



WebAppendix 7. The Balassa-Samuelson Effect, Wealth, and International Price Levels

Table 7.1 in Chapter 7 establishes that price levels are systematically higher in wealthier countries than in poorer ones, when prices are measured in a common currency. This appendix provides a formal analysis of two of the most important explanations of this important regularity. The first, the Balassa-Samuelson effect, was described in Box 7.4 and appeals to differences in labor productivity in traded goods, which are generally correlated with GDP and capital per head. The second section explains how overall wealth may affect the current level of the exchange rate, even if a nation has labour productivity in traded goods comparable to that of its trading partners. The interested reader is referred to Obstfeld and Rogoff (1997) or Heijdra and van der Ploeg (2002) for other perspectives on international price differences.

A7.1. The Balassa-Samuelson Effect

The price level is measured using the consumer price index (CPI). Let the CPI in each country be given by the same (geometrically) weighted average of tradable and nontradable goods prices in the local currency, with weights a and $(1-a)$ reflecting the shares of these goods in the consumption basket:¹

$$(A7.1) \quad P = (P^T)^a (P^N)^{1-a} \quad \text{and} \quad P^* = (P^{T*})^a (P^{N*})^{1-a},$$

where as usual, a star (“*”) denotes foreign country variables. International competition in tradable goods links the tradable goods price to the foreign tradable goods price level in terms of the domestic currency, or the foreign currency:

$$(A7.2) \quad P^T = P^{T*}/S. \quad \text{or} \quad SP^T = P^{T*}.$$

In Chapter 8, this result is called the *Law of One Price*. As in the text, S denotes the nominal exchange rate in “British terms” (e.g. dollar/euro from the perspective of the Euro-area). We learned in Chapter 4 that the real wage paid by profit-maximizing firms is equal to the marginal product of labour, so

$$(A7.3) \quad W^T/P^T = MPL^T \quad \text{and} \quad W^*/P^{T*} = MPL^{T*},$$

¹ The assumption that consumers have similar tastes (that the weights of tradable and nontradable goods are the same across countries) is a useful simplification. The key results of this WebAppendix also hold if tastes differ but the analysis is less tractable.



where W^T denotes the nominal (local currency) wage in the tradable goods sector. This will also hold for nontradable goods:

$$(A7.4) \quad W^N/P^N = MPL^N \quad \text{and} \quad W^{N*}/P^{N*} = MPL^{N*}.$$

Combining (A7.1), (A7.2), and (A7.3) yields the result

$$(A7.5) \quad W^T/(EW^{T*}) = MPL^T/MPL^{T*}.$$

Equation (A7.5) links, via international trade, wages in the tradable goods sector at home and abroad. It is evident that, holding the exchange rate E constant, domestic wages are higher, the higher is local labour productivity in the tradable sector relative to that in the rest of the world.

The next step is to recognize that labor mobility *within a country* will link wages in the traded and nontraded sectors. To make this point in a clear and tractable way, wages are assumed equal in both sectors:

$$(A7.6) \quad W = W^T = W^N \quad \text{and} \quad W^{T*} = W^{N*} = W^*.$$

This is equivalent to assuming that labour is perfectly mobile between tradable and nontradable goods sectors.

Now we are in a position to derive the Balassa-Samuelson result. Using the definitions, the ratio of price levels between home and abroad can be written as

$$(A7.7) \quad \frac{P}{(P^*/S)} = \frac{S(P^T)^a (P^N)^{1-a}}{(P^{T*})^a (P^{N*})^{1-a}} = \left(\frac{SP^T}{P^{T*}} \right)^a \left(\frac{SP^N}{P^{N*}} \right)^{1-a}.$$

Logically, the price ratio depends on relative prices of tradable and nontradable goods in a common currency. But the law of one price (A7.2) implies that the first term of (A7.7) is equal to 1, so

$$(A7.8) \quad \frac{P}{(P^*/S)} = \left(\frac{SP^N}{P^{N*}} \right)^{1-a} = \left(\frac{S(W^N / MPL^N)}{W^{N*} / MPL^{N*}} \right)^{1-a}.$$

The last step incorporates the empirical observation that labor productivity in nontraded goods does not vary much across countries, or at least much less so than in traded goods. As a first approximation,² we assume $MPL^N = MPL^{N*}$. Using this and the wage

² Naturally, this is only an approximation (just like the assumption that wages are equal across sectors) which can be weakened without affecting the central conclusions. The reason often



equalization assumption (A7.6) allows us to rewrite (A7.8) as follows:

$$(A7.9) \quad \frac{P}{(P^*/S)} = \left(\frac{SW^N}{W^N*} \right)^{1-a} = \left(\frac{SW^T}{W^T*} \right)^{1-a} = \left(\frac{SP^T MPL^T}{P^T * MPL^T*} \right)^{1-a} = \left(\frac{MPL^T}{MPL^T*} \right)^{1-a},$$

where we have once again invoked the law of one price for tradable goods (A7.2). Equation (A7.9) is the Balassa-Samuelson effect: countries with higher productivity in tradable goods have higher overall price levels, when measured in the same currency. In Chapters 3 and 4 it was shown that labor productivity depends primarily on capital per worker and the state of technical progress; Balassa-Samuelson effect can thus explain why richer, capital-intensive, and technically sophisticated economies have higher price levels, all other things equal.

A7.2. Wealth, the real exchange rate, and the intertemporal budget constraint³

In this section, we link the real equilibrium exchange rate – measured as the relative price of nontradable goods in terms of tradable goods – to exogenous level of wealth of the economy. This would explain why wealthier countries have appreciated exchange rates, even though productivity in the tradable goods sector is equal to or lower than that of their trading partners –the United Kingdom or Saudi Arabia are two examples. To show this, we invoke intertemporal arguments found in Section 7, maintaining the two-period framework set out in Chapters 4, 5, and 6 of the textbook, but modified to allow for two goods each period. In what follows, we denote tradable and nontradable goods by superscript T and N respectively.

Suppose consumers have preferences over four possible goods – tradable goods today (C_1^T), nontradables today (C_1^N), tradables tomorrow (C_2^T), nontradables tomorrow (C_2^N) – given by the utility function:

$$(A7.10) \quad u(C_1^T, C_1^N) + \frac{u(C_2^T, C_2^N)}{1 + \rho}.$$

As in WebAppendix 6, this utility function is time separable, but each period's utility is a strictly concave function of tradable and nontradable good consumption in each

given is the relative unimportance of capital in nontradable goods production. More generally, it may be seen as a symptom of the “service disease” phenomenon described by Baumol (1967).

³ This section covers mathematically more advanced and difficult material.



period. We assume that $u_1 > 0, u_2 > 0, u_{11} < 0, u_{22} < 0$, and $u_{12} > 0$, where u_i denotes the first partial derivative of $u(.,.)$ with respect to the i th argument, u_{ii} denotes its second partial derivative with respect to i , etc. Tradables and nontradables are normal goods within a period, meaning that for both, the marginal propensity to consume out of wealth will be positive when the real exchange rate is held constant. The distinction between nontraded and traded goods implies that for $t=1, 2$

$$(A7.11) \quad C_t^N = Q_t^N,$$

so only traded goods can be used to honour debt contracts or to extend international credit. The intertemporal budget constraint is given by:

$$(A7.12) \quad C_1^T + \frac{C_2^T}{1+r} = Q_1^T + \frac{Q_2^T}{1+r} + \Gamma_1 + \frac{\Gamma_2}{1+r}$$

where Γ_t stand for exogenous, tradable wealth in period t – petroleum reserves, stocks of precious metals, “unearned” holdings of foreign assets or other exogenous, but tradable national treasures. Without loss of generality we will set $\Gamma_1 = \Gamma$ and $\Gamma_2 = 0$.

In each period, consumers can exchange tradable goods for nontradable goods at price σ_t . Producers are perfectly competitive and their behaviour gives rise to a production possibilities frontier (PPF) for the economy given for $t=1, 2$ by

$$(A7.13) \quad Q_t^T = Q^T(Q_t^N)$$

Consistent with the bowed-out shape of the PPF depicted in Figure 7.3, we assume $Q^{T'} < 0$ and $Q^{T''} < 0$ for possible values taken by Q^N (we use the shorthand $Q^{T'}$ for the first derivative, since there is only one argument). Using the logic of the textbook, the marginal rate of transformation of the economy of tradable goods for nontradable goods, the slope of the PPF, is the absolute value of the first derivative of the PPF, $-Q^{T'}$. In the decentralised market equilibrium under conditions of perfect competition, the marginal rate of transformation in production will equal the relative price of nontradable goods, σ_t , which is also our measure of the real exchange rate. Moving rightwards along the PPF given by (A7.4), increasing production of the nontradable good is associated with a higher marginal rate of transformation and relative price, for reasons outlined in Box 7.2.

The representative consumer chooses $\{C_1^T, C_1^N, C_2^T, C_2^N\}$ to maximize utility (A7.1) subject to the conditions (A7.2) and the budget constraint summarized by (A7.3). First order conditions are for $t=1,2$:



$$(A7.14) \quad \frac{u_2(C_1^T, C_1^N)}{u_1(C_1^T, C_1^N)} = -Q^T(Q_1^N)$$

$$(A7.15) \quad \frac{u_2(C_2^T, C_2^N)}{u_1(C_2^T, C_2^N)} = -Q^T(Q_2^N)$$

$$(A7.16) \quad \frac{u_1(C_1^T, C_1^N)}{u_1(C_2^T, C_2^N)} = \frac{1+r}{1+\delta}$$

$$(A7.17) \quad \frac{u_2(C_1^T, C_1^N)}{u_2(C_2^T, C_2^N)} = \left(\frac{1+r}{1+\delta} \right) \frac{Q^T(Q_1^N)}{Q^T(Q_2^N)}$$

The first two equations are intratemporal (within-period) conditions for utility maximization and are represented by Figure 7.7 in the textbook. Equations (A7.16) and (A7.17) are intertemporal optimality conditions such as those found in Chapter 6 or the WebAppendix 6.⁴

Define the real exchange rate that satisfies the efficiency condition for production in each period:

$$(A7.18) \quad \sigma_1 = -Q^T(Q_1^N)$$

$$(A7.19) \quad \sigma_2 = -Q^T(Q_2^N).$$

Define an equilibrium for the open economy as a set of consumption and production plans $\{C_1^T, C_1^N, C_2^T, C_2^N, Q_1^T, Q_1^N, Q_2^T, Q_2^N\}$, plus a pair of real exchange rates today and tomorrow, σ_1 and σ_2 , for which (A7.14), (A7.15), (A7.16), (A7.17), (A7.18), and (A7.19) hold and the four intratemporal resource restrictions (A7.11) and (A7.13) as well as the intertemporal budget constraint (A7.12) are respected.⁵

Having characterized the equilibrium, we now demonstrate that the equilibrium

⁴ Note that while the interest rate r is the real financial interest measured in terms of tradable goods, one could also define an interest rate in term of the other good as $(1+r)\sigma_1/\sigma_2$, the amount of nontraded goods that the market pays tomorrow for one unit of nontradable output today.

⁵ In fact, we have eleven equations in ten unknowns – how can that be? The answer is that the intertemporal budget constraint (A7.12) was substituted into the maximization problem and holds by assumption. It represents an additional constraint on the problem.



exchange rate which enforces the intertemporal budget constraint will appreciate (σ_1 rises) in response to an exogenous increase in Γ . The accompanying diagrams make the point graphically; what follows is a demonstration of the point formally. Total differentiation of (A7.14), (A7.15) and (A7.16) plus the intertemporal budget constraint (A7.12) results in:

$$\begin{aligned} & u_{21}(C_1^T, C_1^N)dC_1^T + u_{22}(C_1^T, C_1^N)dQ_1^N \\ &= \sigma_1[u_{11}(C_1^T, C_1^N)dC_1^T + u_{12}(C_1^T, C_1^N)dQ_1^N] + u_1 \frac{d\sigma_1}{dQ_1^N} dQ_1^N \end{aligned}$$

$$\begin{aligned} & u_{21}(C_2^T, C_2^N)dC_2^T + u_{22}(C_2^T, C_2^N)dQ_2^N \\ &= \sigma_2[u_{11}(C_1^T, C_1^N)dC_2^T + u_{12}(C_1^T, C_1^N)dQ_2^N] + u_1 \frac{d\sigma_2}{dQ_2^N} dQ_2^N \end{aligned}$$

$$u_{11}(C_1^T, C_1^N)dC_1^T + u_{12}(C_1^T, C_1^N)dQ_1^N = \frac{1+r}{1+\delta}[u_{11}(C_2^T, C_2^N)dC_2^T + u_{12}(C_2^T, C_2^N)dQ_2^N]$$

$$dC_1^T + \frac{dC_2^T}{1+r} = Q^T{}'(Q_1^N)dQ_1^N + Q^T{}'(Q_2^N)dQ_2^N + d\Gamma$$

which is a system of four equations in four unknowns. Rewrite this system in matrix form as $\mathbf{Ax}=\mathbf{b}$, where

$$\mathbf{A} = \begin{bmatrix} u_{21} - \sigma_1 u_{11} & 0 & u_{22} - \sigma_1 u_{12} + u_1 Q^T{}''(Q_1^N) & 0 \\ 0 & u_{21} - \sigma_2 u_{11} & 0 & u_{22} - \sigma_2 u_{12} + u_1 Q^T{}''(Q_2^N) \\ u_{11} & -\frac{1+r}{1+\delta} u_{11} & u_{12} & -\frac{1+r}{1+\delta} u_{12} \\ 1 & \frac{1}{1+r} & -\sigma_1 & -\sigma_2 \end{bmatrix},$$

$$\mathbf{x} = \begin{bmatrix} dC_1^T \\ dC_2^T \\ dQ_1^N \\ dQ_2^N \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ d\Gamma \end{bmatrix}.$$

Here we have made use of (A7.18) and (A7.19). Since $\frac{d\sigma_1}{dQ_1^N} = -Q^T{}''(Q_1^N) > 0$ always,

we need merely to sign $\frac{dQ_1^N}{d\Gamma}$ to sign the total effect of Γ on the real exchange rate



$\frac{d\sigma_1}{d\Gamma} = \frac{d\sigma_1}{dQ_1^N} \frac{dQ_1^N}{d\Gamma}$. Using Cramer's rule, $dQ_1^N = \det(\mathbf{A}_3)/\det(\mathbf{A})$ where \mathbf{A}_3 is equal to \mathbf{A}

with vector \mathbf{b} as its third column. It turns out that for the economy to have a unique equilibrium corresponding to utility maximization, $\det(\mathbf{A}) < 0$ (utility is concave in consumption and production). The determinant of \mathbf{A}_3 is

$-(u_{21} - \sigma_1 u_{11}) \frac{1+r}{1+\delta} [-u_{12}u_{21} + u_{11}u_{22} + u_{11}u_1 Q_2''(Q_2^N)] d\Gamma$; by concavity of periodic

utility, it is also negative, so in equilibrium, $\frac{dQ_1^N}{d\Gamma} > 0$.⁶ It follows that $\frac{d\sigma_1}{d\Gamma} > 0$; all

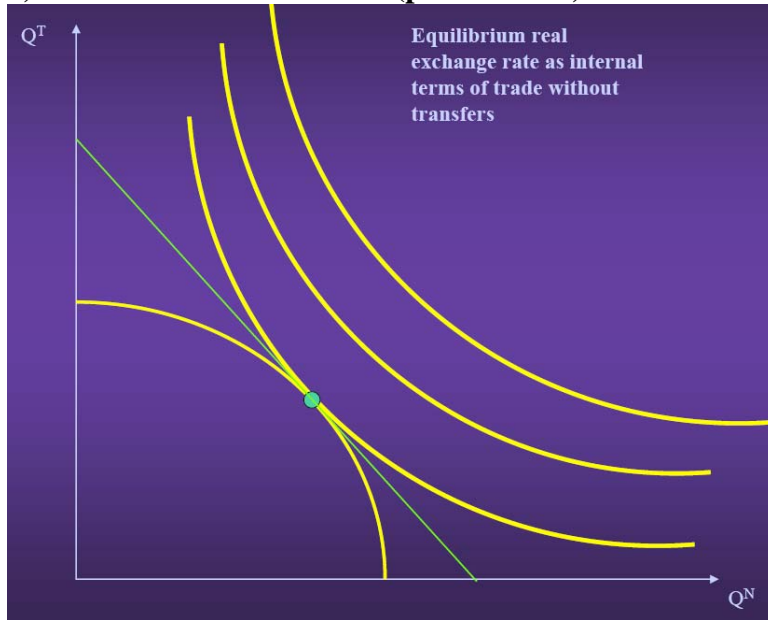
other things equal, countries with more tradable wealth have appreciated exchange rates. Using reasoning similar to that of the previous section, this result will imply that wealthy countries also have higher price levels when measured in a common currency.

Figure A7.1. shows diagrammatically how an increase in wealth, here in the form of a transfer of tradable goods, leads to a real appreciation, a reduction in tradable goods production, an increase in nontradable goods production, and an increase in consumption of both tradable and nontradable goods. This illustrates the mechanism by which the "Dutch disease" operates.

⁶ For a strictly concave function $f(x,y)$, $f_{11}f_{22} > f_{12}f_{21}$.



a) Before increase in wealth (pre-transfer)



b) After increase in wealth (post-transfer)

