the figure at 80 billion humans ever born by mid-1987 – the year in which the world's total population reached 5 billion (Bourgeois-Pichat 1989: 81).

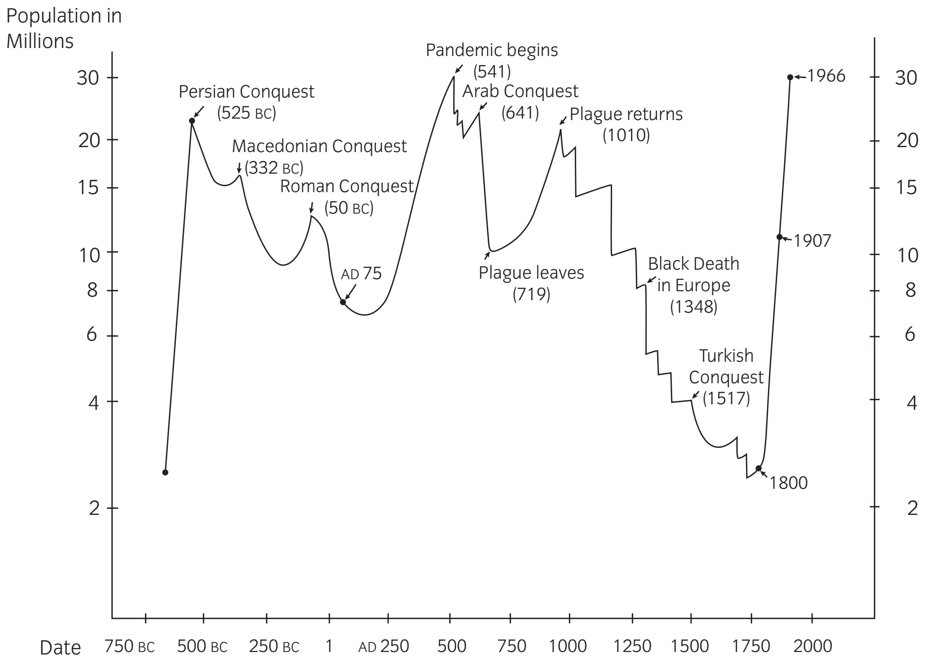


Figure 2.5 The population of Egypt, 664 BC–AD 1966.

Source: Hollingsworth (1969, 311). See also Cox (1970: 311–312).

BOX 2.3 **Growth rates module (*Growth.xls*)**

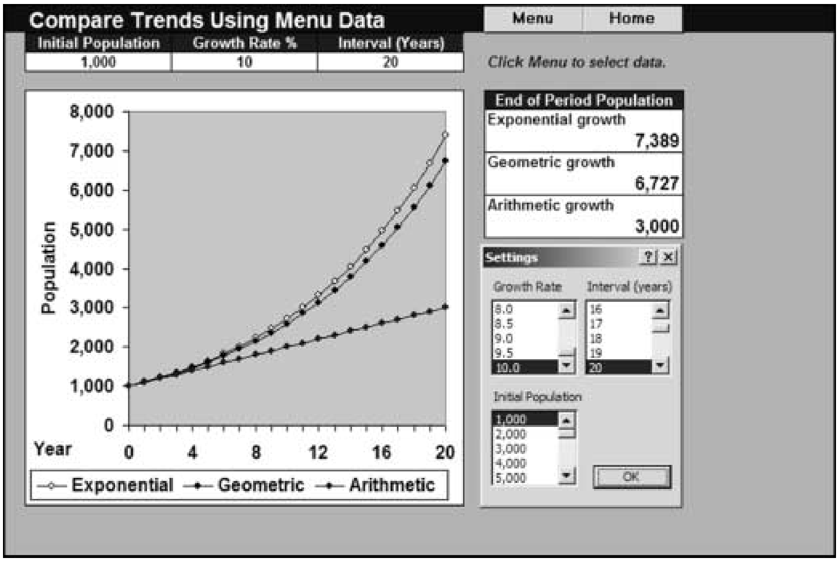
This module compares outcomes of exponential, geometric and arithmetic growth by plotting growth curves and calculating the end-of-period populations. It uses data either selected from menus or typed in. Its main purpose is to provide visual illustrations of the differences between the three concepts of population growth. The module also calculates rates and end-of-period populations from any entered data.

Instructions

To use the module, insert the CD in the drive, start Excel and open the file *Growth.xls*. Enable macros when prompted.

- Click **Menu Data** for comparisons using in-built data.
- Click **Own Data** for comparisons using your own data.
- Click **Rates** to obtain growth rates, or end-of-period populations, from your own data.
- **Home** returns to the opening display.
- **Menu** brings up the settings menu.

See Appendix B for further advice on using the Excel modules.



Compare Trends Using Menu Data

Initial Population	Growth Rate %	Interval (Years)
1,000	10	20

Click Menu to select data.

End of Period Population	
Exponential growth	7,389
Geometric growth	6,727
Arithmetic growth	3,000

Settings

Growth Rate: 10.0

Interval (years): 20

Initial Population: 1,000

continued

BOX 2.3 continued

Illustrative applications

Questions to explore using the module include:

1. Under what circumstances is annual average growth a satisfactory measure of change? Answer: when the interval is short or the growth rate is low. The module illustrates this by showing that the growth curves coincide in such circumstances.
2. In demographic calculations, will the findings be different according to whether the growth rate is arithmetic, geometric or exponential? If so why? Again, the answer depends upon whether the interval is short and the growth rate is low.
3. What is the range of growth rates within which the geometric rate is similar to the exponential rate. Answer: in the range of 1.0 to 4.0 per cent, that is in the usual range of growth rates for national populations. These growth rates are often exceeded, however, for regional and local populations.
4. If a population of 1 million is subject to a growth rate of 2.5 per cent for 50 years, will the population at the end of the period differ appreciably according to whether the growth rate is geometric or exponential?
5. When using growth rates to derive population estimates between two censuses, such as the 1990 and 2000 censuses, which is the most appropriate growth rate to choose? Answer: unless the growth rate was very high, it would make little difference which rate was chosen. Some prefer the geometric growth rate, because it is easier to calculate, while others prefer the exponential rate because it recognizes that, in large populations, growth is compounding continuously.

Another estimate is 60.5 billion ever born by 2000 (Weeks 2002: 17). Such figures vary according to assumptions about the number of years human beings have existed as well as population growth rates through time.

2.5 Growth and replacement

Statistics on population growth or decline conceal most of the details of population change. Studies often need to go beyond appropriate measures of growth or decline to include reference to the components of population changes (see Chapter 1) or the characteristics of the population. In 1990 the population of California was 29.8 million, while in 1999 it reached 33.1 million. The difference between these numbers is California's population growth of 3.3 million. Yet this figure actually comprises only the *net growth*, or the excess of gains over losses. Before any absolute increase in a population occurs, it is necessary to make up losses due to deaths and outward migration. In California, these included 2.0 million deaths in 1990–1999 and a domestic net migration loss of 2.2 million (See http://www.census.gov/population/estimates/state). Consequently, many additions are absorbed in the process of *population replacement*. Vast numbers of replacements may be needed even to maintain a population at its original size. The total gains, or additions, represent the *gross growth* of the population. Thus:

gross growth = gains (survivors from births and migration arrivals)

or:

gross growth = net growth + replacements

Figure 2.6 depicts Price's (1976: A57ff) concepts of net growth and gross growth, illustrating the point that the replacement of losses over time substantially reduces the gains from births and inward migration. Little or no population growth may occur in a community which is nonetheless experiencing major changes in its population membership. To describe such a community as 'static' would be misleading, because it requires ongoing replacements to maintain numbers. Substantial changes in population composition can arise even where there is minimal growth or decline, because net growth is the 'the tip of the iceberg' of gross growth. Similarly, population decline is not an unqualified outcome of losses but of a deficit in ongoing replacement processes. Thus the concepts of net growth, gross growth and replacement are valuable in appreciating the dynamic nature of populations and obtaining more realistic interpretations of changes.

A related concept is *population turnover,* which denotes the sum of all losses and gains:

population turnover = losses (deaths & migration departures)
+ gains (survivors from births and migration arrivals)

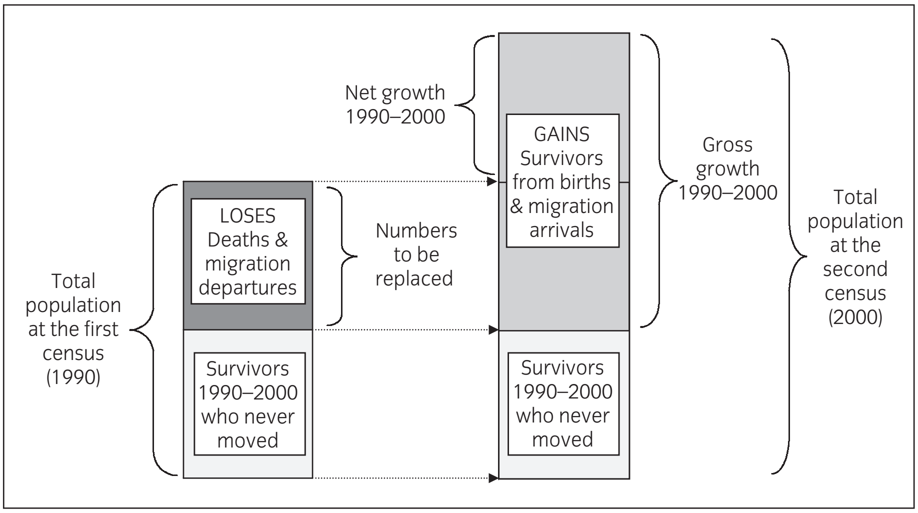


Figure 2.6 Replacement, net growth and gross growth.

Thus, population turnover is equal to losses (the numbers to be replaced) plus gross growth:

population turnover = losses (deaths and migration departures) + gross growth

Population turnover captures the total magnitude of changes in membership that a population experiences – the total of births, deaths, migration arrivals and migration departures – all of which are treated as positive numbers. Shryock and Siegel (1973: 608) describe population turnover as the sum of migration turnover (arrivals plus departures) and natural turnover (births plus deaths). Turnover is even greater in particular population subgroups, such as the aged where it draws attention to:

- the total load that administrative systems bear in maintaining registers of the clientele for pensions and services;
- the difficulties faced in community service provision for the elderly, where the client population is changing continually;
- the frequent changes in the circle of age peers – relatives, friends and acquaintances – with which individual older people must contend.

Overall, an adequate understanding of population dynamics and their implications calls for more than attention to the net growth or net decline in numbers. To obtain a more complete understanding of population change, informative concepts are turnover, replacement, gross growth and net growth.

2.6 Conclusion

Growth and decline are essential topics in many national and sub-national population studies; they often serve as a starting point in identifying demographic trends and their implications for planning and policy making. The concepts discussed in this chapter are necessary to an understanding of the literature on demographic change, as well as to making informed use of the range of different measures of population growth. The same concepts and measures have applications also in the important debate on the impact of population changes on the environment.

In developing plans and policies for the future, growth rates cannot be changed at will since they are dependent on a range of other factors, among which the age structure of the population is very influential. *Zero population growth* (ZPG), or a growth rate of zero, is the best-known demographic goal for minimizing environmental damage. Yet zero population growth is well beyond the reach of the less developed countries, because the form of their age structures is such that decades of substantial growth are inevitable. The next chapter discusses the concepts and methods used to describe and summarize features of the age structures of populations.

Study resources

KEY TERMS

- Arithmetic change
- Average annual increase
- Doubling time
- Exponential change
- Geometric change
- Gross growth
- Growth rate
- Logistic curve
- Net growth
- Percentage change
- Population replacement
- Population turnover
- Zero population growth

FURTHER READING

Barclay, George W. 1966. *Techniques of Population Analysis*. Chapter 7, 'Growth of Population', especially pp. 203–211.

Shryock, Henry S., Siegel, Jacob S. et al. 1973. *The Methods and Materials of Demography*, Volume 2, Chapter 13, 'Population Change', pp. 372–388.

Weeks, John R. 2002. *Population: an Introduction to Concepts and Issues*, Chapter 1, 'Introduction to the World's Population', pp. 28–56.

INTERNET RESOURCES

Subject	Source and Internet address
World population growth	**Musee de l'Homme** http://www.popexpo.net/ **United Nations Population Division (The World at Six Billion)** http://www.undp.org/popin/wdtrends/6billion/ **US Census Bureau (World Population Information)** http://www.census.gov/ipc/www/world.html
Population clocks	**US Census Bureau** http://www.census.gov/main/www/popclock.html

Note: the location of information on the Internet is subject to change; to find material that has moved, search from the home page of the organization concerned.

EXERCISES

1 Name one commonly used demographic measure that is based on the assumption of arithmetic change.

2 Distinguish between an arithmetic progression and a geometric progression.

3 Sketch a diagram to illustrate gross growth and net decline where the end-of-period population is smaller than the initial population.

4 Using information from the table below, answer the following questions, using the geometric growth formula for 4(c)–4(e):

Estimated world population 1960–2000 (millions)

Year	Mid-year population
1960	3037
1970	3696
1980	4432
1990	5321
2000	6067

Sources: United Nations 1983: *Demographic Yearbook 1981*; Population Reference Bureau (1990 & 2000). *World Population Data Sheet*.

(a) What was the percentage change in the world's population in each decade, i.e. (i) 1960–70, (ii) 1970–80, (iii) 1980–90 and (iv) 1990–2000?

(b) What was the average annual numerical increase in the population in each decade?

(c) What was the average annual growth rate per cent (geometric) in each decade?

(d) How long would the world's population take to double if the growth rate continued at the average level for each decade?

(e) Using the average growth rates from (c), estimate the year in which the population reached (i) 4 billion and (ii) 5 billion. Assuming a continuation of the 1990–2000 average growth rate, when would the total reach 7 billion?

5 Repeat questions 4(c) using the formula for the exponential growth rate.

6 Construct a table comparing doubling times using the geometric growth formula and the approximation (70/*r* per cent) for rates from 1.0 to 6.0 per cent in steps of 1.0. Does the approximation give satisfactory results?

7 Complete the following table for a population growing at an annual rate of 3 per cent and comment on why the figures differ.

Population growth 2005–2010

Year	Arithmetic growth	Geometric growth	Exponential growth
2005	1 000 000	1 000 000	1 000 000
2006			
2007			
2008			
2009			
2010			

8 For your own country, make a list of the main printed publications, and/or Web sites, that provide the most recent information on national population numbers and components of growth.

9 The larger the population, the greater the increments needed to sustain a constant growth rate. Demonstrate this with an example.

SPREADSHEET EXERCISE 2: POPULATION GROWTH

Calculate the geometric growth rates for the countries in the table; sort the rows of the table into ascending or descending order by growth rate and plot a bar graph of the rates. The notes below provide a step-by-step approach which assumes that Spreadsheet Exercise 1 has been completed. Appendix C covers the main Excel procedures and is intended to provide further assistance as needed.

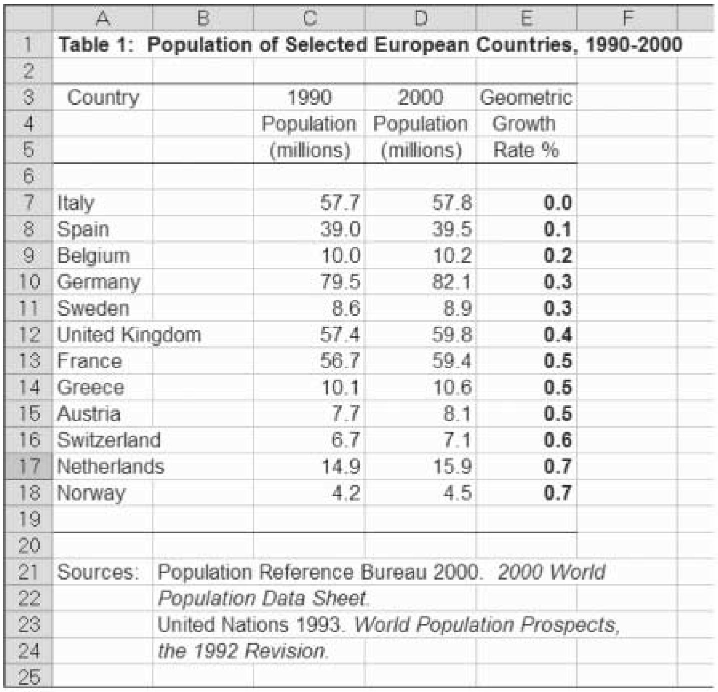


Table 1: Population of Selected European Countries, 1990-2000

Country	1990 Population (millions)	2000 Population (millions)	Geometric Growth Rate %
Italy	57.7	57.8	**0.0**
Spain	39.0	39.5	**0.1**
Belgium	10.0	10.2	**0.2**
Germany	79.5	82.1	**0.3**
Sweden	8.6	8.9	**0.3**
United Kingdom	57.4	59.8	**0.4**
France	56.7	59.4	**0.5**
Greece	10.1	10.6	**0.5**
Austria	7.7	8.1	**0.5**
Switzerland	6.7	7.1	**0.6**
Netherlands	14.9	15.9	**0.7**
Norway	4.2	4.5	**0.7**

Sources: Population Reference Bureau 2000. *2000 World Population Data Sheet.*
United Nations 1993. *World Population Prospects, the 1992 Revision.*

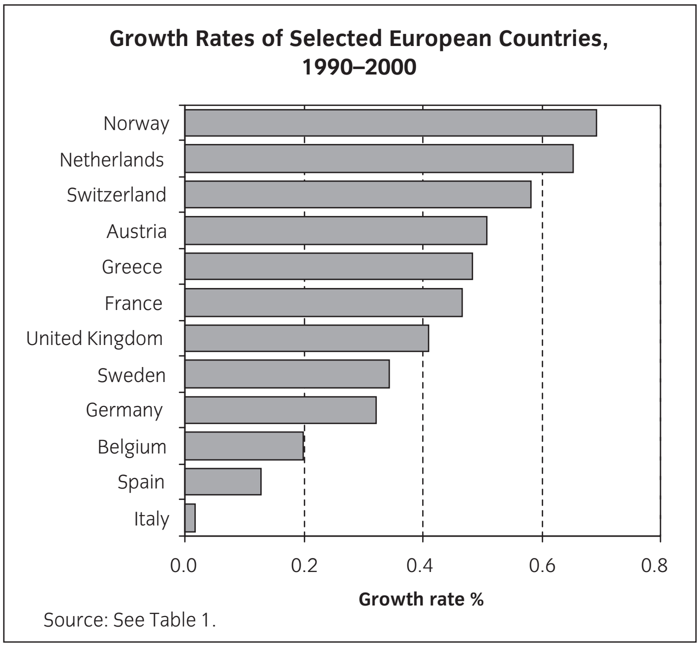
Constructing the table

1. Type Table 1, using formulas to calculate the figures in bold type. Use brackets in the first formula, to ensure the correct order of operations (see Appendix A), then copy the formula down to the other cells (see Appendix C, Section 2.8).

2. Sort the data into ascending or descending order by growth rate:
   - highlight the row numbers to be sorted; choose Data Sort;
   - choose sort by Column E (i.e. the column containing the growth rates);
   - click 'ascending' or 'descending' and click 'OK' or press Enter.
3. Save the spreadsheet, then plot the bar graph of the geometric growth rates.

Plotting the graph

4. Use the mouse to highlight the cells to be plotted, i.e. the country names and the growth rates. If the cells are not in adjacent columns, highlight the first column, then hold down the Ctrl key while highlighting the second column.
5. Click the Chart Wizard button on the standard toolbar. Follow the menu prompts and choose a bar graph (horizontal bars) then click 'Finish', or step through all the formatting options by choosing 'Next'.



Source: See Table 1.

6. Position the graph by dragging it with the mouse. If some of the country names are not displayed, increase the size of the graph by dragging one of the sizing handles (square boxes on the frame around the outside of the graph). If the sizing handles are not displayed, click once on the graph.

7. To position the country names at the low end of the vertical axis:
    - click once on the graph so that a solid box appears around the graph;
    - click the mouse on the vertical axis (a box will appear at each end of the axis);
    - now double click on the vertical axis to bring up the menu for editing the axis; choose Patterns Tick Mark Labels Low (i.e. position the labels at the low end of the scale).
8. To add a title, from the Menu Bar choose Chart Chart Options Titles and type the required title in the box, e.g. Population Growth Rates for Selected Countries, 1990-2000.
9. Label the horizontal and vertical axes in the same way. Note that, in Excel, the normal labelling of the X and Y axes is reversed for horizontal bar charts: the X axis is the vertical axis, while the Y axis is the horizontal axis.
10. To add notes, e.g. 'Source', click near the outer edge of the graph to select the whole graph, then type the information, press Enter and position the text box with the mouse. The text can be moved with the mouse only when there is an edit box around the text. Click on the text if necessary to activate the text box.
11. To change the spacing between bars, click on one of the bars, then double click, or right click, the mouse to bring up the editing menu. Choose Format Data Series Options and change the Gap Width to, for example, 10 or 20.
12. To change the colour or shading:
    - double click one of the bars in the graph to bring up the editing menu;
    - choose Patterns, Area (i.e. choose colour or shading) and click OK;
    - change the plot area shading in the same way (graphs with unshaded backgrounds may print best in black and white).
13. Save and print the spreadsheet.

Review of Excel procedures

- Sorting rows of a table (Data Sort).
- Plotting graphs (highlight data and click the Chart Wizard button).
- Changing the type of chart – click on the chart then choose Chart Chart Type.
- Formatting axes (double click the axis to activate the menu).
- Changing shading (double click a bar to activate the menu).
- Adding titles (Chart Chart Options Titles).