A

a- Prefix meaning 'not'. For example, an asymmetric figure is one which possesses no symmetry, which is not symmetrical.

A The number 10 in hexadecimal notation.

abacus A counting device consisting of rods on which beads can be moved so as to represent numbers.

SEE WEB LINKS

- A description of how one abacus works.

abelian group Suppose that *G* is a *group with the operation $\circ$. Then *G* is abelian if the operation $\circ$ is commutative; that is, if, for all elements *a* and *b* in *G*, $a \circ b = b \circ a$.

Abel, Niels Henrik (1802–29) Norwegian mathematician who, at the age of 19, proved that the general equation of degree greater than 4 cannot be solved algebraically. In other words, there can be no formula for the roots of such an equation similar to the familiar formula for a quadratic equation. He was also responsible for fundamental developments in the theory of algebraic functions. He died in some poverty at the age of 26, just a few days before he would have received a letter announcing his appointment to a professorship in Berlin.

Abel's test A test for the convergence of an infinite series which states that if $\sum a_n$ is a convergent sequence, and $\{b_n\}$ is monotically decreasing, i.e. $b_{n+1} \leq b_n$ for all n, then $\sum a_n b_n$ is also convergent.

above Greater than. The limit of a function at *a* from above is the limit of $f(x)$ as $x \to a$ for values of $x > a$. It is of particular importance when $f(x)$ has a discontinuity at *a*, i.e. where the limits from above and from below do not coincide. It can be written as $f(a+)$ or $\lim_{x \to a+} f(x)$.

abscissa The *x*-coordinate in a Cartesian coordinate system in the plane.

absolute address In spreadsheets a formula which is to appear in a number of cells may wish to use the contents of another cell or cells. Since the relative position of those cells will be different each time the formula appears in a new location, the spreadsheet syntax allows an absolute address to be specified, identifying the actual row and column for each cell. When a formula is copied and pasted to another cell, a cell reference using an absolute address will remain unchanged. A formula can contain a mixture of absolute and *relative addresses.

absolute error *See* ERROR.

absolute frequency The number of occurrences of an event. For example, if a die is rolled 20 times and 4 sixes are observed the absolute frequency of sixes is 4 and the *relative frequency is 4/20.

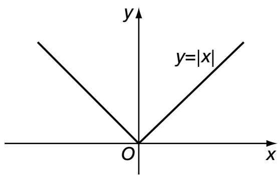
absolute measure of dispersion = MEASURE OF DISPERSION.

absolutely convergent series A series $\{a_n\}$ is said to be absolutely convergent if $\sum_{n=1}^{\infty}|a_n|$ is *convergent. For example, if $a_n = (-1)^{n-1} \times \frac{1}{n}$ then the series is convergent but not absolutely convergent, whereas $a_n = (-1)^{n-1} \times \frac{1}{n^2}$ is absolutely convergent.

absolutely summable = ABSOLUTELY CONVERGENT.

absolute value For any real number a, the absolute value (also called the *modulus) of a, denoted by $|a|$, is a itself if $a \geq 0$, and $-a$ if $a \leq 0$. Thus $|a|$ is positive except when $a = 0$. The following properties hold:

(i) $|ab| = |a||b|$.
(ii) $|a+b| \leq |a| + |b|$.
(iii) $|a-b| \geq ||a| - |b||$.
(iv) For $a > 0$, $|x| \leq a$ if and only if $-a \leq x \leq a$.



absorbing state *See* RANDOM WALK.

absorption laws For all sets A and B (subsets of some *universal set), $A \cap (A \cup B) = A$ and $A \cup (A \cap B) = A$. These are the absorption laws.

abstract algebra The area of mathematics concerned with algebraic structures, such as *groups, *rings and *fields, involving sets of elements with particular operations satisfying certain axioms. The purpose is to derive, from the set of axioms, general results that are then applicable to any particular example of the algebraic structure in question. The theory of certain algebraic structures is highly developed; in particular, the theory of vector spaces is so extensive that its study, known as *linear algebra, would probably no longer be classified as abstract algebra.

abstraction The process of making a general statement which summarizes what can be observed in particular instances. For example, we can say that $x^2 < x$ for $0 < x < 1$ and $x^2 > x$ for $x < 0$ or $x > 1$. Mathematical theorems are essentially abstraction of concepts to a higher level.

abundant number An integer that is smaller than the sum of its positive divisors, not including itself, For example, 12 is divisible by 1, 2, 3, 4 and 6, and $1 + 2 + 3 + 4 + 6 = 16 > 12$.

acceleration Suppose that a particle is moving in a straight line, with a point O on the line taken as origin and one direction taken as positive. Let x be the *displacement of the particle at time t. The acceleration of the particle is equal to $\ddot{x}$ or d^2x/dt^2, the *rate of change of the *velocity with respect to t. If the velocity is positive (that is, if the particle is moving in the positive direction), the acceleration is positive when the particle is speeding up and negative when it is slowing down. However, if the velocity is negative, a positive acceleration means that the particle is slowing down and a negative acceleration means that it is speeding up.

In the preceding paragraph, a common convention has been followed, in which the unit vector **i** in the positive direction along the line has been suppressed. Acceleration is in fact a vector quantity, and in the one-dimensional case above it is equal to $\ddot{x}\mathbf{i}$.

When the motion is in two or three dimensions, vectors are used explicitly. The acceleration **a** of a particle is a vector equal to the rate of change of the velocity **v** with respect to t. Thus $\mathbf{a} = d\mathbf{v}/dt$. If the particle has *position vector **r**, then $\mathbf{a} = d^2\mathbf{r}/dt^2 = \ddot{\mathbf{r}}$. When Cartesian coordinates are used, $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, and then $\ddot{\mathbf{r}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j} + \ddot{z}\mathbf{k}$.

If a particle is travelling in a circle with constant speed, it still has an acceleration because of the changing direction of the velocity. This acceleration is towards the centre of the circle and has magnitude $\frac{v^2}{r}$ where v is the speed of the particle and r is the radius of the circle.

Acceleration has the dimensions LT^{-2}, and the SI unit of measurement is the metre per second per second, abbreviated to 'm s^{-2}.

acceleration–time graph A graph that shows acceleration plotted against time for a particle moving in a straight line. Let $v(t)$ and $a(t)$ be the velocity and acceleration, respectively, of the particle at time t. The acceleration–time graph is the graph $y = a(t)$, where the t-axis is horizontal and the y-axis is vertical with the positive direction upwards. With the convention that any area below the horizontal axis is negative, the area under the graph between $t=t_1$ and $t=t_2$ is equal to $v(t_2)-v(t_1)$. (Here a common convention has been followed, in which the unit vector **i** in the positive direction along the line has been suppressed. The velocity and acceleration of the particle are in fact vector quantities equal to $v(t)\mathbf{i}$ and $a(t)\mathbf{i}$, respectively.)

acceptance region *See* HYPOTHESIS TESTING.

acceptance sampling A method of quality control where a sample is taken from a batch and a decision whether to accept the batch is made on the basis of the quality of the sample. The most simple method is to have a straight accept/reject criterion, but a more sophisticated approach is to take another sample if the evidence from the existing sample, or a set of samples, is not clearly indicating whether the batch should be accepted or rejected. One of the main advantages of this approach is reducing the cost of taking samples to satisfy quality control criteria.

accuracy A measure of the precision of a numerical quantity, usually given to n *significant figures (where the proportional accuracy is the important aspect) or n *decimal places (where the absolute accuracy is more important).

accurate (correct) to *n* decimal places Rounding a number with the accuracy specified by the number of *decimal places given in the rounded value. So $e=2.71828\ldots = 2.718$ to three decimal places and $=2.72$ to two decimal places. $\sqrt{86.56}=9.30076$ is 9.30 correct to two decimal places. Where a number of quantities are being measured and added or subtracted, using values correct to the same number of decimal places ensures that they have the same degree of accuracy. However, if the units are changed, for example between centimetres and metres, then the accuracy of the measurements will be different if the same number of decimal places is used in the measurements.

accurate (correct) to *n* significant figures Rounding a number with the accuracy specified by the number of *significant figures given in the rounded value. So $e=2.71828\ldots = 2.718$ to four significant figures and $=2.72$ to three significant figures. $e^{-3}=0.049787\ldots = 0.0498$ correct to three significant figures. Rounding to the same number of significant figures ensures all the measurements have about the same proportionate accuracy. If the units are changed, for example between centimetres and metres, then the accuracy of the measurements will not be changed if the same number of significant figures is used in the measurements.

Achilles paradox The paradox which arises from considering how overtaking takes place. Achilles gives a tortoise a head start in a race. To overtake, he must reach the tortoise's initial position, then where the tortoise had moved to, and so on **ad infinitum*. The conclusion that he cannot overtake because he has to cover an infinite sum of well-defined non-zero distances is false, hence the paradox.

acre An imperial unit of surface area, which is 4 840 square yards. This is the area of a furlong (220 yards) by a chain (22 yards) which used to be standard units of measurement in the UK. A square mile contains 640 acres. In the metric system a *hectare is approximately 0.4 acre.

action limits The outer limits set on a *control chart in a production process. If the observed value falls outside these limits, then action will be taken, often resetting the machine. For the means of samples of sizes n in a process with standard deviation σ with target mean μ, the action limits will be set at $\mu \pm 3.09\frac{\sigma}{\sqrt{n}}$.

active constraint An inequality such as $y + 2x \geq 13$ is said to be active at a point on the boundary, i.e. where equality holds, for example (6, 1) and (0, 13).

activity networks (edges as activities) The networks used in *critical path analysis where the edges (arcs) represent activities to be performed. Paths in the network represent the precedence relations between the activities, and *dummy activities are required to link paths where common activities appear, but the paths are at least partly independent. While this is a complication, each activity appears on only one edge and the sequence of activities needed is easier to follow than when the vertices are used to represent the activities. Once the activity network has been constructed from the precedence table, the *critical path algorithm (edges as activities) can be applied.

activity networks (vertices as activities) The networks used in *critical path analysis where the vertices (nodes) represent activities to be performed. The edges (arcs) coming out from any vertex X join X to any vertex Y whose activity cannot start until X has been completed, and the edge is labelled with the time taken for activity X. Note that there will often be more than one such activity, in which case each edge will carry the same time. The construction of an activity network requires a listing of the activities, their duration, and the precedence relations which identify which activities are dependent on the prior completion of other activities. With this structure, *dummy activities are not needed, but the same activity will be represented by more than one edge when more than one other activity depends on its prior completion, and the sequence of a

activities is less easy to follow than the alternative structure *activity networks (edges as activities).

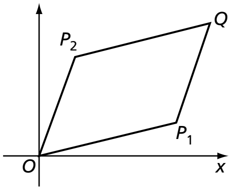
acute angle An angle that is less than a *right angle. An *acute-angled triangle is one all of whose angles are acute.

adders (in combinatorial circuits) The half-adder and full adders are sections of circuits which use a system of logic gates to add binary digits, by using a combination for which the truth table output is identical to the output required by the binary addition.

SEE WEB LINKS

- An article demonstrating how a simple adder works.

addition (of complex numbers) Let the complex numbers z_1 and z_2, where $z_1 = a + bi$ and $z_2 = c + di$, be represented by the points P_1 and P_2 in the *complex plane. Then $z_1 + z_2 = (a + c) + (b + d)i$, and $z_1 + z_2$ is represented in the complex plane by the point Q such that OP_1QP_2 is a parallelogram; that is, such that $\overrightarrow{OQ} = \overrightarrow{OP_1} + \overrightarrow{OP_2}$. Thus, if the complex number z is associated with the *directed line-segment $\overrightarrow{OP}$, where P represents z, then the addition of complex numbers corresponds exactly to the addition of the directed line-segments.

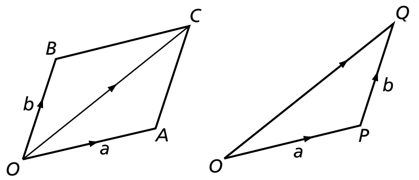


addition (of directed line-segments) *See* ADDITION (of vectors).

addition (of matrices) Let $\mathbf{A}$ and $\mathbf{B}$ be $m \times n$ matrices, with $\mathbf{A} = [a_{ij}]$ and $\mathbf{B} = [b_{ij}]$. The operation of addition is defined by taking the SUM $\mathbf{A} + \mathbf{B}$ to be the $m \times n$ matrix $\mathbf{C}$, where $\mathbf{C} = [c_{ij}]$ and $c_{ij} = a_{ij} + b_{ij}$. The sum $\mathbf{A} + \mathbf{B}$ is not defined if $\mathbf{A}$ and $\mathbf{B}$ are not of the same order. This operation $+$ of addition on the set of all $m \times n$ matrices is *associative and *commutative.

addition (of vectors) Given vectors $\mathbf{a}$ and $\mathbf{b}$, let $\overrightarrow{OA}$ and $\overrightarrow{OB}$ be *directed line-segments that represent $\mathbf{a}$ and $\mathbf{b}$, with the same initial point O. The sum of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is the directed line-segment $\overrightarrow{OC}$, where $OACB$ is a parallelogram, and the SUM $\mathbf{a} + \mathbf{b}$ is defined to be the vector $\mathbf{c}$ represented by $\overrightarrow{OC}$. This is called the parallelogram law. Alternatively, the sum of vectors $\mathbf{a}$ and $\mathbf{b}$ can be defined by representing $\mathbf{a}$ by a directed line-segment $\overrightarrow{OP}$ and $\mathbf{b}$ by $\overrightarrow{PQ}$ where the final point of the first directed line-segment is the initial point of the second. Then $\mathbf{a}+\mathbf{b}$ is the vector represented by $\overrightarrow{OQ}$. This is called the triangle law. Addition of vectors has the following properties, which hold for all **a**, **b** and **c**:

(i) $\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$, the commutative law.
(ii) $\mathbf{a}+(\mathbf{b}+\mathbf{c})=(\mathbf{a}+\mathbf{b})+\mathbf{c}$, the associative law.
(iii) $\mathbf{a}+\mathbf{0}=\mathbf{0}+\mathbf{a}=\mathbf{a}$, where $\mathbf{0}$ is the zero vector.
(iv) $\mathbf{a}+(-\mathbf{a})=(-\mathbf{a})+\mathbf{a}=\mathbf{0}$, where $-\mathbf{a}$ is the negative of **a**.



addition formula *See* COMPOUND ANGLE FORMULA.

addition law If A and B are two events then the addition law states that the $\Pr(A \text{ or } B)=\Pr(A)+\Pr(B)-\Pr(A \text{ and } B)$, or using set notation $P(A\cup B)=P(A)+P(B)-P(A\cap B)$. In the special case where A and B are *mutually exclusive events this reduces to $P(A\cup B)=P(A)+P(B)$.

addition modulo *n* *See* MODULO *N*, ADDITION and MULTIPLICATION.

additive function A function for which $f(x+y)=f(x)+f(y)$. For example, $f(x)=3x$ is an additive function since $f(x+y)=3(x+y)=f(x)+f(y)$ but $g(x)=\sqrt{x}$ is not additive since $\sqrt{x+y}$ is not in general equal to $\sqrt{x}+\sqrt{y}$.

additive group A *group with the operation +, called addition, may be called an additive group. The operation in a group is normally denoted by addition only if it is *commutative, so an additive group is usually *abelian.

additive identity The identity element under an operation of addition, usually denoted by 0, so $a+0=0+a=a$.

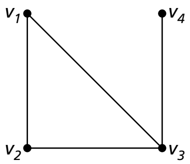
additive inverse *See* INVERSE ELEMENT.

adherent point A point of the *closure of a set.

ad infinitum Repeating infinitely many times.

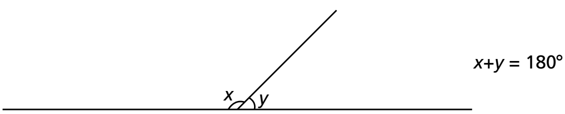
adj Abbreviation for *adjoint.

adjacency matrix For a *simple graph G, with n vertices $v_1, v_2, \ldots, v_n$, the adjacency matrix $\mathbf{A}$ is the $n \times n$ matrix $[a_{ij}]$ with $a_{ij} = 1$, if v_i is joined to v_j, and $a_{ij} = 0$, otherwise. The matrix $\mathbf{A}$ is *symmetric and the diagonal entries are zero. The number of ones in any row (or column) is equal to the *degree of the corresponding vertex. An example of a graph and its adjacency matrix $\mathbf{A}$ is shown below.



$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

adjacent angles A pair of angles on a straight line formed by a line or half-line meeting it. Adjacent angles will add to 180°.



adjacent edges A pair of edges in a graph joined by a common vertex.

adjacent side The side of a right-angled triangle between the right angle and the given angle.

adjacent vertices A pair of vertices in a *graph joined by a common edge.

adjoint The adjoint of a square matrix $\mathbf{A}$, denoted by adj $\mathbf{A}$, is the transpose of the matrix of cofactors of $\mathbf{A}$. For $\mathbf{A} = [a_{ij}]$, let A_{ij} denote the *cofactor of the entry a_{ij}. Then the matrix of cofactors is the matrix $[A_{ij}]$ and adj $\mathbf{A} = [A_{ij}]^T$. For example, a 3×3 matrix $\mathbf{A}$ and its adjoint can be written

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \qquad \operatorname{adj} \mathbf{A} = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}.$$

In the 2×2 case, a matrix $\mathbf{A}$ and its adjoint have the form

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \qquad \operatorname{adj} \mathbf{A} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

The adjoint is important because it can be used to find the *inverse of a matrix. From the properties of cofactors, it can be shown that $\mathbf{A}$ adj $\mathbf{A} = (\det \mathbf{A})\mathbf{I}$. It follows that, when $\det \mathbf{A} \neq 0$, the inverse of $\mathbf{A}$ is $(1/\det \mathbf{A})$ adj $\mathbf{A}$.

adjugate = ADJOINT.

aerodynamic drag A body moving through the air, such as an aeroplane flying in the Earth's atmosphere, experiences a force due to the flow of air over the surface of the body. The force is the sum of the aerodynamic drag, which is tangential to the flight path, and the LIFT, which is normal to the flight path.

affine geometry A geometry in which some properties are preserved by *parallel projection from one plane to another. However, others are not, and in particular *Euclid's third and fourth axioms do not hold.

affine transformation A *transformation which preserves *collinearity and therefore the straightness and parallel nature of lines, and the ratios of distances.

aggregate Census returns are used to construct aggregate statistics concerning a wide range of characteristics such as economic, social, health, education, often grouped by geographical region, gender, age, etc. The name derives from the way they are compiled as counts from a large number of individual census returns. Other examples of aggregate statistics include indices such as the *retail price index.

agree If $f(x)$ and $g(x)$ are defined on a set S, and $f(x) = g(x)$ for all $x \in S$, then f and g agree on the set S.

air resistance The resistance to motion experienced by an object moving through the air caused by the flow of air over the surface of the object. It is a force that affects, for example, the speed of a drop of rain or of a parachutist falling towards the Earth's surface. As well as depending on the nature of the object, air resistance depends on the speed of the object. Possible *mathematical models are to assume that the magnitude of the air resistance is proportional to the speed or to the square of the speed.

Aitken's method (in numerical methods) If an iterative formula $x_{r+1} = f(x_r)$ is to be used to solve an equation, Aitken's method of accelerating convergence uses the initial value and the first two values obtained by the formula to calculate a better approximation than the iterative formula would produce. This can then be used as a new starting point from which to repeat the process until the required accuracy has been reached. While this is computationally intensive, it is the sort of process which spreadsheets handle very easily.

If x_0, x_1, x_2 are the initial value and the first two iterations and $\Delta x_r = x_{r+1} - x_r$, $\Delta^2 x_r = \Delta x_{r+1} - \Delta x_r$ are the forward differences then

$x_4 = x_3 - \frac{(\Delta x_2)^2}{\Delta^2 x_1}$. More generally this will be expressed as

$$x_{r+1} = x_r - \frac{(\Delta x_{r-1})^2}{\Delta^2 x_{r-2}}.$$

aleph Any infinite *cardinal number, usually denoted by the Hebrew letter ℵ. *See also* TRANSFINITE NUMBER.

aleph-null The smallest infinite cardinal number. The cardinality of any set which can be put in one-to-one correspondence with the set of natural numbers. Such sets are said to be *countable or *denumerable. One of the apparent paradoxes in number theory is that the set of rational numbers between 0 and 1, the set of all rational numbers, and the set of natural numbers all have the same cardinality. The symbol $\aleph_0$ is used.

algebra The area of mathematics related to the general properties of arithmetic. Relationships can be summarized by using variables, usually denoted by letters $x, y, n, \ldots$ to stand for unknown quantities, whose value(s) may be determined by solving the resulting equations. *See also* ABSTRACT ALGEBRA and LINEAR ALGEBRA.

Algebra, Fundamental Theorem of *See* FUNDAMENTAL THEOREM OF ALGEBRA.

algebra of sets The set of all subsets of a *universal set E is closed under the binary operations ∪ (*union) and ∩ (*intersection) and the unary operation ′ (*complementation). The following are some of the properties, or laws, that hold for subsets A, B and C of E:

(i) $A \cup (B \cup C) = (A \cup B) \cup C$ and $A \cap (B \cap C) = (A \cap B) \cap C$, the associative properties.
(ii) $A \cup B = B \cup A$ and $A \cap B = B \cap A$, the commutative properties.
(iii) $A \cup \emptyset = A$ and $A \cap \emptyset = \emptyset$, where Ø is the *empty set.
(iv) $A \cup E = E$ and $A \cap E = A$.
(v) $A \cup A = A$ and $A \cap A = A$.
(vi) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, the distributive properties.
(vii) $A \cup A' = E$ and $A \cap A' = \emptyset$.
(viii) $E' = \emptyset$ and $\emptyset' = E$.
(ix) $(A')' = A$.
(x) $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$, De Morgan's laws.

The application of these laws to subsets of E is known as the algebra of sets. Despite some similarities with the algebra of numbers, there are important and striking differences.