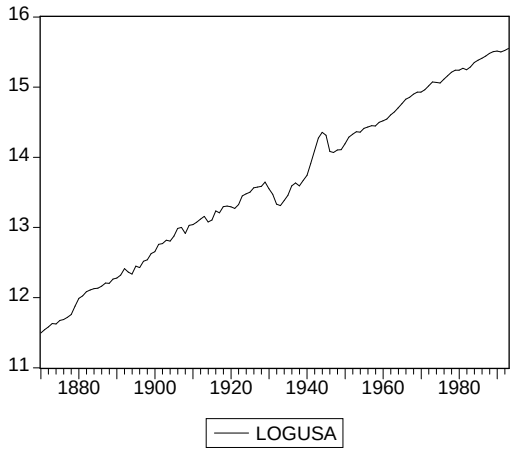
7.20.a. Figure S 7.24 shows the plot of the logarithm of US GNP (in parts (a)-(e) we further denote this series for convenience by y_t, and by Y in figures and tables). As the series has a clear upward trend direction we include a constant and a deterministic trend in the ADF test equation. The lag order p for the test equation can be selected, for instance, by starting with a large order (we choose $p = 5$) and then reducing the order by sequentially testing for the significance of the coefficient ρ_{p-1} of the largest included lag Δy_{t-p+1}. This leads to a test equation with $p = 2$, that is, with Δy_{t-1} as largest included lag, so that the ADF test equation becomes

$$\Delta y_t = \alpha + \beta t + \rho y_{t-1} + \rho_1 \Delta y_{t-1} + \varepsilon_t.$$

Figure S 7.24 shows the output for this test regression. The ADF t-test has value $t = -4.12$ which is smaller than the 5% critical value of -3.45. Therefore the ADF t-test rejects the null hypothesis of a stochastic trend (at the 5% significance level, and also at the 1% significance level). The ADF F-test for the two restrictions that $\rho = 0$ and $\beta = 0$ gives $F = 8.59$ which is larger than the 5% critical value of 6.49 in Table 7.6, so that the null hypothesis of a stochastic trend is also rejected by the ADF F-test. Both tests indicate that the trend is deterministic.



Augmented Dickey-Fuller Unit Root Test on Y

ADF Test Statistic	-4.117386	1% Critical Value*	-4.0355
		5% Critical Value	-3.4469
		10% Critical Value	-3.1482

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(Y)
Method: Least Squares
Date: 08/07/02 Time: 13:23
Sample(adjusted): 1872 1993
Included observations: 122 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Y(-1)	-0.184605	0.044836	-4.117386	0.0001
D(Y(-1))	0.350010	0.085987	4.070501	0.0001
C	2.165163	0.519110	4.170918	0.0001
@TREND(1870)	0.005978	0.001473	4.058175	0.0001

R-squared	0.186811	Mean dependent var	0.032856
Adjusted R-squared	0.166137	S.D. dependent var	0.056714
S.E. of regression	0.051789	Akaike info criterion	-3.051038
Sum squared resid	0.316489	Schwarz criterion	-2.959103
Log likelihood	190.1133	F-statistic	9.035895
Durbin-Watson stat	2.068002	Prob(F-statistic)	0.000020

Figure S 7.24: Part (a) : Plot of time series and result of ADF test

b. For all three subperiods we use the same ADF test equation as in part (a). The resulting values for the ADF t- and F-tests are shown in Table S 7.24 (together with the 5% critical values of Table 7.6 for $n = 50$). This shows that the null hypotheses of a stochastic trend can not be rejected for any of the three subperiods. On the other hand, the result in part (a) indicates that the information over the full sample period suffices to reject this hypothesis.

Period	n	F-test	t-test
1870-1929	60	4.12	-2.81
1900-1949	50	4.59	-3.03
1950-1993	44	5.22	-2.59
5% crit. val.		6.73	-3.49

Table S 7.24: Part (b) : ADF test outcomes for three subperiods

c. The estimation results of the two models are shown in Table S 7.25. The average yearly growth rate in GNP is estimated as 3.3% in the deterministic trend model and as 3.5% in the stochastic trend model.

Dependent Variable: Y
Method: Least Squares
Date: 08/07/02 Time: 14:02
Sample: 1950 1989
Included observations: 40

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	11.58087	0.051383	225.3821	0.0000
@TREND	0.033240	0.000513	64.79778	0.0000

R-squared	0.991031	Mean dependent var	14.88821
Adjusted R-squared	0.990795	S.D. dependent var	0.390340
S.E. of regression	0.037451	Akaike info criterion	-3.682879
Sum squared resid	0.053297	Schwarz criterion	-3.598435
Log likelihood	75.65759	F-statistic	4198.753
Durbin-Watson stat	0.431224	Prob(F-statistic)	0.000000

Dependent Variable: D(Y)
Method: Least Squares
Date: 12/13/02 Time: 14:24
Sample: 1950 1989
Included observations: 40

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.034912	0.004034	8.655360	0.0000

R-squared	0.000000	Mean dependent var	0.034912
Adjusted R-squared	0.000000	S.D. dependent var	0.025510
S.E. of regression	0.025510	Akaike info criterion	-4.474771
Sum squared resid	0.025381	Schwarz criterion	-4.432549
Log likelihood	90.49542	Durbin-Watson stat	1.499485

Table S 7.25: Part (c) : Deterministic trend model (left panel) and stochastic trend model (right panel)

d. The forecast performance of the two models is evaluated in Table S 7.26. The stochastic trend model performs better, as it both has a smaller RMSE (of 7.5%) and MAE (of 7.0%) than the deterministic trend model (with RMSE of 10.0% and MAE of 9.7%).

Forecast Evaluation

Forecast: FC720C1
Actual: Y
Forecast sample: 1990 1993
Included observations: 4

Root Mean Squared Error	0.100212
Mean Absolute Error	0.097134
Mean Absolute Percentage Error	0.625682

Forecast Evaluation

Forecast: FC720C2
Actual: Y
Forecast sample: 1990 1993
Included observations: 4

Root Mean Squared Error	0.075159
Mean Absolute Error	0.070392
Mean Absolute Percentage Error	0.453384

Table S 7.26: Part (d) : Forecast performance of deterministic model (left panel) and of stochastic model (right panel)

e. We discuss one of the possible improvements of the deterministic trend model of part (c). The residuals of this model in Table S 7.25 show strong positive serial correlation (the Durbin-Watson statistic is $DW = 0.43$, corresponding to a first order autocorrelation coefficient of around 0.8). Therefore we include the lagged term y_{t-1} as additional regressor, so that the model equation becomes

$$y_t = \alpha + \beta t + \rho y_{t-1} + \varepsilon_t.$$

The estimated model is shown in Table S 7.27, together with an evaluation of the forecast performance of this model. This model has RMSE of 7.4% and MAE of 6.9%, so that this model performs slightly better than the stochastic trend model of part (c). The estimated value of ρ is 0.74, meaning that the series shows prolonged deviations from the postulated linear trend.

f. To select the appropriate form of the cointegration test we first consider the plots of the series in Figure S 7.25. This shows that, over both periods 1870-1993 and 1950-1993, the four (logarithmic) GNP series display a clear upward trend. Therefore we apply the Johansen trace test for cointegration with intercept and deterministic trend included in the cointegration relation and with a constant (but no deterministic trend) included in the VECM. Further we include four lags (the four 4×1 vectors $\Delta Y_{t-1}, \cdots, \Delta Y_{t-4}$) in the test equation. The results are shown in Table S 7.28. First we consider the full sample period 1870 - 1993. The trace test for the null hypothesis that $r = 0$ (no cointegration) has value $LR = 50.40$ which is smaller than the 5% critical value of 62.99 (see Table S 7.28, and also Table 7.17 for $m = 4$ variables and $r = 0$ so that $(m - r) = 4$). This means that the null hypothesis that

Dependent Variable: Y
Method: Least Squares
Date: 12/13/02 Time: 14:36
Sample: 1950 1989
Included observations: 40

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3.067503	1.088896	2.817077	0.0077
@TREND	0.008276	0.003207	2.580361	0.0140
Y(-1)	0.740388	0.094658	7.821723	0.0000

R-squared	0.996620	Mean dependent var	14.88821
Adjusted R-squared	0.996437	S.D. dependent var	0.390340
S.E. of regression	0.023299	Akaike info criterion	-4.608757
Sum squared resid	0.020086	Schwarz criterion	-4.482091
Log likelihood	95.17515	F-statistic	5454.679
Durbin-Watson stat	1.455542	Prob(F-statistic)	0.000000

Forecast Evaluation

Forecast: FC720E1
Actual: Y
Forecast sample: 1990 1993
Included observations: 4

Root Mean Squared Error	0.073833
Mean Absolute Error	0.069355
Mean Absolute Percentage Error	0.446717

Table S 7.27: Part (e) : Alternative model with forecast evaluation

$r = 0$ can not be rejected, that is, the four series are not cointegrated over this long period of time.

The results for the subperiod 1950 - 1993 are as follows. The trace test for $r = 0$ has value $LR = 76.33 > 62.99$, so that this hypothesis is rejected. Therefore the series are cointegrated, and it remains to determine the number of cointegration relations. The trace test for $r = 1$ gives $LR = 44.37 > 42.44$, so that this hypothesis is rejected, whereas the test for $r = 2$ gives $LR = 23.40 < 25.32$ so that this hypothesis is not rejected (all at the 5% significance level, see also Table 7.17 for the critical values for $(m-r) = 3$ and $(m-r) = 2$). We conclude that there are two cointegration relations, and hence two stochastic trends that drive the four considered series. The graph in Figure S 7.25 indicates that Japan experienced systematically higher growth rates than the other three countries over the period 1950 - 1993, so that possibly one stochastic trend drives the GNP of Japan and the other one drives the GNP of the US, Germany and the UK jointly.

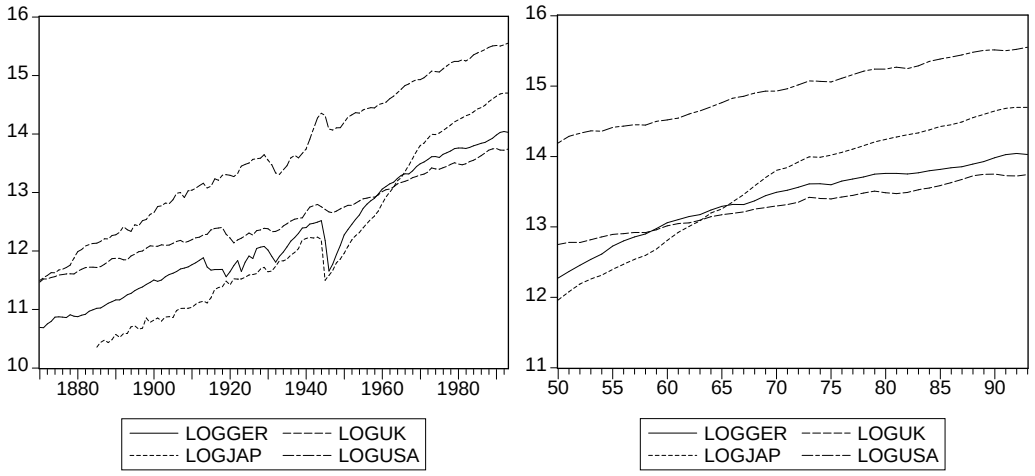


Figure S 7.25: Part (f) : Plots of the four GNP series (in logarithms)

Date: 12/13/02 Time: 14:58
Sample: 1870 1993
Included observations: 104
Test assumption: Linear deterministic trend in the data
Series: LOGGER LOGJAP LOGUK LOGUSA
Lags interval: 1 to 4

Eigenvalue	Likelihood Ratio	5 Percent Critical Value	1 Percent Critical Value	Hypothesized No. of CE(s)
0.198365	50.40244	62.99	70.05	None
0.133360	27.40780	42.44	48.45	At most 1
0.081420	12.52207	25.32	30.45	At most 2
0.034857	3.689791	12.25	16.26	At most 3

*(**) denotes rejection of the hypothesis at 5%(1%) significance level
L.R. rejects any cointegration at 5% significance level

Date: 12/13/02 Time: 14:59
Sample: 1950 1993
Included observations: 44
Test assumption: Linear deterministic trend in the data
Series: LOGGER LOGJAP LOGUK LOGUSA
Lags interval: 1 to 1

Eigenvalue	Likelihood Ratio	5 Percent Critical Value	1 Percent Critical Value	Hypothesized No. of CE(s)
0.516364	76.33014	62.99	70.05	None **
0.379070	44.36752	42.44	48.45	At most 1 *
0.290273	23.39992	25.32	30.45	At most 2
0.172164	8.313394	12.25	16.26	At most 3

*(**) denotes rejection of the hypothesis at 5%(1%) significance level
L.R. test indicates 2 cointegrating equation(s) at 5% significance level

Table S 7.28: Part (f) : Johansen cointegration test for full sample (top panel) and for the subperiod 1950-1993 (bottom panel)