4.14.a. The regression results for the four models are in Table S 4.15. Note that, under the restriction $\beta_4 + \beta_5 = 0$, the model becomes $y = \beta_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4(x_4 - x_5) + \varepsilon$, with corresponding regressors a constant, x_2, x_3 and $(x_4 - x_5)$. The resulting sum of squared residuals (SSR) and the ML estimates $s^2_{\text{ML}} = \frac{\text{SSR}}{n} = \frac{n-k}{n} s^2$ are summarized in Table S 4.16.

Dependent Variable: Y
Method: Least Squares
Date: 02/06/01 Time: 09:53
Sample: 1 474
Included observations: 474

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.079647	0.314798	6.606288	0.0000
X2	0.023268	0.003870	6.013129	0.0000
X3	0.821799	0.036031	22.80783	0.0000
X4	0.048156	0.019910	2.418627	0.0160
X5	-0.042369	0.020342	-2.082842	0.0378

R-squared	0.804117	Mean dependent var	10.35679
Adjusted R-squared	0.802446	S.D. dependent var	0.397334
S.E. of regression	0.176603	Akaike info criterion	-0.619330
Sum squared resid	14.62750	Schwarz criterion	-0.575435
Log likelihood	151.7812	F-statistic	481.3211
Durbin-Watson stat	1.813375	Prob(F-statistic)	0.000000

Dependent Variable: Y
Method: Least Squares
Date: 02/06/01 Time: 09:53
Sample: 1 474
Included observations: 474

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.932281	0.307832	6.277073	0.0000
X2	0.023378	0.003883	6.020717	0.0000
X3	0.836406	0.035468	23.58226	0.0000
X4	0.039600	0.019551	2.025468	0.0434

R-squared	0.802305	Mean dependent var	10.35679
Adjusted R-squared	0.801043	S.D. dependent var	0.397334
S.E. of regression	0.177229	Akaike info criterion	-0.614342
Sum squared resid	14.76280	Schwarz criterion	-0.579226
Log likelihood	149.5990	F-statistic	635.7996
Durbin-Watson stat	1.815691	Prob(F-statistic)	0.000000

Dependent Variable: Y
Method: Least Squares
Date: 02/06/01 Time: 09:53
Sample: 1 474
Included observations: 474

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.646916	0.274598	5.997550	0.0000
X2	0.023122	0.003894	5.938464	0.0000
X3	0.868505	0.031835	27.28174	0.0000

R-squared	0.800579	Mean dependent var	10.35679
Adjusted R-squared	0.799733	S.D. dependent var	0.397334
S.E. of regression	0.177812	Akaike info criterion	-0.609870
Sum squared resid	14.89166	Schwarz criterion	-0.583534
Log likelihood	147.5393	F-statistic	945.4206
Durbin-Watson stat	1.821092	Prob(F-statistic)	0.000000

Dependent Variable: Y
Method: Least Squares
Date: 02/06/01 Time: 09:53
Sample: 1 474
Included observations: 474

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.065371	0.308208	6.701228	0.0000
X2	0.023239	0.003863	6.014920	0.0000
X3	0.823543	0.035176	23.41193	0.0000
X4-X5	0.045340	0.015612	2.904278	0.0039

R-squared	0.804095	Mean dependent var	10.35679
Adjusted R-squared	0.802845	S.D. dependent var	0.397334
S.E. of regression	0.176425	Akaike info criterion	-0.623438
Sum squared resid	14.62912	Schwarz criterion	-0.588323
Log likelihood	151.7549	F-statistic	643.0411
Durbin-Watson stat	1.813057	Prob(F-statistic)	0.000000

Table S 4.15: Part (a) : Regression results of the four models

Model	SSR	s^2_{ML}	log-likelihood
unrestricted	14.628	0.0309	151.78
$\beta_5 = 0$	14.763	0.0311	149.60
$\beta_4 = \beta_5 = 0$	14.892	0.0314	147.54
$\beta_4 - \beta_5 = 0$	14.629	0.0309	151.75

Table S 4.16: Parts (a) and (b) : SSR , s^2_{ML} and log-likelihood of the four regression models

b. The log-likelihood is calculated according to (4.30), so that

$$l(\beta, \sigma^2) = -\frac{n}{2}\log(2\pi) - \frac{n}{2}\log(\sigma^2) - \frac{1}{2\sigma^2}(y - X\beta)'(y - X\beta).$$

Because $(y - Xb_{ML})'(y - Xb_{ML}) = SSR = ns^2_{ML}$, it follows that the log-likelihood evaluated at the ML estimates is equal to

$$l(b_{ML}, s^2_{ML}) = -\frac{n}{2}\log(2\pi) - \frac{n}{2}\log(s^2_{ML}) - \frac{n}{2} = -672.58 - 237\log(s^2_{ML}).$$

The resulting log-likelihood values are reported in the last column of Table S 4.16. The results are equal to the values reported in the EViews panels in Table S 4.15.

The log-likelihood values can be used to perform the LR-tests. For $H_0 : \beta_5 = 0$ this gives

$$LR = 2(151.78 - 149.60) = 4.36, \qquad P = 0.037.$$

The P-value is obtained from the $\chi^2(1)$-distribution (in EViews: `1 - @cchisq(4.36,1)`). For $H_0 : \beta_4 = \beta_5 = 0$ the result is

$$LR = 2(151.78 - 147.54) = 8.48, \qquad P = 0.014.$$

As the null hypothesis concerns two restrictions, the P-value is now obtained from the $\chi^2(2)$ distribution (in EViews: `1 - @cchisq(8.48,2)`). Finally, for $H_0 : \beta_4 + \beta_5 = 0$ the result is

$$LR = 2(151.78 - 151.75) = 0.05, \qquad P = 0.819.$$

Here the P-value is obtained from the $\chi^2(1)$ distribution.

If we use a significance level of 5%, then the restricted models (i) and (ii) are rejected but the restriction (iii) that $\beta_4 + \beta_5 = 0$ is not rejected.

c. The results of the Wald tests are reported in Table S 4.17. These are the outcomes of the EViews Wald tests based on the OLS estimate s^2 of the variance σ^2 instead of the ML estimate $s^2_{ML} = \frac{n-k}{n} s^2$ (as $n = 474$ and $k \leq 5$ in all four models, the difference between the two estimates of σ^2 is very small).

The conclusions are the same as in part (b), that is, the model restrictions (i) and (ii) are rejected and the model (iii) is not rejected.

Wald Test: Equation: EQUNRES			
Null Hypothesis:	C(5)=0		
F-statistic	4.338232	Probability	0.037807
Chi-square	4.338232	Probability	0.037266

Wald Test: Equation: EQUNRES			
Null Hypothesis:	C(4)=0 C(5)=0		
F-statistic	4.234946	Probability	0.015038
Chi-square	8.469892	Probability	0.014481

Wald Test: Equation: EQUNRES			
Null Hypothesis:	C(4)+C(5)=0		
F-statistic	0.052075	Probability	0.819590
Chi-square	0.052075	Probability	0.819491

Table S 4.17: Part (c) : Wald tests for the three restricted models

d. The auxiliary regressions required for the LM test are in Table S 4.18. Here 'RESR1' denotes the residuals of the restricted model (i) with $\beta_5 = 0$, and 'RESR2' and RESR3' are defined in a similar way for models (ii) and (iii) respectively. The corresponding LM test statistics are reported in Table S 4.19. The conclusions are the same as in parts (b) and (c), that is, models (i) and (ii) are rejected and model (iii) is not rejected.

e. For hypothesis (i) there holds (after rounding, and using the χ^2 value for the Wald test) $P_{LM} = P_{LR} = P_W = 0.037$, for hypothesis(ii) $P_{LM} = 0.015 > P_{LR} = P_W = 0.014$, and for hypothesis (iii) $P_{LM} = P_{LR} = P_W = 0.819$. The F-tests in Section 3.4.2 provided P-values $P_F = 0.038$ for model (i), $P_F = 0.016$ for model (ii) and $P_F = 0.821$ for model (iii). The outcomes of all four tests (LM, LR, W and F) are very much the same and hence they will lead to the same conclusions, independent of the chosen significance level. For instance, at the 5% significance level model (iii) is not rejected but models (i) and (ii) are rejected.

Dependent Variable: RESR1
Method: Least Squares
Date: 02/06/01 Time: 11:03
Sample: 1 474
Included observations: 474

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.147366	0.314798	0.468128	0.6399
X2	-0.000110	0.003870	-0.028372	0.9774
X3	-0.014607	0.036031	-0.405409	0.6854
X4	0.008556	0.019910	0.429706	0.6676
X5	-0.042369	0.020342	-2.082842	0.0378

R-squared	0.009165	Mean dependent var	1.36E-15
Adjusted R-squared	0.000715	S.D. dependent var	0.176666
S.E. of regression	0.176603	Akaike info criterion	-0.619330
Sum squared resid	14.62750	Schwarz criterion	-0.575435
Log likelihood	151.7812	F-statistic	1.084558
Durbin-Watson stat	1.813375	Prob(F-statistic)	0.363497

Dependent Variable: RESR2
Method: Least Squares
Date: 02/06/01 Time: 11:03
Sample: 1 474
Included observations: 474

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.432731	0.314798	1.374631	0.1699
X2	0.000146	0.003870	0.037735	0.9699
X3	-0.046706	0.036031	-1.296249	0.1955
X4	0.048156	0.019910	2.418627	0.0160
X5	-0.042369	0.020342	-2.082842	0.0378

R-squared	0.017739	Mean dependent var	1.05E-15
Adjusted R-squared	0.009362	S.D. dependent var	0.177436
S.E. of regression	0.176603	Akaike info criterion	-0.619330
Sum squared resid	14.62750	Schwarz criterion	-0.575435
Log likelihood	151.7812	F-statistic	2.117473
Durbin-Watson stat	1.813375	Prob(F-statistic)	0.077595

Dependent Variable: RESR3
Method: Least Squares
Date: 02/06/01 Time: 11:03
Sample: 1 474
Included observations: 474

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.014276	0.314798	0.045351	0.9638
X2	2.98E-05	0.003870	0.007691	0.9939
X3	-0.001744	0.036031	-0.048399	0.9614
X4	0.002815	0.019910	0.141402	0.8876
X5	0.002972	0.020342	0.146085	0.8839

R-squared	0.000111	Mean dependent var	7.92E-16
Adjusted R-squared	-0.008417	S.D. dependent var	0.175865
S.E. of regression	0.176603	Akaike info criterion	-0.619330
Sum squared resid	14.62750	Schwarz criterion	-0.575435
Log likelihood	151.7812	F-statistic	0.013019
Durbin-Watson stat	1.813375	Prob(F-statistic)	0.999665

Table S 4.18: Part (d) : Auxiliary regressions for LM test

Model	R^2	$LM = nR^2$	P-value
model (i) ($\beta_5 = 0$)	0.00917	4.344	0.037
model (ii) ($\beta_4 = \beta_5 = 0$)	0.01774	8.408	0.015
model (iii) ($\beta_4 + \beta_5 = 0$)	0.00011	0.053	0.819

Table S 4.19: Part (d) : R^2 of auxiliary regression, LM test statistic and P-value